Recent papers by Ady [1], Fisher and Temin [3], and Labys [4] have provided some interesting insights into the supply of various commodities, particularly in regard to clarifying the dynamic aspects of this type of analysis. In this note it will be claimed that by following the spirit, though somewhat diluting the letter of these expositions, a useful diagrammatic device can be developed.

The fundamental assumption is that (on the basis of some highly visible institutional observations) supply cannot or will not be adjusted instantaneously to changes in demand. In terms of Figure 1, if there is a price change from \( P_0 \) to \( P^* \), supply will not move to \( S^*_t \) immediately, but will only go part of the way.

As shown in the figure, following a change in price from \( P_0 \) to \( P^*_t \), short run supply is \( S_1 \), and over time there is an asymptotic movement to \( S^*_t \). An intermediate supply curve is \( S_1 \), with supply \( S_2 \), and price still constant at \( P_t \). The adjustment equation is obviously \( S_t = S_{t-1} + \lambda (S^*_t - S_{t-1}) \), and the justification for this can be seen if this expression is written as a difference equation, or:

\[
S_t - (1 - \lambda) S_{t-1} = \lambda S^*_t.
\]
This can be solved by the usual devices to give:

\[ S_t = (S_0 - S^*_t) (1 - \lambda)^t + S^*_t. \]  

(2)

Here we obviously have an asymptotic movement to \( S^*_t \), where \( S_{t-1} = S_t = S^*_t \). If we now relate \( S^*_t \) to price by a linear relation (as in Fig. 1) of the type \( S^*_t = a + bP_t^* \), this can be multiplied through by \( \lambda \) and put in (1). This gives:

\[ S_t = \lambda a + (1 - \lambda) S_{t-1} + \lambda bP^*_t. \]  

(3)

This relationship will now be presented employing a diagrammatic device of the type commonly associated with multiplier analysis. This is shown in Figure 2. Note in Figure 2a that the production equilibrium is \( S_{t-1} = S_t = S^* \); the slope of ZZ', which is the coefficient of \( S_{t-1} \), is \( (1 - \lambda) \); and the intercept is a function of \( P^*_t \). As shown here the initial equilibrium price is \( P_o \).

If we now have an increase in price to \( P_1 \) (due, e.g., to an increase in demand), ZZ' shifts up. The new equilibrium is \( S^*_t \), but, as postulated above, this cannot be reached instantaneously. In the first period the increase in production is \( T'T' \), while in the second it is \( T'A \), and so on. We thus have a multiplier sequence consisting of:

\[ \lambda b (P_1 - P_o) [1 + (1 - \lambda) + (1 - \lambda)^2 + ...] = \frac{\lambda b (P_1 - P_o)}{1 - (1 - \lambda)} = b (P_1 - P_o). \]  

(4)

This serves to show that \( \lambda b (P_1 - P_o) \) is equal to \( T'T' \), while \( \lambda (1 - \lambda) (P_1 - P_o) \) is equal to \( T'A \), and so on. It is also interesting to observe that if producers were willing to supply the increase in demand \( \Delta S = S^* - S \) by drawing down inventories, as well as from current production, the amount drawn down would be \( T'T'' \) in the first

![Fig. 2a](image-url)
period, AT" in the second period, and so forth. Obviously this can also be summed in a series of the type shown above.

Similarly an argument might be advanced that production could in fact be increased by TT" (to S") in the present period if consumers were willing to pay a price corresponding to the intercept Z". This means that \( \lambda b (P_x - P_o) = \Delta S \), with \( P_x \) the price in question, or \( P_x = P_o + \Delta S / \lambda b \) if the (essentially) linear assumptions that are being used here continue to hold. Alternatively, this could be the price necessary for producers to provide consumers with the increase \( \Delta S \), with TT' coming from production, and T'T" coming from inventories. Obviously there are many combinations of production and inventory behavior here, both for the present period and over time.

Finally, as a more substantial variation on the above theme, some preliminary results from a large scale investigation of the world copper market will be offered. In the equations below \( S_t \) is the supply of newly mined copper in thousands of metric tons, expressed as an index with 1963 the base year. \( P_t \) is the price of refined copper on the London Metal Exchange, while \( P_{ut} \) is the "American" copper price (computed as an average of several prices); and these prices are also put in index form, with 1963 as base year. D is a dummy which is unity for the years given, and zero otherwise. "t" ratios are in parenthesis.

**Chile**

\[
\log S_t = 0.1236 + 0.7694 \log S_{t-1} + 0.2890 \log P_{t-1} \quad R^2 = 0.920
\]

\[(8.412) \quad (2.670)\]
Canada
\[ \log S_t = 0.21511 + 0.7949 \log S_{t-1} + 0.2365 \log P_{ut} \]
\[ \bar{R}^2 = 0.8114 \]
\[ (6.876) \quad (2.245) \]

Peru
\[ \log S_t = -0.2896 + 0.7240 \log S_{t-1} + 0.542 \log P_t + 0.210 D_s \]
\[ \bar{R}^2 = 0.902 \]
\[ (7.00) \quad (1.851) \quad (2.750) \]

Congo (Kinshasa)
\[ \log S_t = 0.4144 + 0.7204 \log S_{t-1} + 0.1726 \log P_t + 0.031 D \]
\[ \bar{R}^2 = 0.947 \]
\[ (8.854) \quad (3.12) \quad (1.869) \]

United States
\[ \log S_t = 0.9751 + 0.5190 \log S_{t-1} + 0.3100 \log P_{t-1} - 0.02713 D \]
\[ \bar{R}^2 = 0.6501 \]
\[ (3.40) \quad (2.17) \quad (1.00) \]

The above investigations employed annual data for the years 1950—1969. The elasticities calculated from the above equations agreed very closely with elasticities taken from a large number of sources by Takeuchi [6].

It should be admitted, however, that the sort of analysis given here probably holds better for metals than for "soft" commodities, since in the former case changes in price are probably best explained via shifts in demand triggered by changes in an aggregate variable, such as industrial production; and with the low price elasticity common to many metals the changes in price initiating from the producers’ side (in response e.g. to changes in inventories) would not disturb the equilibrium position \( S^* \). But still, in so far as the basic assumptions used in the above exercise hold, Figure 2 can be worked over by almost the entire range of multiplier theory, with the probability that other results useful to the analysis of the commodity markets can be obtained.

Notes
1. The fundamental reference here, of course, is Nerlove [5].
2. The relationship between \( S_t \) and previous prices can be derived in many ways. One of the simplest is to begin with the estimating equation \( S_t = \sigma_0 + \sigma_1 S_{t-1} + \sigma_2 P_{t-1} + u_t \), where \( u_t \) is the disturbance term. If \( S_t \) is lagged and replaces \( S_{t-1} \) in the same expression, the resulting equation is \( S_t = \sigma_0 + \sigma_1 S_{t-2} + \sigma_2 P_{t-2} + u_{t-1} + \sigma_2 P_{t-1} \). Repeated application of this procedure yields:
\[ S_t = \frac{\sigma_0}{1-\sigma_1} + \sigma_2 \sum \sigma_i P_{t-1-i} + v_t. \]
The disturbance term is \( v_t = u_t + \sigma_t u_{t-1} + \sigma_t^2 u_{t-2} + \ldots \), but this can be lagged, multiplied by \( \sigma_t \), and subtracted from \( v_t \) to give \( u_t = v_t - \sigma_t v_{t-1} \). This merely confirms the auto-correlation in \( u_t \) which the estimating procedure (in this case a variant of maximum likelihood) must take into account. As for elasticities, with the estimating equation expressed in logarithmic form we have \( \sigma_2 \) for the short run elasticity and \( \sigma_2 / 1 - \sigma_4 \) for the long run. It should also be apparent that in the introductory analysis of this note little is changed if the term "equilibrium" is replaced by "expected".

With trend results taken from equations of the above sort, as well as detailed investment plans of the major copper producers, I have calculated that supplies of newly mined copper are expanding at a rate of about 6 per cent in the less developed countries and about 4½ per cent in the United States and Canada. Then taking the growth of industrial production in the major consuming countries as 5½ per cent (which is a weighted average) and taking the elasticity of demand with respect to industrial production in these countries at about 0.75 — and given no major changes in the institutional structure of the world copper market — copper prices during the next few years seem almost certain to center around the 50 cents/pound level. Other information relating to these matters can be found in Banks [2].

References


A diagrammatic presentation of dynamic supply behavior

Summary

The purpose of this paper is to present a simple diagrammatic device that can be used in the analysis of the supply side of a commodity market. The mathematical expression relevant to the diagram is derived, and is shown to be similar to the algebraic content of the simple Keynesian model. Finally, some econometric results taken from a preliminary study of the world copper market are given as an illustration of the applicability of the model to an important (commodity) market.
Eine graphische Präsentation von dynamischem Angebotsverhalten

Zusammenfassung

Zweck dieses Artikels ist es, eine einfache diagrammatische Methode darzustellen, die in der Analyse der Angebotsseite eines Warenmarktes verwendet werden kann. Der mathematische Ausdruck zu diesem Diagramm wird abgeleitet und zeigt eine Ähnlichkeit mit dem algebraischen Inhalt des einfachen Modells von Keynes. Schliesslich werden einige ökonometrische Ergebnisse aufgezeigt, die einer vorläufigen Untersuchung des Welt-Kupfermarktes entnommen sind und zur Illustration der Anwendbarkeit des Modells auf einen wichtigen (Waren-)Markt dienen.

Une présentation graphique de la réaction dynamique de l’offre

Résumé

Le but de cet article est de présenter une simple méthode graphique qui peut servir pour l’analyse de l’offre d’un marché de marchandises. L’expression mathématique applicable au diagramme est dérivée, et elle est démontrée être similaire à la contenance algébrique du modèle simple de Keynes. Quelques résultats économétriques, pris d’une étude préliminaire du marché mondial du cuivre, sont finalement présentés pour illustrer l’applicabilité du modèle à un marché de marchandises important.