
Entsprechend dem Ingress zum Landwirtschaftsgesetz besteht die grundlegende Zielsetzung unserer Agrarpolitik darin, «einen gesunden Bauernstand und im Dienste der Landesversorgung eine leistungsfähige Landwirtschaft zu erhalten und sie unter Wahrung der Interessen der schweizerischen Gesamtwirtschaft zu fördern.» Angesichts der aktuellen Schwierigkeiten der schweizerischen Agrarpolitik und angesichts konstruktiver ausländischer Vorbilder ist es daher angebracht, die Wege zur optimalen Verwirklichung dieser Zielsetzung erneut unvorhergesehen durchzudenken.


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Theory of Inflation and Quantity-Theory

Dr. Karl Brunner, Los Angeles *

A scientist paging the literature of monetary theory will probably experience a growing sense of frustration. For every plausible proposition he will find a plausible counterproposition. One may wonder whether any "knowledge" can be achieved in the face of such diversity and volatility of opinions. Apparently, either we seem to acknowledge intellectual fashions as arbiter of essentially scientific issues or drift towards solipsism and "knowledge by revelation and through personal grace". As a result, problems of major importance remain unsolved. Opinions and plausible arguments are offered in abundance on whether or not the money-supply is a "residual" quantity in the economic process, on the existence and nature of an association between general price movements and the money-supply, or on the crucial features of the inflationary mechanism.

In recent years two books have been published which collide in all major respects, particularly in the answers supplied to the problems indicated. On one side we have Bent Hansen with a study of the theory of inflation which completely neglects monetary variables. Friedman and his workshop, on the other side, assign to the money-supply a "vital role" in the movements of the economy as a whole 1.

I. Economic Analysis and Knowledge

More fundamental than the divergent theses on the significance of monetary variables is the conflict in the general nature of the respective supporting structures. The book edited by Friedman substantially reflects modern scientific methodology 2, whereas Hansen engages essentially in a form of linguistic exercise. The latter's general procedure

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* The author is associate professor of economics at the University of California at Los Angeles. This paper was written in the academic year 1957/58 with the benefit of a Ford Foundation Faculty Research Fellowship.


2 The book consists of 4 contributions plus an introduction by Friedman. The contributions by E.M. Lerner, "Inflation in the Confederacy 1861–1865", and by J.J. Klein, "German Money and Prices 1932–1944" are mainly descriptive in the sense of a systematic discourse of materials relevant for judging some general proposition, without explicitly stating this proposition. The contributions by P. Cagan, "The Monetary Dynamics of Hyperinflation", and by R.T. Selden, "Monetary Velocity in the United States", are of a significantly higher order of empirical-analytical work. Observations are assembled to test hypotheses and to judge the validity of specific formulations. But I hasten to add that all four contributions reveal an excellent craftsmanship, are of a high order of quality, and with a thoughtful and very careful argument on methods applied and data use. The only qualification to my statement in the text on the positive scientific character of the book relates strangely to some parts of Friedman's introduction, particularly on p.3 which extoll the virtues of a "flexible and sensitive tool" for interpretation (consider also the last sentence on p.3).
reveals the schism between the notion of "theory" generally accepted in science on the one side and the notion often prevalent in economics, on the other. A "theory" of "hypothesis" in a scientifically relevant sense is a set of general statements with a specified logical structure and constructed in such a way that explicit rules of identification are formulated as an integral part of the hypothesis. The possibility of falsifying this theory by confrontation with observations is a basic requirement of scientific methodology.

Another usage of the term has spread in economics. It may refer to a purely formal structure with no concern for empirical relevance. The choice of names for the variables in the scheme often falsely suggests an empirical content where none exists. Or it indicates a "framework of analysis" in the following sense: Variables are enumerated and the existence of a broad class of relations postulated. But the characteristic aspect of the "framework" notion is actually the manner in which the formal apparatus is used, namely by ex post adaptations to whatever has been observed. The a priori class of relations has usually been conceived broad enough, so that any observations of the relevant variables can be fitted ex post into the structure.

The unfortunate consequences for the progress of our knowledge of such a linguistic conception of "theory" and "theoretical analysis" can be clearly seen in Hansen's book. The quantity-theoretical approach, for instance, is rejected as unsuitable for an analysis of inflation. This rejection is apparently based on a statement which establishes that an excess demand for commodities and factors (a "monetary pressure of inflation") does not imply an excess supply of money – only of money and claims. This assertion is derived from a definitional relation similar to the "Walras identity". This definition can be supplemented by a set of relations in such a manner that the "monetary pressure of inflation" and the excess supply of money are unambiguously associated. No information about the "suitability" of such an approach can be squeezed out of a manipulation of definitions. Whether a quantity-theoretical approach has a valid content can only be decided by an empirical test. But Hansen is not concerned with such tests. Their place is taken by a priori discussions of the reasonableness of assumptions. As we can easily conceive plausible arguments in support of or against any set of assumptions, the procedure only creates the impression of a thoughtful realism which is misleading.

Another example is Hansen's criticism of the Keynesian model for its neglect of the factor market. This neglect is considered a sufficient reason to reject this model. Again, rejection is not based on a confrontation of two alternative structures with the relevant set of observations – it suffices to state that something which conceivably may be quite irrelevant has not been considered explicitly. Hansen's critique and rejection of the

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1 This justification is on p. 8. There is another, rather ludicrous justification on p. 1. The argument on p. 192–196 concerning "Walras Law" may be construed as another justification, but still without empirical relevance.

2 Op. cit. p. 228 "... we will discuss these assumptions more thoroughly and try to discover whether it is possible beforehand to say that they may be taken as a reasonable picture of the world we live in." Also, "The only way to settle this question definitely is by testing the hypothesis empirically, but as long as this has not been done, we have to try to discuss it more generally."

3 Op. cit. p. 19 "... the disregard of the possibility of excess demand in the factor markets in their analysis of inflation is quite a serious deficiency compared with the Wicksellian analysis." A hypothesis may completely disregard the factor-market, as e.g. Cagan's hypothesis, and still explain the price-behavior under hyperinflation satisfactorily. Suppose our problem is to explain the behavior of x, and we formulate some hypothesis \( f(x, a) = 0 \) with "f" a function and "a" a parameter. This hypothesis completely neglects a set of circumstances expressed by a parameter b, together with another function g. Thus, one could suggest among many alternative hypotheses \( g(x, a, b) = 0 \). The possibility of formulating such alternatives, contrary to Hansen's apparent
investment-savings relation proceeds in the same fashion. He shows that the investment-savings gap is equal to the inflationary gap plus some additional terms. And then he concludes "...the difference between planned savings and planned investment ceases to be of immediate independent interest in the problem of inflation." But such a conclusion simply does not follow from the definitional relation between the investment-savings gap and the inflationary gap. It presupposes in addition a validated scientific hypothesis (i.e. an economic law) about the mechanism generating inflationary gaps. But Hansen was neither concerned with empirical evidence nor with the meaningfulness of his structures. In this manner Hansen substitutes a game with the following rules for the construction of hypotheses and their confrontation with appropriate observations: (a) a statement asserting some general connection of events is "deficient" if you can show that within some framework this connection possibly does not hold, (b) a framework is "deficient" if you can show that some "event" has not been incorporated into the framework. Even by staying within the confines of this game one would naturally conclude that Hansen's analysis is deficient. The uselessness of the procedure should be apparent if we consider that all scientific achievements in any branch of knowledge would be "deficient" according to the rules of this game.

II. Contributions to "Positive Economics"

The spirit of the Chicago workshop on monetary theory is radically different. Here we find a concentrated and conscious effort to enlarge our knowledge. General ideas are transformed into carefully constructed hypotheses, available data assembled and evaluated, and ultimately the hypotheses weighted and judged in terms of these observable data. The positive contributions, and some further problems raised by the investigations presented in the book will be discussed in three sections.

1. Theory of Demand

The contributions by Cagan and Selden are concerned with the formulation of useful demand hypotheses for money. Both use general demand theory as a starting point for their discussion. This leads Cagan to a hypothesis explaining the behavior of real money-belief, provides no basis to reject the first hypothesis. There is only one way for adequate judgment: confront the implications of each one with the observations on \( x \).
balances under hyper-inflation and Selden to a general procedure accounting for the behavior of income velocity.

a) The Case of Hyper-Inflation: General demand theory suggests to Cagan that the real balances \( \frac{M}{P} \) (\( M \) = money supply, \( P \) = price level) are a function of real wealth, real income, and the returns of alternative forms of holding wealth which determine the cost of holding money. A casual inspection of data indicates that the very substantial variations in real balances during hyper-inflation could barely be explained by the relatively negligible variations in real wealth and real income. Among the costs of holding money appears one element with sizable changes: the cost arising from holding money in lieu of commodities and claims to commodities, measured by the depreciation in the value of money. Thus, an idea emerges connecting the demand for real balances under hyper-inflation with the rate of change in prices or expectations of this rate. This idea leads Cagan to a hypothesis explaining the demand for real balances. It can be restated in the following manner: if there exists a hyperinflation, then \( q \). And \( q \) is a symbol for the conjunction of the following statements:

\[
\begin{align*}
a) \quad & \frac{d}{dt} \frac{M}{P} = \pi \left( \log \frac{M^d}{P} - \log \frac{M}{P} \right), \\
b) \quad & \pi \text{ is of large order relative to } \frac{d}{dt} \frac{M}{P}, \\
c) \quad & \log \frac{M^d}{P} = -\alpha E - \gamma + \varepsilon, \\
d) \quad & \varepsilon_t \text{ is normally and independently distributed for every } t, \\
e) \quad & \frac{dE}{dt} = \beta (C - E), \\
f) \quad & C = \frac{d}{dt} \log P, \\
g) \quad & \alpha \text{ and } \beta > 0,
\end{align*}
\]

where \( M \) = money supply, \( P \) = price level, \( E \) = expected rate of change in prices, \( C \) = actual rate of change in prices, \( M^d \) = desired money-balances. Identification rules for \( M, M^d, P, \) and \( C \) have been carefully specified by Cagan and belong to the hypothesis. \( M^d, E, \) and \( \varepsilon \) have no observable counterpart; they belong to the category of "theoretical terms"\(^1\) and thus are defined implicitly by the system in terms of the other variables\(^2\).

Statements "a" and "b" imply that \( M^d \) is approximately equal to \( M \). This permits Cagan to substitute \( M \), an observable magnitude for the theoretical magnitude \( M^d \).

\(^1\) Consider the clarification of the nature of "theoretical terms" by Braithwait in *Scientific Explanation*, Cambridge 1955, chapter II, p. 50.

\(^2\) The meaning of eq. "a", "c", and "e" should be clearly noticed: eq. "a" asserts that any discrepancy between actual and desired real balances is eliminated by further price changes which move actual balances in the desired direction. Equation "c" formulates that the elasticity of desired real balances with respect to the expected rate of price change is (negatively) proportional to the expected rate of price change. The higher the expected rate of increase in \( P \), the smaller the desired real balances. And eq. "e" states that expectations concerning future price changes are adjusted according to the current discrepancy between expected and actual rates of price change.
Also, statements "c" and "e" imply that

$$\log \left( \frac{M}{P} \right)_t = -\alpha \beta e^{-\beta t} \int_{-T}^{t} C(x) e^{\beta x} dx - \gamma + \epsilon_t,$$

provided additional specifications are made, so that the constant of integration vanishes. With the aid of an additional rule the integral in the last expression can be approximated by a finite sum. Thus, ultimately we obtain the following implication of the hypothesis

$$\log \left( \frac{M}{P} \right)_t = -\alpha \frac{1 - e^{-\beta t}}{e^{\beta t}} \sum_{x=-T}^{t} C(x) e^{\beta x} - \gamma + \epsilon_t,$$

which contains directly observable magnitudes (except for $\epsilon$). Statement $e$ implies that the expected rate of price change $E$ is an exponentially weighted average of past observed rates of change of prices. Consequently, the demand for real balances is made ultimately dependent on a range of past price-experiences. Cagan is at pains to demonstrate that a simple hypothesis which declares real balances to be a function of the current rate of price change implies a self-generating process of hyper-inflation. His empirical results also indicate that the introduction of lags through the relation between $E$ and past $C$'s by means of statement $e$ leads to a significantly better explanation of observable price-behavior than a no-lag hypothesis.

Data from 7 different hyper-inflations were assembled and subjected to a careful analysis to uncover inherent biases and to evaluate their reliability. Estimates of $\alpha$ and $\beta$ were derived by least-squares procedure. These estimates exhibit considerable variation as between the seven cases and Cagan establishes that these differences cannot well be assigned to random variability. Consequently, systematic differences in the values of $\alpha$ and $\beta$ between different situations must be expected. This indicates that the theory actually under consideration is a two-parameter family of elements, each element characterised by a definite value of $\alpha$ and $\beta > 0$. This class is completely specified by statements "a" to "g" together with the rules of identification.

A comparison of the time-series of estimated real-balances with the time-series of actual real balances confirms the broad outline of the hypothesis and so does the scatter

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1 Cagan also shows that exactly the same result can be derived by eliminating $E$ and statement $b$, i.e. by assuming in lieu of $b$ and $c$ a statement $b'$ which asserts $\pi$ to be of a similar order as $\frac{dM}{dt}$ and $c'$ which asserts the dependence of $\frac{Md}{P}$ on $\frac{d}{dt} \log P$. The $E$-lag is then replaced by an equivalent $\pi$-lag. op. cit. p. 75.

2 The evidence which leads him to reject a simple hypothesis of the form

$$\frac{M}{P} = f \left( \frac{d}{dt} \log P \right),$$

consists of a straight comparison of time-series of actual real balances, desired real balances according to his hypothesis, and of the rate of change in prince-level. Another piece of evidence can be found in the circumstance that in a self-generating inflation $\log P$ grows at an exponential rate; actually not even $P$ continuously grew at such a rate. The reader should note that the simple hypothesis involves $M$, and not $Md$.

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of $E$ and $\frac{M}{P}$ around the regression line. Also, both pieces of evidence indicate the necessity to reconsider some parts of the hypothesis. First, the scatter of $E$ and $\frac{M}{P}$ shows that towards the end of hyper-inflations real-balances tend to be much larger than the amount predicted from $E$. Cagan considers two changes in the original structure: (a) statement "c" should be replaced by a curvilinear expression, (b) when currency-reform begins to be talked about, $E$ is not solely determined by past observations of price-behavior. A detailed investigation of the evidence establishes considerable support for these two revisions.

Second, the comparison of estimated and actual time-series of real balances reveals a marked serial correlation of the random residuals. Estimated real balances tend in general to be too low at the beginning and too high at the end of the process. This suggests another revision of the original hypothesis, namely, that the "coefficient of expectation" $\beta$ is not constant but varies in a systematic manner during hyper-inflations. Cagan investigates whether $\beta$ is rising significantly with the process of hyper-inflation. Such an increase reflects a shortening of the lag of expectations behind actual price-movements. Special tests for the Hungarian I, Polish, Russian, Greek, and Austrian experience confirm the suggested revision. The sample from the Hungarian II case is too small to admit a useful test and while Germany's case is unclear, it provides no support for a strong rejection of the revision.

b) The "Normal" Case: The general schema of demand theory permits a clear specification of the essential difference between Cagan's and Selden's work. Cagan restricts his attention to a situation characterised by extreme variations of a single variable in the scheme. Selden is concerned with situations where variations of the relevant variables are of approximately equal order. Analysis of velocity under such conditions is significantly more complex. Still, as a result of Selden's patient efforts some patterns do emerge and his careful analysis is perhaps most important as a clarification of some of the foundations which will facilitate subsequent systematic research. Some major contributions and issues raised by Selden's work will be mentioned in the following passages.

The choice of a useful concept of velocity is one of the most pressing problems in the field. This choice involves two steps: first, a decision as between two classes of concepts: income velocity and transaction velocity – and second, a decision within each class. A priori arguments against income velocity by Keynes and Marget do not clarify the usefulness of the two classes of concepts in terms of their relative explanatory power. As a matter of fact, in view of the substantial difference in the supply of reliable data relevant for the two concepts, the requirements of empirical science will usually induce us to select income-velocity. Unfortunately, this class is quite large. According to Selden it contains 45 different elements. This abundance results from differences in definitions and identification rules. Evidently, an analysis of various concepts in terms of the relative validity of the hypotheses built around them still forms a major research problem. Selden does not consider this problem. He assembles the concepts, compares carefully their definitions, the data used, the handling of these data, and presents a reasoned a priori presumption in favor of some specific one.

The velocity-measures available are investigated for the existence of some general patterns. Two types are distinguished: a secular decline and the short-run variations. If differences in the underlying period and definitions are eliminated a secular decline emerges clearly in both transaction- and income velocities. Selden's detailed analysis also establishes that no single linear trend can usefully approximate this secular decline. It apparently slows down towards the end of the 19th century. Recent decades show no
evidence of it. The short-run patterns consist of a positive cyclical association of income-velocity and income, a seasonal movement of income-velocity and a wartime reaction. These patterns pose some difficult problems of interpretation, and a velocity-theory able to explain these movements has still to be constructed 1.

A simple idea has often been advanced to account for the behavior of velocity. It postulates that velocity is positively related to the cost of holding money, and that variations of the latter sufficiently explained the behavior of the former. As the identification rules are an essential part of the specific content of a hypothesis, Selden uses two measures of velocity and eight measures of "cost of holding money", and consequently considers 16 hypotheses based on the same general idea. The variation in cost of holding money explains in no case more than 58% of the variation in velocities. And in a number of cases the correlation has the wrong sign. If we consider that markets are not perfectly and immediately adjusted and data not ideal, we may feel a multi-dimensional measure of the cost of holding money to be more appropriate. Consequently, Selden tries a multivariate formulation. The results remain poor and supply no evidence supporting the simple idea. As a result of this negative outcome Selden engaged in a more general approach. His procedure is of great importance for future systematic investigations in the field, particularly, as it reveals the methodological problems encountered at every step of our search for valid hypotheses. The following discussion consists to a large extent of a logical interpretation of Selden’s elaboration of velocity-theory 2.

First, we notice a classification of phenomena considered relevant for the problem under consideration: to explain the observable behavior of the amount of dollars held in the average per dollar of transaction. The very formulation of this problem contains already an implicit decision as to the relative promise of various approaches. An alternative procedure, in general not equivalent to Selden’s choice, would be a classification designed to organize phenomena relative to the behavior of real balances. Selden presents no case for his choice. The formal structure of the organisation-scheme, suggested by the classificatory arrangements implicit in general demand-theory, is characterised by the following categories: cost of holding money, cost of money substitutes, cost of money complements, real income, expectation, tastes and technical conditions.

The second step in the construction of the hypothesis is the careful description of identification rules associated with each category. Selden’s discussion of this point reveals


(1) $V_y$ (income velocity) generally rises during business expansions and falls during contractions.

(2) a seasonal low appears to occur in the first quarter and a high in the last quarter of each year.

(3) during both world wars $V_y$ apparently rose initially, fell during years of full military effort, and rose again in the early postwar period.

Selden emphasizes at another place that the fall in $V_y$ during World War II was probably smaller than biased data would indicate. The cyclical and secular patterns of $V_y$ raise difficult questions. According to general demand theory the secular decline is associated with the decrease in the cost of holding money and the increase in real wealth. But the cycles show a positive association of $V_y$ and real wealth (or income). Friedman suggested a solution by means of his "permanent income" hypothesis. The disappearance of the trend in recent decades while real wealth still rose substantially also demands an explanation. Apparently, various compensating factors have been at work for a future searching analysis to be uncovered.

2 Selden’s analysis is concentrated in sections B, C, and D of chapter V and particularly in the historical analysis of chapter VI.
the many protracted and difficult problems we are bound to meet in a search for optimal rules. Only for the first, second, and fourth category are specific rules formulated. Consequently, the classification scheme actually used for analysis has been reduced to three categories.

A last step is the specification of the sign of association between the categories of the classification and the variable under consideration. The resulting structure, while not rich in meaning, is still a definite, scientifically relevant hypothesis. As a matter of fact, in a field where hardly anything significant exists, the building of scientific knowledge has to begin with this phase of "qualitative" hypotheses. Success on this level will lead us naturally to formulations of richer hypotheses containing quantitative information of various sorts. For instance, considerations of orders of magnitudes could be imposed on the qualitative hypotheses. Selden attempted to go immediately a step further and considered a "quantitative" hypothesis in form of a linear relation connecting transaction-velocity (represented by deposit-turnover) with the measures representing cost of holding money, cost of money substitutes, and real income per capita. The correlation-coefficient associated with the regression is very high (.94). Still, the result conflicts with the hypothesis, because the estimated coefficient of the cost of holding money is negative and revision is indicated. The rejection of the quantitative hypothesis does not specify the precise points needing revisions. The classification-scheme may have to be changed, conceivably enlarged, or some rules of identification reconsidered. Selden points particularly to the identification-rule attached to the "cost of holding money" category. This suspicion is strengthened by the observation that the contribution of the cost of holding money measure to the total correlation is negligible. This is consistent with the notion that this measure is related with categories of opposite association-signs. Consequently, the cost of holding money measure is deleted, and the resulting "quantitative" hypothesis only contains cost of money substitutes and real income per capita. This corresponds to a "qualitative" hypothesis consisting only of two categories. The high correlation coefficient (.92) obtained for the period 1919–1951 looks very good, but is it sufficient to accept the hypothesis?

Selden carefully avoids such hasty judgements. To evaluate the validity of his schemes he engages in a detailed "historical analysis" of subperiods. This historical analysis is actually a confrontation of the two-and-three-category "qualitative" hypotheses with the appropriate observations. To incorporate income-velocity in his analysis a relation between the two velocities was required. This relation was defined in terms of two ratios: the ratio of final output prices to prices of non-income transactions and the ratio of real income to non-income real transactions. This relation determines the relative variation of the two velocities, but is unable to transform by itself the relative variation into absolute changes of the two velocities. The absolute variations in transactions-velocity are already taken care of by the hypothesis constructed for this purpose.

1 Op. cit. p.214. "This (i.e. the negative coefficient for the cost of holding money) is not altogether surprising, since a decline in bond yields has at least two aspects: money is less expensive to hold in terms of sacrificed income, but less cash need be held, since credit is available on better terms. There is no reason to expect these opposing tendencies to offset each other precisely." Thus, Selden, suggests that bond yield may be related to two distinct categories of the scheme with opposite signs of association: the cost of holding money, and the cost of money substitutes (options to borrow). Very likely his choice of identification-rule for the cost of holding money is inadequate.

2 Op. cit. p. 219. Selden hints at a connection between the relative importance of medium of exchange and store of value function and the nature of this imputation. But his argument is sketchy and not integrated into the hypothesis under consideration.
If we combine the transactions-velocity hypothesis with a "qualitative" hypothesis of the velocity-relation we obtain a "qualitative" hypothesis of income-velocity.\footnote{1}

The analysis of subperiods erodes the explanatory value of the linear regression substantially. Also, the two "qualitative" hypotheses do not hold up well.\footnote{2} Selden concludes that short-term movements (of velocity) are more closely related to tastes, the factors responsible for non-proportionality of velocities (i.e., their relative variation) and the cost of money substitutes. And the detail of Selden's exemplary evaluations strongly indicates that revisions of the "qualitative" hypothesis in both the classificatory scheme and the identification-rules are unavoidable before some degree of success in the construction of velocity-theory can be achieved.\footnote{3}

\footnote{1} It should be noticed that the "qualitative" hypothesis explaining the velocity-relation has not been fully constructed by Selden. The classificatory scheme is clearly specified. But a complete and formal statement of identification rules is missing. His imaginative discussion does contain specific examples and suggestions which would lead us to such rules. But further research is still needed on this point. Selden's analytical handling of $V_y$ is actually not quite clear. He starts out with a hypothesis for $V_t$ and then discusses the relation between $V_t$ and $V_y$. In the application of the scheme he usually analyses first the velocity-relation and then tries to impute the relative variations to $V_t$ or $V_y$. This imputation is not based systematically on his $V_t$ hypothesis, but introduces at times ad hoc considerations.

\footnote{2} For the period 1839–1919 the hypothesis is consistent with observations. For 1919–1929 the hypothesis is contradicted. The observations of 1929–1933 and of 1933–1943 are inconclusive, and those of 1943–1946 from inconclusive to negative. More convincing is the story of the last period 1946–1951.

\footnote{3} The revision in the classificatory scheme essentially means an attempt to incorporate "tastes" and "expectations". The major problems are obviously useful identification rules for these categories. But the situation is not at all hopeless, provided some ingenuity is exercised. An alternative "qualitative" hypothesis which should be evaluated is of the following form:

$$\frac{M}{P} = -a_0 r + a_1 s - a_2 m + a_3 X + a_4 T + a_5 W,$$

where $r$ is a measure of the cost of holding money, $s$ a measure of the cost of money substitutes, $m$ a measure of the cost of money complements, $X$ = real income, $T$ = real volume of transactions, $W$ = real wealth. All coefficients $a_i$ are positive. From this we derive

$$\frac{1}{V_t} = \frac{M}{PT} = -a_0 \frac{r}{T} + a_1 \frac{s}{T} - a_2 \frac{m}{T} + a_3 \frac{X}{T} + a_4 + a_5 \frac{W}{T},$$

$$\frac{1}{V_y} = \frac{M}{Y} = \pi \left[ -a_0 \frac{r}{X} + a_1 \frac{s}{X} - a_2 \frac{m}{X} + a_3 + a_4 \frac{T}{X} + a_5 \frac{W}{X} \right],$$

where $V_y$ = income-velocity, $V_t$ = transaction-velocity, $Y$ = money value of income, $P$ = price-level associated with total transactions, $P_y$ with final output, and $\pi$ is defined by $P = \pi P_y$. This structure incorporates the velocity-relation as an integral part and separates the effect of non-proportionality on $V_t$ and $V_y$. Non-proportionality of price-levels affects only $V_y$; non-proportionality of transaction-types affects both $V_y$ and $V_t$, but each with opposite sign. As a matter of fact we have:

$$\frac{dV_y}{d\pi} < 0 \quad \text{and} \quad \frac{dV_t}{d\pi} = 0,$$

$$\frac{dV_y}{dt} > 0 \quad \text{and} \quad \frac{dV_t}{dt} < 0,$$

where $t$ is defined by $X = tT$. 
2. Theory of Supply

The major part of the "Studies in the Quantity Theory of Money" is concerned with an enquiry into the demand for money. Nowhere do we find an explicit analysis of the supply-function of money. Still, the contributions do contain important fragments of money-supply theory.

a) Lerner and Klein: In the course of a purely descriptive analysis of money-supply components and bank expansion Lerner presents some data which permit, with the aid of a simple scheme, some inferences about supply patterns. This scheme involves the three magnitudes \( r, k, \) and \( B \), defined in footnote 1. Variations in specie \((S_p + S_b)\) were probably small relative to the changes in \( B \) determined by government note issue. Thus, the monetary base rose approximately by 800 mill. dollars from June 1861 to January 1864. With a constant monetary multiplier deposits (plus bank notes) would have risen by at least 1200 mill. dollars. Actually, deposits increased only by approximately 150 mill. dollars.

The material presented by Lerner indicates that the deposit and monetary multiplier decreased substantially during the war. The banks' reserve quotient seems to have moved up to around 50\% \(^2\). Also, the public's currency quotient \( k \) rose spectacularly from a likely .2 to approximately 2.8. This behavior of the public and the banks reduced the deposit multiplier from around 2 to somewhere near 0.3. These changes transformed the multiple expansion process into a "fractional" process, and we notice that deflationary behavior of public and banks dampened to a large extent the government's inflationary policy.

1 Op. cit. p.169/70. The scheme is as follows:

1. \( R = rD \),
2. \((S_p + N_p) = kD\),
3. \( R = S_b + N^b \),
4. \( B = S_p + S_b + N_p + N^b \),

where \( R = \) banks' reserves, \( D = \) deposits plus bank note liabilities, \( N_p = \) government notes held by public, \( N^b = \) government notes held by banks in form of reserves, \( S_p = \) specie held by banks in reserves (coin or bullion), \( S_b = \) coin held by public, \( B = \) monetary base. We obtain then the money-supply function:

\[
M = \frac{1+k}{r+k} B,
\]

where \( k \) = currency quotient of public, \( r = \) reserve quotient of banks. \( k \) and \( r \) determine the "monetary multiplier" \( \frac{1+k}{r+k} \), which is a measure of the monetary systems expansive (or contractive) power. The deposit multiplier is given by \( \frac{1}{r+k} \).

2 Op. cit. p.170. "The most important reason was that commercial banks had no central bank to support them during a crisis. Southern bankers expected mass withdrawals whenever Union troops approached."

3 \( k = 0.2 \) for middle 1861 is a guess based on the US currency-quotient for the decade and the coin-note proportion in currency. \( r = 0.3 \) for middle 1861 is also a guess, very likely too high. These guesses imply a monetary base of approximately 60 mill. $ at the beginning of war. Consequently the monetary base in January 1864 must have settled around 880 mill $. With \( r = 0.5 \) in January 1864 bank reserves must have been around 135 mill. $. Upon subtracting reserves from the base we obtain 745 mill. $ as an estimate of currency held by the public. Other informations needed to compute the multipliers can be found in the table on p.169.

4 With a constant multiplier the rise in the money-supply would have been nearly twice as large as actually occurred, with a corresponding additional increase in prices.
Klein’s work on “German Money and Prices” is less informative about the supply function. No information is presented on the monetary base and the banks’ reserve quotient. A currency quotient and a savings-deposit quotient can be computed. The currency quotient rises very slowly from .442 in 1932 to .462 in 1939, drops to .394 in 1941 and rises to .503 in 1944. Thus, in the first two years of war the currency quotient — contrary to US experience — contributed to raise the expansive power of the monetary system. The savings-deposit ratio increased throughout the period of 1932-1944. It started at 1.11 and ended up at 1.68. This trend implied an increase in the monetary multiplier. Also, it can be computed that the expansive effect of the increase in the savings-deposit quotient outweighed the contractive effect of the rise in the currency quotient during the war.

b) Cagan: A complete supply theory can be constructed from Cagan’s analysis of the implicit tax on money-balances imposed by inflation. We start with a simple money-supply function $M = mB$, with $m =$ the monetary multiplier, and $B =$ the monetary base. By logarithmic differentiation we obtain $\dot{M} = m\dot{B} + mB$. It is assumed that whatever variations of $m$ may occur in hyper-inflation, they will be of small order relative to the variations of $B$. Consequently, we assert that $\dot{M} = m\dot{B}$.

The next step is the derivation of an expression which explains the behavior of $\dot{B}$. This is achieved with the aid of the following considerations: Government operations absorb a proportion $q$ of current real income $x$. $q$ is taken as a government action parameter. We therefore write $R = qx$, where $R$ are the real expenditures of government. These expenditures are covered in two ways: by taxes and by the issue of money. This latter operation is essentially equivalent with an increase in the monetary base. Consequently, we state the relation $RP = T + \dot{B}$. Financial and administrative convenience seduces the government to rely mostly on $\dot{B}$ as against $T$, i.e. the tax parameter $t$, defined by $\frac{T}{PR}$ is small and also as a result of important lag features, falling. By combining the various elements, we obtain a relation

$$\dot{B} = qPX(1-t),$$

which explains the motion-pattern of the monetary base in terms of two government policy parameters $q$ and $t$, current price-level $P$, and current real income $X$. As the relative variations of the latter are small, $X$ is taken to be fixed. And by a further substitution we derive

$$\dot{M} = mqPX(1-t).$$

1 The following scheme underlies the discussion:

$$M = \frac{1 + k + s}{r(1 + s) + k} B,$$

$M =$ money-supply (currency + all deposits), $k =$ currency quotient, $s =$ savings-deposit quotient, $r =$ reserve quotient. $B =$ monetary base. Data specifying $k$ and $s$ are on p. 135. If $r = 0.2$ and assumed to be constant, the monetary multiplier stayed around 3 from 1932 to 1938 and moved to approx. 3.3 in 1944.

2 Op. cit. p. 77, section VII.
A second relation between $M$ and $P$ can be gained from Cagan’s demand hypothesis. It has the following general form

$$P = \varphi \left[ M, \frac{\partial M(t)}{\partial t}; \alpha, \beta \right]$$

and declares that $P$ in $t$ depends on $M$ in $t$ and the pattern of past $\dot{M}$. The nature of the dependence is shaped by the “coefficient of expectation” $\beta$ and the “reaction-coefficient of real balances relative to expectations” $\alpha$. The two equations jointly determine the motion of $M$ and $P$ in terms of government policies expressed by $q$ and $t$, the monetary multiplier $m$, and the public’s behavior patterns as revealed by $\alpha$ and $\beta$. In this manner, money-supply theory has been integrated into a complete system which accounts for the observable behavior of prices, real balances, and the monetary system under hyper-inflation.

3. Nature and Validity of the Quantity-Theory

a) The Criterion of a Quantity-Theory: Any attempt to evaluate the validity of the quantity-theory runs into serious difficulties as to its precise meaning. “The quantity theory of money is a term evocative of a general approach rather than a label for a well defined theory.” The vague generic quality of the term is substantiated by Friedman’s implicit admission that the Chicago tradition has not supplied a definite operational statement. Friedman’s contribution to the symposium, apart from a discussion of the formal scheme of the theory of demand for money, tries to specify the characteristic features of a quantity-theoretical approach, an approach insisting on the “relevance of money”. Friedman’s specifications are somehow contradictory. On p. 4 he writes: “The quantity theory is in the first instance a theory of the demand for money. It is not a theory of output, or of money income, or of the price-level. Any statement about these variables requires combining the quantity-theory with some specifications about the conditions of supply of money and perhaps about other variables as well.” But with this interpretation there is no issue between a “Keynesian-theory” and the “quantity-theory”, the latter becomes a part of the first. Also, it is possible to construct models containing the “quantity-theory” which imply that the money-supply “does not matter”.

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1 Among the data presented by Cagan we find the ratio of government money to total money-supply. This magnitude approximates the reciprocal of $m$. Cagan’s analysis of the implicit tax, while interesting, suffers from some obscure points. The most important of these refers to his interpretation of $M \frac{d}{dt} \log P$ as the total real revenues collected by the inflation tax on money balances (p. 78). I think it is misleading to identify what one may call an inflation tax on money-holders with real revenues collected by the government and flowing from this tax. The real revenues “collected” are the real resources made available to the government. They have been made available as a result of a contraction of real resources absorbed by the private sector due to the reallocation of resources as between public and government. This reallocation is carried out when the government bids the resources away from the public by means of an increase in prices associated with the spending of the newly created base money. The total loss in the public’s real wealth exceeds the loss in non-monetary real wealth resulting from the reallocation by a magnitude measuring the inflation-induced loss in monetary real wealth. The magnitude isolated by Cagan as “real revenues collected by the government” can be shown to be a component of the latter quantity. Hence, the ratios computed with the “real revenues” as a numerator are barely significant.

On p. 15 Friedman actually reconsiders his earlier position. Now he states three features defining the quantity-theoretical approach:

i) the existence of a stable demand function for money and that this function “plays a vital role in determining variables that he (i.e. the economist) regards as of great importance for the analysis of the economy as a whole, such as the level of income or of prices”.

ii) the existence of a supply function of money which contains variables not appearing in the demand function, i.e. no non-trivial linear combination of the two functions should be able to reproduce the general form of either function. In addition, the supply-factors should not be systematically connected with variables in the demand function.

iii) the rate of interest is not determined by the interaction of the supply of and demand for money.

These suggestions contribute significantly to the clarification of the nature of the quantity theoretical approach. In particular, they indicate that we are not concerned with a single specific hypothesis, but with the common properties of a class of monetary hypotheses. But the conditions seem partly dependent, as condition iii) is already implied in condition i), if the “vital role” of money is suitably formulated and they also need an adequate formalization. As a further step in the required clarification process I submit the following criterion defining the class of quantity-theoretical hypotheses:

a) A proper subsystem of the hypothesis not containing the demand function for money cannot be solved for money income and money prices.

b) A proper subsystem not containing the money-supply function cannot be solved for the money-supply.

c) There exists no supplementary system which solves simultaneously exogeneous variables of the supply function and any set of variables from other relations in the hypothesis.

b) Cagan’s evaluation of the validity of a quantity-theoretical hypothesis: In a previous section a complete monetary theory involving money-supply and prices had been constructed from Cagan’s analysis. This section shows the manner in which Cagan combines his demand hypothesis with some additional specifications in order to derive a quantity-theoretical hypothesis of the motion of prices under hyper-inflation. It is important to realize the logic of Cagan’s general procedure. We have a system of three relations:

i) \( \log \frac{M}{P} = d(E, p) \),

ii) \( p = a \) constant,

iii) \( \dot{E} = h(C - E) \),

where \( p \) is a vector of magnitudes of general importance for demand behavior. It is then postulated that no significant variation takes place in \( p \). This is a short-cut statement of additional relations which determine \( p \). By substituting ii) into i) and then differentiating with respect to time and also solving ii) and i) for \( E \) in terms of \( \frac{M}{P} \) we can replace \( \dot{E} \) and \( E \) in iii) by expressions only containing observable variables. As a result we obtain a first-order differential equation in \( \frac{d}{dt} \log P \) which is linear under Cagan’s hypothesis.
The general form is:

\[ a_0 \frac{d}{dt} \log P + a_1 \frac{d}{dt} \log M + a_2 \log M + a_2 \log P = 0. \]

This can be solved for \( \log P \) and the solution appears in the form

\[ P = \varphi \left[ M, M(t); a, \beta \right]. \]

If we add the previously derived money-supply function, we have again the complete system. Conditions i) and ii) of the above advanced criterion are evidently satisfied. Condition iii) appears in the nature of a postulate which permits certain mathematical and statistical operations.

c) Lerner and Klein: The two contributions on "German Money and Prices 1932 to 1944" and "Inflation in the Confederacy 1861–1865" essentially describe some important material and analyse with care and ingenuity the reliability of price and money series. Klein notes the large discrepancy between the movement of prices and money-supply in the period 1938–1944. Is such a discrepancy consistent with the "quantity-theory" and the hypothesis of a stable demand function? An imaginative analysis of data by Klein results in corrected price-series. There still remains a discrepancy in the order of magnitude of wartime changes in \( M \) and \( P \), though substantially reduced. The discussion of German

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1 The last expression which determines the motion of \( P \) in terms of \( M \) and past behavior of \( M \) is used by Cagan to derive some extremely interesting results on the question whether inflation will become self-generating or not. Closely associated is the issue whether a rise in velocity, once started, will continue, independent of the behavior of money-supply. Cagan's analysis of the price-equation, noted in the text, enables us to state the following propositions:

a) if \( \alpha \beta > 1 \) and \( M \) rises at less than an exponential rate, \( \log P \) and \( \log V_Y \) will approximate an exponential growth;

b) if \( \alpha \beta > 1 \) and \( M \) rises at less than an exponential rate, \( P \) and \( V_Y \) will continue, once started, their upwards motion, independent of the behavior of \( M \);

c) if \( \alpha \beta \leq 1 \) then a continuous increase in \( P \) and \( V_Y \) occurs if, and only if, there is a continuous increase in \( M \);

d) if \( \alpha \beta \leq 1 \) and \( M \leq 0 \) for all \( t \geq t_0 \), there exists some \( \tau > 0 \) so that for all \( t \geq t_0 + \tau \), \( \dot{P} \leq 0 \), and \( \dot{V_Y} \leq 0 \).

The condition \( \alpha \beta \leq 1 \) is thus of central importance for inflation theory. Cagan presents strong evidence that the alternative hypothesis \( \alpha \beta > 1 \) must be rejected. Point estimates of \( \alpha \beta \) based on the whole period of hyperinflation give two cases of \( \alpha \beta > 1 \) and five of \( \alpha \beta < 1 \). Point estimates based on the later part of the hyperinflation period give one case only of \( \alpha \beta > 1 \). Important is the circumstance that the confidence-intervals for the whole period have all lower limits below 1. Another piece of evidence is the comparison of a price-series computed from the price-equation with the estimated values of \( a \) and \( \beta \) inserted with the actual price-series. For the cases with \( \alpha \beta > 1 \) the estimated series diverges substantially from the actual one. This may sound surprising, but the reader should carefully note Cagan's explanation on p. 72 which turns on the fact that the original demand model which was used to estimate \( a \) and \( \beta \) and the logically implied price-model are actually two different regression-models. A last piece of evidence noted is the fact that \( P \) did not always rise at an exponential rate, but if \( \alpha \beta > 1 \), then even \( \log P \) should rise at an exponential rate.
economic policies certainly suggests that a forceful control-system will generate a negative
association of money-supply and velocity changes\(^1\).

Lerner's description of the Confederacy's currency reform in 1864 contains some
relevant information. This currency reform immediately reduced the money-supply and
broke the inflationary expansion for several months. The currency reform took effect in
May 1864 and prices immediately slid downwards until December 1864. These obser-
vations are consistent with Cagan's results and strongly confirm a quantity-theoretical
hypothesis.

III. Concluding Remarks

Perhaps the most important aspect of the "Studies" is the general spirit of the book,
permeated by an imaginative and relentless drive to contribute to the science of economics.
What are now, in summary, these contributions?

i) A demand hypothesis for money under hyper-inflation with satisfactory empirical
results.

ii) An empirically satisfactory quantity-theoretical hypothesis to explain the
connection of prices, money-supply, and income-velocity under hyper-inflation.

iii) A confirmation of a quantity-theoretical hypothesis by Lerner's material on the
Confederate currency reform.

iv) It is strongly suggested by Klein's description that the application of an effective
control system makes a quantity-theoretical hypothesis inapplicable.

v) Selden presents strong evidence against a simple "cost of holding money" hy-
pothesis of velocity.

vi) Selden's discussion of a general approach to velocity-theory, particularly the
methodological implications concerning the importance of adequate classificatory schemes
and particularly the overriding need for a systematic analysis of identification rules.

vii) Friedman's contribution to a clarification of the logical structure of a quantity-
theoretical hypothesis.

viii) Points i) and ii) have wider ramifications. Selden's analysis reveals the diffi-
culties met by any evaluation of the demand effect of variations in the return of various
assets when such variations are of similar order with purely random influences. Cagan's
successful isolation of one particular return, and the clear evidence as to the connection

\(^1\) What happens can be stated in the following manner: By logarithmic differentiation of the
"equation of exchange", we obtain \(m + v = p + x\), where the small letters indicate relative
changes in \(M, V, P,\) and \(X\). As \(x\) is of relatively minor order under inflation, we shorten to:
\(m + v = p\). The last variable is limited by the control-system to \(\bar{p}\) and thus we have \(v = \bar{p} - m\).

\(m\) is determined by the government's financial policy working through \(b = \left(\frac{d}{dt} \log B\right)\). And thus,
demand for \(M\) becomes specified in terms of the nature of the control system and the supply
factors of \(M\). This seems to violate condition iii) of our criterion of a quantity-theoretical hypo-
thesis. In such a situation the quantity-theoretical hypothesis still suggests a measure of likelihood
of the joint survival of the existing control system and monetary system in form of the quantity
\(\int (m - \bar{p}) \, dx\). The larger this index, the greater the possibility that something will "give" in the
existing control or monetary institutions.
between this return and demand for real balances strengthens quite generally the case for the existence of a systematic connection between asset-yields and desired money-balances. Further, Cagan's successful application of a quantity-theoretical hypothesis to hyper-inflation and Lerner's evidence also bear on the issue of the relative validity of the class of monetary hypotheses, called "quantity-theory", as against various types of money or interest "residuality" hypotheses prevalent in the literature.

And as to Hansen's book, I repeat: No knowledge can be gained by playing "linguistic games" with economic sounding names 1. The development of economics will depend substantially on the ratio of the two types of books published in the future, and we can only hope that the "Studies" type will enjoy a growing share 2.

1 It should be mentioned in this context that an occupation with formal structures is not necessarily ill applied or useless. The transformation of "ideas" into operational hypotheses is usually an intricate and laborious analytical job. At some stages of this transformation a concentration on formal structures is unavoidable. But this occupation occurs as a part of the transformation which gives it a definite guidance. If it occurs independent of such a transformation process, the danger of a game attitude becomes quite serious. And it does not help to add hopefully that the formal apparatus will be useful — somehow.

2 I want to emphasize that my short review of the "Studies" in the "Journal of Finance" did not sufficiently appreciate some of the positive contributions made, such as ii), iii), and vii) above.