Economic Stability under Conditions of a "Commodity Trap"

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Introduction

A concept which has recently been discussed within the framework of macroeconomics is that of the commodity trap. To understand what is meant by the commodity trap, consider an interest-sensitive investment function such as

\[ I = I(i), \]

where \( I \) is real investment and \( i \) the rate of interest. The relationship between these two variables is the usual one:

\[ I'(i) < 0. \]

As \( i \) rises, \( I \) declines. At some rate of interest, say \( i^* \), net investment is zero and gross investment equals capital depreciation. At higher rates of interest, gross investment is lower yet and capital depreciation exceeds gross investment. At some "high" rate of interest, say \( i^{**} \), gross investment may fall to zero. Postulate a parameter for real government spending \( G \), such as

\[ G = G_0. \]

Consider a diagram plotting \( i \) vertically and \( I \) and \( G \) horizontally. Horizontally summing the \( I \)- and \( G \)-schedules yields an \( I + G \)-schedule which is "kinked", i.e., vertical everywhere above \( i^{**} \) and negatively sloped below \( i^{**} \). Next, postulate the following equations to round out our commodity market:

\[ S = S(Y_d), \]
\[ T = T_0, \]
\[ I(i) + G = S(Y_d) + T, \]

where \( S \) is real saving, \( Y \) real income, \( Y_d \) disposable real income, and \( T \) real taxes. The "kinked" \( I + G \)-schedule implies a vertically kinked IS-schedule, such as that shown in Figure 1. The vertical section of the IS-schedule is denoted as the commodity trap and the rate of interest at which the "kink" in the IS-curve forms is the commodity trap rate of interest.

It is the purpose of this paper to consider the possible implications of the commodity trap for economic stability. In particular, this paper investigates the local stability properties of an economic system, under conditions of a commodity trap, for a specific dynamic adjustment mechanism, i.e., a tâtonnement mechanism. The stability problem is considered within the framework of a purely qualitative \((+, -, 0)\) environment: no quantitative information is presumed available. Stability, should it occur, must be qualitative stability: stability determined solely by sign patterns of elements within the matrices encountered in the analysis.
Section I below briefly considers the meaning of stability of a system. Section II considers a basic theorem on sign stable matrices and offers a proposition on the sufficient conditions for sign stability of an indecomposable real 2 x 2 matrix. Section III investigates stability of equilibrium within the context of a commodity trap.

I.

Let \( y_1, y_2, \ldots, y_m \) be the values of \( m \) economic variables and let \( \alpha \) be a shift parameter. Assume \( m \) functional relationships linking \( \alpha \) and the \( y_j \)'s with the \( i \)th relationship being given by \( f_i(y_1, \ldots, y_m, \alpha) \). For a given value of \( \alpha \), say \( \alpha^1 \), an equilibrium position is defined as a set of values \( y'_1, \ldots, y'_m \) such that

\[
    f_i(y'_1, \ldots, y'_m, \alpha^1) = 0 \quad \text{for } i = 1, \ldots, m. \tag{1}
\]

Let a small change in \( \alpha \) occur. The change in the equilibrium values of the variables is given by

\[
    \sum_{i=1}^{m} f_{ij} \frac{dy'_i}{d\alpha} = -f_{i\alpha} \quad \text{for } i = 1, \ldots, m, \tag{2}
\]

where \( f_{ij} = \frac{\partial f_i}{\partial y_j} \) and \( f_{i\alpha} = \frac{\partial f_i}{\partial \alpha} \). Here, \( f_{ij} \) and \( f_{i\alpha} \) are evaluated at the equilibrium position \( (y'_1, \ldots, y'_m, \alpha^1) \). Let us rewrite (2) in matrix form, with \( C = [f_{ij}] \); this yields
with the terms in brackets being \( m \times 1 \) vectors.

We assume that a dynamic adjustment process determines the time paths of the variables \( y_1, \ldots, y_m \) when the system is out of equilibrium. The adjustment process may be expressed as

\[
\frac{dy_i}{dt} = d_i \alpha_i (y_1, \ldots, y_m, \alpha^j) \quad \text{for } i = 1, \ldots, m, \tag{4}
\]

where \( t \) is time and \( d_i \), a positive constant for each \( i \), the speed of adjustment for the \( i \)th variable. We assume that in a sufficiently small neighborhood of equilibrium the linear terms of a Taylor series expansion approximate the adjustment process. Thus, (4) becomes

\[
\frac{dy_i}{dt} = d_i \sum_{j=1}^{m} f_{ij} (y_j - y_j^i) \quad \text{for } i = 1, \ldots, m. \tag{5}
\]

Letting \( x_i = y_i - y_i^i \), (5) can be expressed in matrix notation as

\[
x = DCx, \tag{6}
\]

where \( D \) is a diagonal matrix with \( d_i \) in the \( i \)th diagonal position, \( C = [f_{ij}] \), and \( x = [x_i] \).

Stability of the system (6) is a situation wherein, for arbitrary initial values of deviations from equilibrium (in a sufficiently small neighborhood of equilibrium) we have

\[
\lim_{t \to \infty} x_i = 0 \quad \text{for all } i = 1, \ldots, m. \tag{5.5}
\]

The system (6) is stable in this sense if and only if the real parts of all the characteristic roots of \( DC \) are negative.6

II.

Underlying the analysis in this paper is the notion that matrices are stable under certain conditions. The matrices with which we are to be concerned specify qualitative information (+, −, 0) only. Thus, we are interested particularly in the conditions under which a matrix with only sign information specified is stable. The necessary and sufficient conditions for sign stability of an indecomposable real \( m \times m \) matrix \( E \) with elements \( a_{ij} \), where the subscript \( i \) denotes the row and the subscript \( j \) denotes the column of the element in question, are given as
Condition (1): \( a_{ij}a_{ji} \leq 0 \) for \( i \neq j \).

Condition (2): \( i \neq i_2 \neq \ldots \neq i_m, a_{i_1i_2} \neq 0, a_{i_2i_3} \neq 0, \ldots, a_{i_{m-1}i_m} \neq 0 \) implies \( a_{i_1i_2} = 0 \) for any \( m > 2 \).

Condition (3): \( a_{ii} \leq 0 \) for all \( i \), \( a_{kk} < 0 \) for some \( k \).

Condition (4): There exists a non-zero term in the expansion of \( |E|^7 \).

Our fundamental interest in Section III below is with a 2 x 2 indecomposable real matrix. We then offer the following proposition.

**Proposition.** Let \( E \) be a signed indecomposable real 2 x 2 matrix. The sufficient conditions for \( E \) to be sign stable are:

Condition A: \( a_{11}, a_{22} < 0 \).

Condition B: \( |E| > 0 \).

Proof: Given \( a_{11} < 0 \) and \( a_{22} < 0 \), then \( a_{ij} \leq 0 \) for all \( i \) and \( a_{kk} < 0 \) for some \( k \). This implies Condition (3) is satisfied. Given \( |E| > 0 \), Condition (4) is satisfied. Since \( |E| > 0 \) and since this is known only on the basis of qualitative information, clearly \( a_{12}a_{21} \leq 0 \). Thus, Condition (1) is met. Given \( E \) as a 2 x 2 matrix, Condition (2) may be ignored.

III.

We may now consider the stability of an economic system under conditions of a *commodity trap*. Adopt the following notation in addition to that provided in the Introduction:

\[
\begin{align*}
Y_D &= \text{aggregate real demand} \\
Y_S &= \text{aggregate real supply} \\
N_D &= \text{demand for labor} \\
N_S &= \text{supply of labor} \\
N &= \text{level of employment} \\
M_D &= \text{real demand for money} \\
p &= \text{price level} \\
L &= \text{speculative demand for money (in real terms)} \\
M &= \text{nominal money stock} \\
k &= \text{reciprocal of income velocity}^{10} \\
x &= \text{exports (real)} \\
w &= \text{money wage rate} \\
m^* &= \text{real imports}
\end{align*}
\]

In our system, \( w \) is treated as exogenous, while the price level and interest rate are taken as the dependent variables in the model. Our system may be summarized by

\[
E_Y = Y_D - Y_S = C(Y) + I(i,Y) + G + x - m^*(Y) - Y_S(N) \\
E_M = M_D - M/p = kY + L(i) - M/p,
\]
where $E_Y$ is the excess demand for output and $E_M$ is the excess demand for money. Equilibrium is a set of values for the dependent variables such that

$$E_Y = 0$$  
$$E_M = 0.$$  

Setting $E_Y = 0$ and $E_M = 0$, we differentiate (7) totally. This yields

$$\frac{\partial I}{\partial i} + \left( \frac{\partial Y_s}{\partial N} \cdot \frac{\partial N_p}{\partial p} \right) \left( \frac{\partial C}{\partial Y} + \frac{\partial I}{\partial Y} - \left( \frac{\partial M^*}{\partial Y} - 1 \right) \right) \, dp =$$

$$- dw \left( \frac{\partial Y_s}{\partial N} \cdot \frac{\partial N_p}{\partial p} \right) \left( \frac{\partial C}{\partial Y} + \frac{\partial I}{\partial Y} - \left( \frac{\partial M^*}{\partial Y} - 1 \right) \right) - dG - dX$$

A tâtonnement adjustment process determines the time path of the dependent variables. Adjustment equations are given by

$$\frac{dp}{dt} = d_1 \, E_Y$$
$$\frac{di}{dt} = d_2 \, E_M.$$  

Since (10) can be approximated linearly in a sufficiently small neighborhood of equilibrium by a Taylor series expansion, it can be rewritten as

$$\frac{dp}{dt} = d_1 \, a_{11} \, (p - p') + d_1 \, a_{12} \, (i - i')$$
$$\frac{di}{dt} = d_2 \, a_{21} \, (p - p') + d_2 \, a_{22} \, (i - i'),$$

$i'$ and $p'$ being the equilibrium values of $i$ and $p$, respectively. We now let $C = [a_{ij}]$ for $i, j = 1, 2$ and let $D$ be a $2 \times 2$ diagonal matrix with $d_1$ and $d_2$ the diagonal elements. Assume the marginal propensity to spend locally to be positive but less than unity. Then, given $i^{**}$ as the commodity trap rate of interest, we must consider two cases: (1) when $i > i^{**}$, and (2) when $i < i^{**}$. In the first case, $\partial I/\partial i = 0$; in the second case, $\partial I/\partial i < 0$. When $i > i^{**}$ [case (1)], the sign pattern of $DC$ is given by

$$DC = \begin{bmatrix} - & 0 \\ + & - \end{bmatrix}.$$  

When $i < i^{**}$ [case (2)], the sign pattern of $DC$ is

$$DC = \begin{bmatrix} - & - \\ + & - \end{bmatrix}.$$  

Applying our Proposition from Section II to (12) and (13), we find that our economic system — inclusive of the commodity trap — is stable.
Notes

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4 Related to result (2), see Samuelson, op. cit., p. 259, equation (4).

5 That is, \( \lim_{t \to \infty} y_t = y_t^{*} \).

6 Related to this Section, see Ragnar Frisch, "On the Notion of Equilibrium and Disequilibrium", Review of Economic Studies, III (1936), pp. 100–105; Quirk and Ruppert, op. cit., pp. 312–313; or Samuelson, op. cit., pp. 258–263.

7 See Quirk and Ruppert, op. cit., p. 320, for these conditions. The proposition offered below of course follows directly from these conditions.

8 Retaining Condition B and replacing Condition A by

\[ \text{Condition A': } (a_{11} + a_{22}) < 0 \]

yields necessary but not \( a \text{ priori} \) sufficient conditions for \( E \) to be sign stable.

9 See Quirk, op. cit., for the analytical foundation and point of departure for this Section.

10 Alternately, \( \frac{\partial M}{\partial Y} \) may be substituted for \( k \) in the analysis which follows.

11 In equations (11), the values of the coefficients are given by

\[
\begin{align*}
a_{11} &= \left( \frac{\partial Y_1}{\partial N} \cdot \frac{\partial N}{\partial p} \right) \left( \frac{\partial C}{\partial I} + \frac{\partial I}{\partial Y} \cdot \frac{\partial M^*}{\partial Y} - 1 \right) \\
a_{12} &= \left( \frac{\partial I}{\partial I} \right)
\end{align*}
\]
Stabilität unter dem Gesichtspunkt der "Güterfalle"

Zusammenfassung

In diesem Aufsatz werden die Eigenschaften der "Güterfalle" in bezug auf die Stabilität eines Modells der offenen Volkswirtschaft (oder einer Volkswirtschaft mit interregionalem Handel) untersucht. Dieser Spezialfall tritt bei einer geknickten IS-Kurve auf, d.h. die Kurve verläuft vertikal oberhalb eines bestimmten Zinssatzes und fallend unterhalb dieses Zinssatzes. Dabei wird gezeigt, dass die "Güterfalle" keine Instabilität verursacht, sofern die einheimische marginale Konsumneigung, d.h. die gesamte marginale Konsumneigung minus die marginale Konsumneigung für Importe zwischen 0 und 1 beträgt.

Stabilité économique dans le cas d’une "trappe de biens"

Résumé

L'article analyse les implications d’une stabilité macro-économique de la ”trappe de biens”. Dans ce cas spécial, la courbe IS fait un angle: elle est verticale en dessus d’un certain taux d’intérêt et elle tombe en dessous de ce taux d’intérêt. On montre ensuite que l’existence d’une ”trappe de biens” n’est pas une source d’instabilité si la consommation marginale indigène, c’est-à-dire la consommation marginale totale moins la consommation marginale pour les importations, se trouve entre 0 et 1.

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Summary

This paper investigates the implications for macroeconomic stability of the commodity trap, a concept involving a commodity-market-equilibrium (IS) curve which is "kinked", i.e., vertical above a certain rate of interest and negatively sloped below that rate of interest. The investigation considers a macro-system engaging in international (or interregional) trade. It is shown that the commodity trap is not a source of instability, so long as the marginal propensity to spend domestically, i.e., the marginal propensity to spend less the marginal propensity to import, lies between zero and unity.