The Spatial Dispersion of Production: A Hypothesis

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1. Introduction

This paper is intended to contribute to current understanding of the spatial dispersion of productive activities. The central phenomenon to be explained is the occurrence of industrial «centres» or «agglomerations», usually indicated by different names depending on their size: cities, towns, villages and so on. The characteristic which interests the author most is the size distribution of these centres which seems to show a more or less constant character in most countries and regions. This attempt to find an economic explanation implies the intuitive opinion that economic factors are the most powerful influence shaping the size distribution of industrial centres.

The model presented in this paper is believed to contain some of the essential features of an explanation, i.e. the model represents an optimum distribution of centre sizes. However, it is as yet only a hypothesis, as indicated by the title of this note. We are not yet able to present a consistent piece of scientific analysis. Some of our propositions can be deduced from our assumptions, but there are lacunae in between.

It is possible that this particular model may not lead to a correct theory of spatial dispersion. However, we are publishing this preliminary sketch in the hope that discussion by a wider circle of economists may help us to discover the proper approach.

2. Provisional Basic Assumptions

We will start out by formulating some provisional assumptions delimiting the problem in what we hope is a useful way. Since we have not yet been able to prove that our hypothetical solution is correct, these basic assumptions may have to be changed on the basis of later findings. This is why they are called provisional.

We will assume a closed economy on a homogeneous piece of land – a square – of constant fertility with agricultural production distributed evenly over it. Agricultural enterprises are the only ones bound to the spot. All other activities are organized into «industrial» enterprises. Each enterprise produces one good and is characterized by an index h, which we take equal to 0 for agriculture and from 1 to H for industries. For each industry h there is a minimum size of enterprise (s_h) for which production costs are at a minimum; they remain at this minimum when the size of the enterprise is greater than s_h.

The unit of quantity of each product is chosen to make the price equal to the unit of value; quantities therefore also represent values. This means we are assuming that all goods are uniformly priced, and that the cost of transporting the goods is paid by the producer. All commodities are assumed to be consumer goods, and the incomes of all citizens are assumed to be spent in the same way, i.e. fixed proportions α_h of income Y are spent on good h. For any value of total income Y, therefore, the quantity of good h demanded $(\alpha_h Y)$ is known. Since the minimum size of each type of enterprise is also given, we know the number of such enterprises n_h needed to serve the country. We now arrange the industries in the order of falling n_h:

$$n_1 > n_2 > n_3 \ldots\ldots\ldots > n_H \qquad (2.1)$$

combining any two industries with equal n_h into one single industry[1]. Somewhat more formally, we may also say, in stead of (2.1):

$$h_1 > h_2 \text{ for any } n_{h_1} < n_{h_2} \qquad (2.2)$$

The integers h thus defined will be called the «ranks» of the industries and we will speak of industries of lower and higher ranks or briefly of lower and higher industries.

$$\text{It will be finally assumed that } n_H = 1 \qquad (2.3)$$

i.e. that there is only one enterprise in the «highest» industry. If there were more, the country could be split up into smaller countries which satisfy the condition (2.3).

3. The Optimum Problem

The enterprises of the various industries introduced in section 2 must still be assigned a location. This location should be such that some optimum condition is met. We propose the condition that total costs of production and transportation of the country must be a minimum within the framework of the provisional basic assumption.

If there exists only one industry – apart from agriculture – i.e. if $H=1$, it is obvious that the n_1 enterprises should be spread «evenly» over the surface

[1] We will assume that each value n_h is at least several multiples of the next-rank n_{h+1} and deal with industries with «about equal» rank h as if they were one size group.

of the country, leaving to each an equal market area as far as this is geometrically possible. With a square country and n_1 equal to some i^2 where i is an integer, this possible. If n_1 cannot be written i^2 there will be interesting, but may we suggest, second-order problems of how the solution will look.

When $H > 1$ the problem becomes more complicated. One must now admit the possibility of «clustering» or «agglomeration», and the concept of industrial «centre» now appears as a combination of enterprises, workers and the corresponding families located on virtually the same spot, and commonly called a city, town or village.

The problem becomes to indicate how many industrial centres of what enterprise composition and at what locations will fulfil the optimum condition.

4. Suggested Solution (the Hypothesis)

The hypothesis we have suggested is an incomplete solution to the problem. It specifies only the types and composition of centres, not their location. It is assumed that their locations can be found by a variation process (cf. 8). The hypothesis consists of three elements:

(I) Each centre containing an industry of rank h, also contains all lower industries (with ranks $< h$).

(II) The total output of the industries of all ranks, except the highest present in the centre, is consumed by the centre's population. In other words, only the product of the highest industry is exported to pay for the imports of agricultural products as well as for products of a higher rank which are needed by the centre.

(III) The centre contains only one enterprise of highest rank.

Defining the rank h' of a centre as the rank of the highest industry in the centre, we may summarize the above three elements by saying that the industries h contained in the centre are $1 \leqq h \leqq h'$ and that industries $1 \leqq h < h'$ do not export, whereas the number $n_h^{h'}$ of enterprises h in centre h' satisfies the condition $n_{h'}^{h'} = 1$.

In what follows we will indicate by $N_{h'}$ the number of centres h' and by $Y_{h'}$ the joint income of the $N_{h'}$ centres h'.

Implicit in our suggested model is a definition of «servicing» or «secondary» (as opposed to «pushing» or «primary») industries. This definition appears to depend on the rank of the centres, and is applied to all industries of lower rank (except agriculture).

5. Notation: summary

The symbols used have been summarized here for the reader's convenience, and some identities have been added.

Industries and goods are indicated by an index h for which we have

$$0 \leqq h \leqq H \tag{5.1}$$

where 0 represents agriculture.

Centres are indicated by their rank h' for which we also have

$$0 \leqq h' \leqq H \tag{5.2}$$

if rural areas together are given the rank 0.

Goods produced in the other centres $1 \leqq h' \leqq H$ include all h satisfying

$$1 \leqq h \leqq h' \tag{5.3}$$

Total demand for good h equals $\alpha_h Y$, where Y is total income of all centres:

$$Y = \sum_{0}^{H}{}_{h'} Y_{h'} \tag{5.4}$$

Our analysis being static it is assumed that

$$\sum_{0}^{H}{}_{h} \alpha_h = 1 \tag{5.5}$$

The number of centres h' is represented by $N_{h'}$, with

$$N_H = 1$$

The number n_h of enterprises of any type h follows from total demand and minimum optimum size s_h:

$$n_h = \frac{\alpha_h Y}{s_h} \tag{5.6}$$

The index h has been so attributed to the various industries that

$$n_h > n_{h+1} \text{ for } 1 \leqq h \leqq H \tag{5.7}$$

The size Y of the country is such that $n_H = 1$.

6. Some Consequences of the Hypothesis

From the elements I, II and III of the hypothesis we can calculate, for any given Y, α_h and n_h the figures $N_{h'}$ and $Y_{h'}$, i.e. the size distribution of centres as well as the distribution of income (or production) among centres.

In order to do so we first calculate the incomes $Y_{h'}$ of each family of centres with rank h', starting with $h' = 0$ which represents rural areas. Since agricultural products all originate from this «centre» and since total demand for agricultural products is $\alpha_0 Y$, we have

$$Y_0 = \alpha_0 Y \tag{6.1}$$

Next, we consider $h' = 1$. These centres (the «smallest villages», each consisting of one industrial enterprise $h = 1$) produce good 1, export part of it to rural areas and must import their demand for all other goods. Total imports can be written $(\alpha_0 + \alpha_2 + \alpha_3 + \ldots\ldots\ldots \alpha_H)\, Y_1$ and are equal to the value of their exports which must be $\alpha_1 Y_0$. Hence

$$\alpha_1 Y_0 = Y_1 \left(\sum_2^H {}_h\, \alpha_h + \alpha_0\right) \tag{6.2}$$

Since (cf. [5.5]) the α_h add up to 1, this may be written:

$$\alpha_1 Y_0 = (1 - \alpha_1)\, Y_1 \tag{6.3}$$

from which we derive, first, that

$$Y_1 = \frac{\alpha_1 Y_0}{1 - \alpha_1} = \frac{\alpha_0 \alpha_1 Y}{1 - \alpha_1} \tag{6.4}$$

and, secondly,

$$Y_0 + Y_1 = \frac{\alpha_0}{1 - \alpha_1}\, Y \tag{6.5}$$

Considering now the size class of centres $h' = 2$, we know that they export product 2 to centres 0 and 1 in quantities and value $\alpha_2\,(Y_0 + Y_1)$. Also they must import all goods $3 \leqq h \leqq H$ of which the value is $Y_2\,(\alpha_0 + \alpha_3 + \alpha_4 + \ldots\ldots + \alpha_H)$. Balance of payments equilibrium requires that:

$$\alpha_2\,(Y_0 + Y_1) = (1 - \alpha_1 - \alpha_2)\, Y_2 \tag{6.6}$$

from which we can calculate Y_2 and the incomes of other size groups. We obtain:

$$Y_2 = \frac{\alpha_0 \alpha_2 Y}{(1 - \alpha_1)(1 - \alpha_1 - \alpha_2)} \tag{6.7}$$

and

$$Y_0 + Y_1 + Y_2 = \frac{\alpha_0}{1 - \alpha_1 - \alpha_2} Y \tag{6.8}$$

Formulae (6.5) and (6.8) can be easily generalized and we obtain:

$$Y_0 + Y_1 + \dots Y_{h'} = \sum_{0}^{h'} {}_{h''} Y_{h''} = \frac{\alpha_0 Y}{1 - \alpha_1 - \alpha_2 \dots \alpha_{h'}} \tag{6.9}$$

from which all $Y_{h'}$ can be easily derived. Thus, we find:

$$Y_4 = \frac{\alpha_0 Y}{1 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4} - \frac{\alpha_0 Y}{1 - \alpha_1 - \alpha_2 - \alpha_3} = \frac{\alpha_0 \alpha_4 Y}{(1 - \alpha_1 - \alpha_2 - \alpha_3)(1 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4)} \tag{6.10}$$

It can also easily be seen that application of (6.9) to $h' = H$ yields the necessary result:

$$Y_0 + Y_1 + \dots + Y_H = \sum_{0}^{H} {}_{h'} Y_{h'} = Y \tag{6.11}$$

The results may also be illustrated by a numerical example. The data are the following:

$H = 5; \alpha_0 = \alpha_3 = \alpha_4 = \alpha_5 = 0.2; \alpha_1 = \alpha_2 = 0.1; Y = 1000$
$n_1 = 900; n_2 = 200; n_3 = 39; n_4 = 10; n_5 = 1.$
The results are: $N_1 = 200; N_2 = 50; N_3 = 13; N_4 = 5; N_5 = 1$
$Y_0 = 200;\ Y_1 = 22;\ Y_2 = 28;\ Y_3 = 83;\ Y_4 = 167;\ Y_5 = 500.$

Total number of enterprises in each size group of centres and by type of good produced:

Good / Centre Rank	1	2	3	4	5	Number of centres
1	200	.	.	.	.	200
2	25	50	.	.	.	50
3	75	17	13	.	.	13
4	150	33	6.5	5	.	5
5	450	100	19.5	5	1	1
All	900	200	39	10	1	

The example given above displays some of the inconsistencies which will be mentioned later: e.g. it will not always be possible to obtain an integer for the number of each type of enterprises in each individual centre. In one way or another we must introduce slight deviations from the theory if the data do not permit us to apply the exact solution.

7. Some Possible Inconsistencies and Some Alternative Models

As just stated, there will be inconsistencies for considerable intervals of our variables, for example, the number of enterprises (calculated from the $Y_{h'}$ and $\alpha_{h'}$ with the aid of the s_h) and the $N_{h'}$ may not always be an integer. This does not seem to be a major difficulty, but it does mean that a more sophisticated formulation of the hypothesis is necessary.

There may be many other and more damaging inconsistencies between the hypothesis and the provisional basic assumptions, if the optimum principle is assumed. There may also be inconstistencies between the various provisional basic assumptions themselves. Thus, an attempt to reconcile the optimum principle of minimum total cost with the basic assumption that enterprises are of equal size irrespective of centre in which located may be impossible. It may also be necessary to assume that every enterprise exports something. The latter assumption may lead to an alternative hypothesis in which such exports are recognized and specified. However, it is the author's belief that the hypothesis presented here may still emerge as a limiting case. Another example of an inherent inconsistency is the assumption about prices, unless we make this a matter of imposed government regulation. All these and many other subjects are parts of a research programme which we will describe in the next section.

8. Programme of Testing the Hypothesis

The Netherlands Economic Institute is now carrying out a programme directed towards testing the hypothesis given above. A set of basic assumptions is being sought from which, with the aid of the optimum condition of minimum total cost, the hypothesis can be proved. In broad outlines this is seen to consist of a series of proofs that certain classes of variations will always lead to higher total costs; provisionally it is taken for granted that this is synonymous with higher transportation costs. Among the possible variations there will be included the following classes: (i) variations in location of the centres described by the hypothesis; (ii) variations in the size of the centre in which a given enterprise is located; (iii) multiple variations of both kinds. There may be more classes as well.

It is possible that the basic assumptions may have to be supplemented with assumptions about marginal costs or about variables which affect costs. For example production costs might also depend on the size of the centres in which the enterprise is located.

Some provisional results of this programme will be published elsewhere.

9. Some Suggestions as to Approximations

The author feels that the programme, if it is to succeed at all, may have to make use of approximative techniques as intermediate steps. Some examples of these are listed below.

For the calculation of the minimum or maximum distances involved in variations of location of enterprises we may make assumptions about the shape of «marketing areas» for certain enterprises. For example, we might assume that these are squares or circles, etc. Such assumptions will be better approximations for some sets of n_h than for others (cf. section 3).

The numbers n_h themselves could be replaced by more convenient sets, by admitting small (whatever its meaning) deviations in s_h.

To simplify the calculation of distances, we may also specify the nature of the road system: e.g. it may consist of the rectangular network of roads found in many American towns.

Several further specifications of costs, which may also represent simplifications, are conceivable. An extreme which almost, but by no means completely, seems to beg the question, is the assumption of a sharp minimum of production costs for the size s_h.

10. Desirable Generalizations

It is perfectly clear that once we have succeded in proving one form or another of the hypothesis, there will be an urgent need for greater generalization of the model. Possible additions to the model can be easily listed since everybody is aware of the complexities of reality compared with the oversimplification which our model presents. Thus, not only agriculture, but, say, mining or harbour activities should be added to the model as further types of production «bound to the spot». Exports and imports of the country as a whole should be introduced. Variations in the fertility of the soil, the presence of natural transportation facilities or centers situated in traditional locations are further examples. Clearly some of these elements will soon lead us out of the realm where general theories can be used at all, and where statistical «methods» will have to take their place.