A Diagrammatic Exposition of the Money Supply Process

By Karl Brunner, Rochester

I. Introduction

Three questions guide the analysis of monetary policy: How to interpret policy or monetary trends, how to adjust policy instruments (open market portfolio, requirement ratios, discount rate, ceiling rates, etc.) within given institutional arrangement, and how to determine and implement appropriate monetary institutions. Adequate answers to these questions are a necessary condition for rational policy and effective monetary control. Adequate answers are in particular a necessary condition for any successful anti-inflationary program. The apparent intractibility of the inflation problem in 1973 has been substantially conditioned by inadequate answers and probably by an inadequate recognition of the questions. This failure is not confined to inflations. The single most important element in the complex of circumstances shaping the Great Depression of the early 1930’s was the systematic misinterpretation of monetary policy and monetary events by the authorities. The importance of the question concerning a reliable interpretation of monetary trends was recognized at the time by a few policymakers on rare occasions. But the majority failed to understand its importance and thus emerged a period of drift and inaction guided by severe misinterpretations of evolving events.

The remaining two questions concern two distinct strategy problems confronting the policymaker. One question bears on the best use of available policy instruments relative to specified objectives within the given institutional arrangements. The other problem addresses on the other hand the structural design of the monetary and financial system and considers in particular the constraints imposed
by policymakers or legislators. It is motivated by the expectation that the usefulness of monetary policy and the effectiveness of monetary controls depend significantly on the institutional arrangements imposed on the system.

The problem of reliable interpretation and the first strategy problem have been extensively discussed over the past ten years\(^1\). This strategy problem has traditionally been analyzed, however, within the framework of a money market model of the money supply process\(^2\). It is not clear at this stage how sensitively the properties of the money market model affected the strategy patterns determined in recent analysis. The policy analysis should thus be suitably extended to an alternative theory of the money supply process. Major progress could be achieved with the demonstration that the basic strategy patterns are quite robust with respect to alternative theories. Moreover, the second strategy problem has barely been examined systematically thus far in the literature. The experience of the past five years confronted Central Banks with various aspects of this strategy problem. The controllability of the money stock was lowered and the inflation problem accentuated by a rigid commitment to fixed exchange rates and a "classic" Central Bank regime supplying base money to a large extent via bank borrowing. Traditional institutions conditioning the interrelations between Central Bank, commercial banks and credit market, reenforced by recent modifications and complications, substantially lower the degree of control over the money stock. An adequate analysis of the second strategy problem thus requires an analysis explicitly addressed to the role of the credit market as the centerpiece of the money supply process. It follows that the money market theory of the money supply process cannot cope sufficiently with these issues. It omits the credit market from its description with an implicit postulate that money substitutes only with financial assets of a similar risk class. The market for these financial assets occurs thus as a mirror image of the money market and requires no further attention. An alternative theory postulates that money substitutes in all directions over the whole spectrum of assets. In particular, it is postulated, that money exhibits substitution relations also with real assets. The hypothesis of a generalized substitution relation

This paper is based on work jointly developed with Allan H. Meltzer. The importance of his indirect contribution should be heavily emphasized. I also wish to acknowledge the financial support extended to the project by a grant from the National Science Foundation.


482
over all assets implies that the credit market assumes significance beyond a mere mirror image of the money market. It emerges with an independent function and requires thus an explicit incorporation in the description of the money supply process.

The present paper discusses a credit market theory of the money supply process which has been extensively used in our previous work. It has been offered as an alternative to the traditional free reserve hypothesis cultivated for many years by the Federal Reserve System and also offers a more relevant alternative, in our judgement, to the money market theory recently adopted by the Federal Reserve System. The current paper elaborates our analysis with the aid of a detailed diagrammatic exposition. It is hoped that this exposition may clarify some major issues bearing on the nature of the transmission mechanism, the role of institutional arrangements and the relevance of "reverse causation". Lastly, I also hope that the diagrammatic presentation contributes to a better understanding of the Brunner-Meltzer analysis of monetary processes.

II. The Description of the Model

The description proceeds with two major simplifications. The present paper is confined to a closed economy. This simplification is introduced with a full recognition of the importance of interdependent credit markets in many open economies. Still, the simplification offers some tactical advantages. It is advisable to examine first the basic structure in the absence of interdependent credit markets. This is particularly useful as major issues in recent controversies do not depend on the openness of an economy. Moreover, extensions of the analysis to subsume interdependent credit markets benefit substantially from a clarification of the basic structure. The nature of the generalization and the specific consequences of interdependent credit markets become more transparent in this manner. The reader should only be cautioned to suspend far-reaching policy conclusion bearing on the controllability of the money stock in an open economy from the present paper. Two additional papers will develop the required analytic and diagrammatic extensions for two distinct types of interdependent credit markets in open economies. One will examine interdependent credit markets with imperfect substitutability between securities denominated in different currencies and issued in different countries. Another paper considers interdependent credit markets involving perfect substitutability between such securities.

The second simplification pertains to the essentially generic nature of the presentation. It is not addressed immediately to the money supply process evolving within the specific institutions of a particular country. The paper offers in this respect essentially a schema which can be, and actually has been implemented in
constructions of specific empirical hypothesis bearing on a particular money supply process. The schema offers a framework allowing the institutional specifications necessary for suitable applications. This property of the schema accounts for its usefulness and relevance.

The framework introduced to the reader jointly determines money stock, banks' total earning assets ("bank credit"), the market rate of interest and the market value of real capital. These magnitudes emerge from a process shaped by the behavior of the monetary authorities supplying base money, influenced by the behavior of the banks absorbing and allocating assets and setting conditions on their liabilities, and also affected by the behavior of the public supplying assets to banks, demanding money and allocating assets between various types of liabilities offered by the financial system. This programmatic description can be implemented at various levels. The presentation begins immediately at a relatively aggregative level to prevent an inordinate length. Background information and higher level optimizing theories explaining some of the macro patterns are relegated to suitably detailed chapters of a later monograph.

1. The Multiplicative Decomposition of the Banks' Portfolio Absorption

The Banks total portfolio absorption $I^2$ is introduced as the product of an asset multiplier $a$ and the monetary base $UB$ adjusted for bank borrowing (unborrowed monetary base). We write thus

$$I^2 = a(i, P, d + h, c, ...) \cdot UB$$

$$a_1, a_2, a_3 > 0 > a_3.$$  

The source component of the unborrowed base $UB$ consist of the cumulated sum of "liberated reserves", the Central Bank’s claim against the government sector, claims on the private non-banking sector or assets acquired from this sector. No foreign reserves need enter our simplified world of a closed economy. The cumulated sum of "liberated reserves" can be expressed as the negative value of an integral with an integrand equal to the product of the time rate of change in

$^3$ The programmatic description and the subsequent analysis demonstrate that a frequently encountered juxtaposition of "multiplier analysis" and an "asset preference" analysis misses completely the relevant issues. The subsequent multiplier analysis is explicitly based on "asset preferences". The advantage of the multiplier analysis developed along the lines indicated in this paper over standard alternatives is best recognized by comparison with the Tobin-Brainard procedure. The multiplier formulation permits much tighter constraints substantially beyond the forest of signs in a Jacobian. It produces thus empirical hypotheses with substantial content, which, moreover, do not require construction de novo after minor institutional changes.
requirement ratios multiplied with the appropriate deposit volume. The asset multiplier $a$ is defined by an expression determined by the specification of the unborrowed base

$$a = \frac{(1 + t + d) \ [1 + n - (r - b)]}{(r + t - b) \ (1 + t + d) + k}$$

where $k = \text{the public's currency ratio}$, $t = \text{the public's time deposit ratio}$, $d = \text{the Treasury's deposit ratio}$, $r = \text{the banks reserve ratio}$, $b = \text{the banks' borrowing ratio}$, $n = \text{the bank's non-deposit liability ratio}$, and $t = \text{the ratio of cumulated liberated reserves to total bank deposits}$. The asset multiplier is thus exhibited as a rational function of specific allocation parameters characterizing relevant behavior aspects of banks and public. The currency ratio $k$, the time deposit ratio $t$ and the non-deposit liability ratio (all defined relative to demand deposits held by the public) describe the public's allocation or "asset preference" patterns affecting the money supply process. The banks' behavior is described by the reserve ratio $r$, the borrowing ratio $b$ and the conditions governing the issue of liabilities to the public. Lastly, the Treasury deposit ratio $d$ reflects the Treasury's behavior bearing on its pattern of payments and receipts.

For each allocation ratio we should distinguish explicitly a "desired" and an actual value. The term "desired" should not be burdened with any particular interpretations of volition, calculated deliberation or conscious and deliberate planning. It means nothing more than a magnitude responsive in one form or another to specified stimuli. These stimuli consist in general of suitable wealth variables and appropriate yields and costs. Some of the behavior responses also reflect relevant constraints. This holds in particular for the desired reserve ratio $r^*$

$$r^* = g(i, rrd, rrt, ...)$$

$$\varepsilon(g, i) \leq 0; l \geq \varepsilon(g, rrd) > \varepsilon(g, rrt) > 0$$

which depends on the interest rate $i$ with a negative elasticity and the reserve requirements $rrd$ or $rrt$ on demand and time deposits with a positive elasticity. A complete analysis bearing on the behavior of the reserve ratio involves arguments beyond reserve requirements and interest rates. We omit, however, a detailed description of the reserve ratio. Such a description is unavoidable in the context of a definite empirical analysis of a specific money supply process. Moreover, the

---

4 The public's actual allocation ratios are measured by the following expressions:

$$k = \frac{CP}{DP}, \quad t = \frac{TP}{DP}, \quad n = \frac{N}{TD}$$

$CP = \text{currency held by public}$, $TP = \text{time deposits held by public}$, $DP = \text{demand deposits held by public}$, $N = \text{non-deposit liabilities held by the public}$, and $TD = \text{total deposits}$. Note also that $\varepsilon(y, x)$ denotes the elasticity of $y$ with respect to $x$. 
description of the banks' desired reserve behavior can be anchored in a higher level theory explaining the banks optimal reserve inventories in the face of stochastic reserve flows. But this is unnecessary for present purposes and the potential relevance of the analysis does not depend on inclusion or omission of these underlying micro-foundations.

The banks’ desired borrowing ratio \( b^* \) depends foremost on the interest rate \( i \) and the total cost of borrowing \( d + h \), represented by the

\[
b^* = \beta(i, d + h); \epsilon(\beta, i) > 0 > \epsilon(\beta, d + h)
\] (4)

sum of a discount rate and a measure \( h \) of the administrative cost of harrassment imposed on borrowing banks by the Central Bank. The second component \( h \) usually does not occur in European monetary systems. The US banks were exposed on the other hand to \( h \) for many years. The magnitude \( h \) seems to depend essentially on the difference between market rates and discount rate. The operation of \( h \) explains the occasionally low discount rates relative to Federal funds rate and other market rates associated with comparatively moderate levels of bank borrowing.

The public's desired time deposit and non-deposit liability ratios \( t^* \) and \( n^* \) depend mostly on wealth and interest rates, in particular the rates offered on these instruments (\( i_t \) on time and \( i_n \) on non-deposit liabilities), the market rate \( i \), and the market value \( P \) of real capital. We write thus

\[
t^* = t^*(i_t, i, P, W^t, W^n)
\] (5)

\[
t^*_t, t^*_s, t^*_x, t^*_y > 0 > t^*_z
\]

\[
n^* = n^*(i_n, i, P, W^n, W^n)
\] (6)

\[
n^*_t, n^*_s, n^*_x, n^*_y, n^*_z > 0 > n^*_w.
\]

The own rates on time deposits and non-deposit liabilities exhibit positive elasticities [i.e., \( \epsilon(t^*, i_t) > 0 < \epsilon(n^*, i_n) \)].

The market rate \( i \) occurs with a negative elasticity, whereas \( P \) appears again with a positive elasticity. The latter reflects the circumstance that an increase in \( P \), holding all other variables constant, is equivalent to a fall in the real rate of return on real capital. The resulting substitution raises \( t^* \) and \( n^* \). Both human wealth \( W_h \) and non-human wealth \( W^n \) affect the allocation ratios positively.

---

The costs and yields which affect the public's desired currency ratio $k^*$ exhibit in the USA little association with current credit market processes. Some European systems on the other hand show a negative dependence of $k^*$ on the interest rate it on time deposits. This dependence reenforces somewhat the feedback from credit market processes via the allocation parameters on the asset multiplier. The desired currency ratio $k^*$ depends in general on wealth and the relative costs of holding or using demand deposits and currency. Increased social mobility increases for instances the information cost attached to the use of check payments and thus lowers the relative cost of using currency. Similarly, computerization of checking transfers at retail outlets lowers the relative cost of using demand deposits. We postulate thus

$$k^* = x (i, uc, ud, hc, hd, W^h, W^n)$$

(7)

$\varepsilon(k, i) \sim 0$ in USA and negative in some other countries

$$\varepsilon(x, uc), \varepsilon(x, hc) < 0 < \varepsilon(x, ud), \varepsilon(x, hd)$$

where $uc$ and $ud$ denote the user cost of currency and deposit, $hc$ and $hd$ designate the respective holding costs. The Treasury deposits ratio will not be further explained and occurs as a policy parameter. This need not be so, but the marginal productivity of explaining this parameter appears to be small for horizons extending beyond a quarter.

The characterization of the behavior patterns underlying the asset multiplier is terminated with the introduction of the banks' supply behavior of liabilities

$$it = h^1(i, c); h^1_1 > 0 \leq h^1_2$$

(8)

$$in = h^2(i, ...); h^2_1 > 0$$

(9)

where $c$ denotes a ceiling rate imposed on time deposits. The occurrence of $c$ implies that $\varepsilon(it, i)$, the response of it to $i$, converges to zero as $i$ increases relative to a fixed $c$. This implies furthermore a reversal in the sign of the total interest elasticities of $t^*$. This result affects some patterns in our subsequent diagrammatic exposition bearing on the analysis of ceiling rates and requires an explicit larification at this stage. The total elasticity of $t^*$ denoted with a bar above $\varepsilon$, i.e., $\bar{\varepsilon}$ with respect to $i$ can thus be presented as follows

$$\bar{\varepsilon}(t, i) = \varepsilon(t, it) \cdot \varepsilon(it, i) + \varepsilon(t, i).$$

The first term is positive and the second is negative. At comparatively low rates the first term prevails and $\bar{\varepsilon}(t, i) > 0$. At relatively high rates (relative to $c$) the first term vanishes and $\bar{\varepsilon}(t, i)$ approximates $\varepsilon(t, i) < 0$. A more extensive analysis penetrating beyond the immediate association with the current credit market processes would also acknowledge the cost effect of reserve requirements on the behavior functions 8 and 9. Larger (marginal) reserve requirements increase the
wedge between yields on earning assets (summarized by \( i \)) and the yield offered on liabilities. A raise in reserve requirement on these particular liabilities thus lowers (in the sense of a partial derivative) the respective yields offered by banks on time deposits or non-deposits liabilities\(^6\).

Completion of the link between the assetmultiplier introduced in equation (1) and the desired allocation ratios describing the banks’ and the public’s behavior patterns require one more analytical step. This step also connects the assetmultiplier function in equation (1) with the assetmultiplier in equation (2). The allocation ratios occurring in expression (2) introducing the assetmultiplier are actual ratios. The expression stated in equation (2) is thus purely analytic and as such, devoid of empirical content. On the other hand, the desired allocation ratios are, per se, purely theoretic terms with no directly observable counterpart. The linkage is assured by means of suitable correspondence sentences connecting actual (observable) and theoretic terms. The simplest form of such sentences is used for our purposes. They occur in the form of equilibrium conditions which assert the equality of actual and desired ratios. These equilibrium conditions impose important constraints on the joint covariation of observable magnitudes and introduce the necessary empirical content. They imply in conjunction with equation (2) the assetmultiplier function occurring in equation (1). The assetmultiplier can thus be exhibited as a function of \( i \) and \( P \) and other variables operating via the desired allocation parameters on actual values and the multiplier \( a \), once the equilibrium conditions \( k = k^*, n = n^*, b = b^* \), etc., are specified\(^7\).

The occurrence of the sum \((r + \ell)\) in the multiplier still requires some explanation. Its occurrence is a necessary consequence of the decision to include the cumulated sum of “liberated” reserves into the base UB. The composition of the

---

\(^6\) Demand deposits suffer in the USA a ceiling of zero. No interest payments are admitted. Benjamin Klein argued however that banks replace interest payment with provisions of valuable services to deposit owners and submitted evidence attesting the dependence of this implicit deposit yield on current market condition. Incorporation of this service yield or an explicit pecuniary yield offered by many European banks involves only a minor modification. We can add a function \( y = h^3(i, rrd, ...) \), with \( h^3_1 > 0 > h^3_2 \), and let \( y \) denote an implicit or explicit yield depending on the institutional context. The explicit formulation of the demand for money and the description of \( k^* \) would thus also include \( y \). The possibly negative dependence of \( k^* \) on market rates mediated via \( y \) would be reenforced by a dependence operating also via \( y \) on demand deposits. Moreover, the interest elasticity of money demand considered in our subsequent discussion appears in the nature of a total elasticity jointly reflecting a direct (partial) effect and an indirect effect via \( y \). We obtain thus \( e(A, i) = e(\lambda, y) \cdot e(y, i) + e(\lambda, i) \). The first term is positive and the second term is negative. The usual formulation of the interest elasticity of money demand is thus a net elasticity numerically smaller and algebraically larger than the partial interest elasticity \( e(\lambda, i) < 0 \).

\(^7\) The reader should be cautioned that the relevance of an analysis involving equilibrium conditions cannot be judged by prior declarations to the effect “whether or not the real world is in equilibrium”. Such statements involve a serious logical confusion. They seem to use statements of the object language and to refer to real phenomena. This is however not the case. They are thoroughly elliptical statements of the meta language about a theory formulated in the object language. The general situation can be described as follows: We have first the metalinguistic sentence,
base and the constitution of the multiplier cannot be determined independently once the magnitude to be addressed (in this case total earning assets of banks) has been fixed. Every choice of the base involves thus implicitly a different decomposition of the relative changes in total earning assets between relative changes of the multiplier and relative changes of the base. Each decomposition is equally correct analytically speaking, but not equally useful to organize a comprehension of our phenomenon. The particular decomposition chosen in our work satisfies a specific requirement. It allows a more effective separation between two major groups of influences affecting the money supply process. The comparative role of monetary authorities, or of public and banks formed the subject of many controversies over the past years. Proponents of a "reverse causation" attribute the observed correlation between income and money to the public's money demand or loan demand, or to investors' "asset preferences" bearing on the choice between market instruments and time deposits, or lastly to the banks adjustment in their reserve positions in response to varying market pressure. Monetarists emphasize on the other side the behavior of the Central Bank and the role of its policy conception governing its standard responses to evolving circumstances.

These are serious issues and the correct answers yield important implications for policymaking. It is thus useful to construct a framework which decomposes the money stock in the most informative manner into two components. One reflects completely the behavior of the authorities and the other describes approximately (and dominantly) the variations due to the public's and banks' behavior. Our formulation assures this purpose. The unborrowed base essentially results from the Central Banks behavior, irrespective whether or not this behavior and the associated actions be subsumed under "policy". The monetary multiplier on the other hand mirrors dominantly the public's and the bank's behavior. The required separation was achieved by means of the term \( r + \epsilon \) in the multiplier and the associated inclusion of the cumulated sum of "liberated" reserves into UB. Changes in requirement ratios thus necessarily modify UB but do not change the sum \( r + \epsilon \). An increase in the requirement ratios raises \( r \) and lowers \( \epsilon \) with a matching magnitude. It follows that the monetary multiplier does not respond to variations in requirement ratios and this policy action of Central Banks is completely channeled via the base UB. We noted above, however, that the multiplier reflects in contrast to the base approximately or dominantly the public's and the banks' behavior. The

"Theory T in language L has a property E". It should be noted that the property E is a property of T in L and not of the extra-linguistic "real world". Whenever T is a well accepted theory, elliptical usage transfers the property E to the world. But such transfers frequently yield a confusing use of language. Furthermore, if one objects to T one frequently hears the announcement "the world does not have property E", when the correct statement is "T in L with property E must be rejected". But this is an assertion about the relative cognitive status of T and requires suitable evidential support. No immediate impression can tell us whether a theory containing equilibrium conditions is empirically unsustainable. Immediate impressions also indicate that the sun rotates around the earth.
reader should be cautioned that discount policy, expressed both explicitly by variations in \(d\) or implicitly by variations in \(h\), operates via the borrowing ratio on the multiplier. Similarly, changes in the ceiling rate \(c\) on interest payments on time deposits work their effects via the time deposit ratio on the multiplier. Still, we may state that in the absence of changes in borrowing costs \((d + h)\) and with constant ceiling rate \(c\) variations in the multiplier completely result from the public's and the bank's behavior.

The decomposition yields moreover another useful result. An inspection of our subsequent response patterns demonstrates the pervasive occurrence of various elasticities of the assetmultiplier (and the monetary multiplier \(m\) to be introduced subsequently). The reader may examine the interest elasticity \(\varepsilon(a, i)\) of the assetmultiplier \(a\) as an important example. It is defined as a linear combination of five interest elasticities associated with the allocation parameter \(r^*, b^*, n^*,\) and \(t^*\). We obtain thus

\[
\varepsilon(a, i) = \varepsilon(a, k^*)\varepsilon(k^*, i) + \varepsilon(a, r^*)\varepsilon(r^*, i) + \varepsilon(a, b^*)\varepsilon(b^*, i) + \varepsilon(a, t^*)\varepsilon(t^*, i) + \varepsilon(a, n^*)\varepsilon(n^*, i).
\]

The coefficients of this combination are all elasticities of the assetmultiplier with respect to a specific allocation parameter. Most fortunately, a detailed study of these elasticities for the US monetary system demonstrates that the larger valued elasticities [e.g., \(\varepsilon(a, r)\) and \(\varepsilon(a, t)\)] exhibit over the postwar period a coefficient of variation for most years less than 1%. Substantial coefficients of variation are confined to elasticities with very small values. These patterns can be usefully exploited to establish empirical regularities. We observe in particular for the US case that the first term of the expression defining \(\varepsilon(a, i)\) vanishes [because \(\varepsilon(k, i) \approx 0\)]. Moreover, in low interest regimes the second term dominates the remainder. The other terms reinforce this effect somewhat and the interest elasticity of the assetmultiplier is substantially positive. In high interest periods the second term vanishes and the fourth term turns negative. This negative value is offset by positive values of the third and last term. The result is a numerically small interest elasticity \(\varepsilon(a, i)\) in high interest regimes essentially due to the ceiling rate and the nature of US discount policy. Under some European regimes the sign of \(\varepsilon(t, i)\) is not reversed at high interest rates and \(\varepsilon(a, b)\) increases by a large margin above the US counterpart value. The interest elasticity of the assetmultiplier produced by a "European Central Bank" regime exceeds thus by a large multiple the corresponding elasticity for the US monetary system.

The reader should carefully note that the approximate elimination of the authorities behavior from the multiplier only holds for the monetary multiplier introduced in section 4. The numerator of the asset multiplier \(a\) contains \(r\) but not the sum \(r + t\). The latter occurs however in the denominator. It follows that a change in requirement ratios modifies the assetmultiplier but not the monetary multiplier. Changes in requirement ratios thus induce larger absolute and percentage changes in "bank credit" than in the money stock.
2. The Public's Asset Supply Function

The banks' absorption of assets expressed by their portfolio $a$ and UB is juxtaposed to the (stock) supply of assets by the public to the banks. This asset supply function $\sigma$ forms an additional behavior pattern of the public entering our description of the money supply process. This stock supply is interpreted as the sum of the public's desired loan liability position $L^p$ and the public's implicit (stock) supply of securities to the banks. The stock supply $\Sigma$ can thus be written as follows

$$\Sigma = L^p + S^g + S^p_1 + I^p - \delta$$

(10)

where $S^g$ is the outstanding stock of government securities outside the government sector; $S^p_1$ is the stock of privately issued securities one period before. $I^p$ designates the new issues of private securities in the current period; and lastly, $\delta$ denotes the private sector's desired stock of securities, i.e., the private sector's desired portfolio absorption of securities. The last four terms (i.e., $S^g + S^p_1 + I^p - \delta$) thus describe the stock of securities not absorbed by private portfolios and thus forming an implicit supply to the banks. All terms of the expression defining $\Sigma$ with the exception of $S^g$ and $S^p_1$ depend on current credit market conditions and with the exception of $I^p$ also on the public's human and non-human wealth.

Our analysis proceeds on a thoroughly aggregative level. This does not imply that allocative aspects involving the distribution between loans and securities are irrelevant. The analysis is based, however, on the postulate that these allocative patterns are irrelevant with respect to aggregative patterns and the average level of interest rates. This is simply an empirical assertion consistent with elementary price theory and not an Ontological statement about the "True Structure of the Real World". The basic postulate is certainly consistent with the proposition that the allocative aspects disregarded in our analysis are relevant with respect to allocative issues. The reader is reminded, however, that this paper is not concerned with allocative aspects in the banking sector beyond the partition of total assets between reserves and earning assets or the issue of the distinct liabilities listed. The terms on the right side of equation (10) are summarized for our purposes by a function $\sigma$ with the arguments introduced in equation (11)

$$\Sigma = \sigma(i \pi, p, ap, P, S^g + S^p_1, e, W^n, W^h)$$

(11)

$\sigma_1, \sigma_2 < \sigma_3, \sigma_4, \sigma_5, \sigma_6$

There do occur issues in monetary policy which involves allocative aspects. The ceiling on loan expansion or loan portfolios forms a case in point. It will be shown in another paper on "The Money Supply Process in an Open Economy: The Case of Interdependent Creditmarkets with Imperfect Substitutability", that our postulate of "aggregative irrelevance of allocative aspects" can be analytically justified in terms of a disaggregated model.
where \( \pi \) is the anticipated rate of inflation, \( p \) the level of current output prices, \( a_p \) is the *anticipated* output price level, \( P \) the market value of a unit of real capital, and \( e \) is the anticipated net real return on capital per unit of real capital\(^9\). The terms \( W^n \) and \( W^h \) describing human and human wealth respectively were previously introduced. Non-human wealth is defined by the expression

\[
W^n = PK + v(i, \tau^n)S^\varepsilon + (1 + \omega)UB
\]

where \( K \) is the stock of real capital, \( v \) the market value per unit of security, \( \tau^n \) a parameter summarizing the tax schedule on income from non-human wealth; lastly, \( \omega \) refers to the banking system's net worth multiplier. It is easily demonstrable that \( W^n \) is identical with the public's net worth. Human wealth depends on anticipated output \( a_y \), the anticipated price level \( a_p \) and a tax parameter \( \tau^h \) summarizing the tax schedule on income from human wealth. This formulation presupposes some pattern of regularity concerning the distribution of real income between labor and real capital ownership. The elasticities \( e(\sigma, W^h) \) and \( e(\sigma, W^n) \) are constituted by two offsetting components. An increase in either wealth component raises the public's loan liability position \( L_p \) and also the public's demand for securities \( S^\delta \). The wealth elasticities of \( \sigma \) are thus constrained to a negligible magnitude. The reader should also note that the wealth terms were not explicitly listed in the multiplier function. Both asset and monetary multiplier depend on both wealth terms via currency ratio \( k \) and time deposit ratio \( t \). In either case the effect via \( t \) dominates the effect via \( k \). The specification of the two multipliers implies a relatively larger (algebraically and numerically) responsiveness of the assetmultiplier to wealth. In particular, the wealth elasticities of the monetary multiplier remain negligible.

The two building blocks introduced thus far are depicted in the left side panel of graph 1A. The horizontal axis measures the volume of bank earning assets (i.e., \( \Sigma \) or \( \Pi \)) and the vertical axis is scaled by the interest rate \( i \). The \( \Sigma \)-line represents the public's asset supply and the \( \Pi \)-line the banks' asset absorption. The slopes of the two lines reflect the interest elasticities of the assetmultiplier \( a \) and of the asset-supply \( \sigma \). The position of the \( \Pi \) line changes with modifications of the base UB and other non-interest arguments affecting \( \Pi \). Similarly, the position of the \( \Sigma \)-line changes with variations in the stock of securities, the price-level \( p \), the anticipated price-level \( a_p \) or the anticipated real net yield \( e \) on real capital. The reader should note that the curvature of the \( \Pi \)-line reflects the operation of the ceiling rate which

\(^9\) The reader will note that \( \sigma \) depends on the real rate \( (i - \pi) \) but not the asset multiplier. This follows from the fact that the banks range of asset choice essentially excludes real assets (with some qualification) and that the public's allocation parameters \( k, t, \) and \( n \) involve assets with identical position relative to the inflation risk.
adjusts the sign of the interest elasticity \( \varepsilon(a, i) \) according to the position of the market rate \( i \) relative to the ceiling rate.

3. The Creditmarket and the CM Line

We possess at this stage the elements for the first equation of our description of the system's assetmarkets. Equation (12) describes the creditmarket and the proximate determination of the interest rate \( i \) by the interaction of the banks portfolio absorption and the public's asset supply.

\[
a(i, P, e, ...). UB = \sigma(i - \pi, p, ap, P, S^g + S^p, e, ...).
\]

This equation is represented in Graph 1 by the CM curve in the right side panel. Equation (12) determines an implicit function between \( i \) and \( P \). This function is depicted in Graph 1. The position of the curve depends on \( UB, p, ap, S^g + S^p, e \) and \( \pi \). The subsequent discussion reveals the nature of this dependence. At this stage we direct the reader's attention to the slope properties of the CM curve. These slope properties operate with a crucial significance on the system's response patterns and thus require some explicit attention. Suitable derivation from equation (12) yields immediately the slope of CM expressed as elasticity \( \varepsilon(i, P/CM) \):

\[
\varepsilon(i, \frac{P}{CM}) = -\frac{\varepsilon(CM, P)}{\varepsilon(CM, i)} < 0.
\]

The terms forming the ratio are defined as follows

\[
\varepsilon(CM, P) = \frac{d}{d \log P} \log \frac{aUB}{\sigma} = \varepsilon(a, P) - \varepsilon(\sigma, P) > 0
\]

\[
\varepsilon(CM, i) = \frac{d}{d \log i} \log \frac{aUB}{\sigma} = \varepsilon(a, i) - \varepsilon(\sigma, i) > 0.
\]

These definitions have the following economic interpretation: \( \varepsilon(CM, P) \) exhibits the assetprice elasticity of the excess supply of bank credit on the creditmarket and \( \varepsilon(CM, i) \) expresses the interest elasticity of the same excess supply. Both elasticities are by construction positive. Moreover, it is postulated that the assetprice elasticity is less than the interest elasticity, i.e., \( 0 < \varepsilon(CM, P) < \varepsilon(CM, i) \). It follows that all along the CM curve its elasticity is less than unity in absolute value.
4. The Multiplicative Decomposition of the Money Stock

The two components of the second assetmarket equation required for our analysis involve expressions describing the money stock generated by the monetary system and the money demand. Equation (11) introduces the description of the money stock $M$ as the

$$M = m \cdot UB \quad (14)$$

product of a monetary multiplier and the unborrowed monetary base $\cdot UB$. The monetary multiplier is defined according to equation (15)

$$m = \frac{1 + k}{(r + t - b)(1 + t + d) + k}. \quad (15)$$

It appears again as a rational function of allocation parameters. The reader should note that both multipliers $a$ and $m$ possess the same denominator. Nevertheless, the two multipliers are very different functions of the allocation ratios. A comparison of elasticities with respect to $x = r, b, k, t, d, n$ demonstrates this difference. The elasticities of $m$ with respect to $r, b, k,$ and $d$ are numerically smaller than the corresponding elasticities for $a$. Moreover, the elasticities with respect to $t$ and $d$ are positive for $a$ and negative for $m$. Lastly, variations in $n$ affect $a$, but not $m$. It follows that cyclic variations of the underlying behavior parameters generate very distinct movements of the two multipliers. These differences between the two multipliers also explain the more sensitive exposure of earning assets to influences from the real sector in the US system. The reader should note that earlier remarks concerning the relation between actual and “desired” values of the allocation parameters established by means of equilibrium conditions extend to the present case. The conjunction of (15) and these equilibrium conditions thus implies that the monetary multiplier can be written as a function of the arguments explaining the desired allocation parameters. We write thus

$$m = m(i, P, e, ...). \quad (16)$$

The expression $M = m(i, P, e, ...) \cdot UB$ is represented by the $M$ curve in Graph 1B in the left side panel. It should be noted that the curvature depends substantially on the Central Bank regime instituted in a particular country. This dependence is most usefully demonstrated by means of the interest elasticity of $m$ described in (17).

$$\varepsilon(m, i) = \varepsilon(m, k) \cdot \varepsilon(k, i) + \varepsilon(m, r) \cdot \varepsilon(r, i) + \varepsilon(m, b) \cdot \varepsilon(b, i) + \varepsilon(m, t) \cdot \varepsilon(t, i). \quad (17)$$
The elasticity $\varepsilon(k, i)$ vanishes usually in the absence of special institutions which offer attractive liabilities to lower income and smaller business groups. The elasticity $\varepsilon(r, i)$ in the second term depends critically on the banking structure and the banks' accessibility to credit markets involving very low transaction and information costs. The third term depends particularly on prevalent Central Bank practices. With a «classic European regime» relying largely on the supply of base money via bank borrowing in one form or another the elasticity $\varepsilon(m, b)$ is a large multiple of its values found in the US-British system. The supply of base money via bank borrowing contributed in the USA since the 1920's a very modest portion of total base money issued. The last term depends substantially on the operation of ceiling rates. Without such constraints the last term is negative and contributes to lower $\varepsilon(m, i)$ below $\varepsilon(a, i)$. With ceiling rates imposed the last term is positive whenever $i$ is sufficiently large relative to $c$. Moreover, the magnitude of the last term is conditioned by the degree of competition prevailing in the banking system. This degree affects the speed of adjusting interest rates on time deposits to changing market conditions. A larger speed lowers the positive value of the total interest elasticity in the absence of ceiling rates.

5. The Demand for Money

The $\lambda$-curve in the left side panel of 1B represents the demand for money expressed by equation (18)

$$M = \lambda(i, p, ap, P, e, W^n, W^h)$$

(18)

the coefficients $\lambda_1, \lambda_3, \lambda_5 < 0 < \lambda_2, \lambda_4, \lambda_6, \lambda_7$.

Money demand depends positively on output price level $p$ and the asset price level $p$, and negatively on interest rate, anticipated price level and the anticipated real net yield on real capita. The reader should note that money demand $\lambda$ and asset supply $\sigma$ exhibit very different properties and form thus independent behavior functions.

6. The Money Market and the MM Line

Equation (19) describes the equilibrium condition of the money market and defines a second implicit function involving $i$ and $P$. This implicit function is represented by the MM curve in the right side panels of Graphs 1A and 1B. The slope of the curve corresponds to the elasticity $\varepsilon(i, P | MM)$ defined by equation (19a) $m(i, P, e ... ) \cdot UB = \varepsilon(i, P, e, W^n, W^h)$

$$\varepsilon(i, P | MM) = - \frac{\varepsilon(MM, P)}{\varepsilon(MM, i)} > 0$$

(19a)
where
\[ \varepsilon (\text{MM}, P) = \frac{d}{d \log P} \log \frac{mB}{\lambda} = \varepsilon (m, P) - \varepsilon (\lambda, P) < 0 \]
\[ \varepsilon (\text{MM}, i) = \frac{d}{d \log i} \log \frac{mB}{\lambda} = \varepsilon (m, i) - \varepsilon (\lambda, i) > 0. \]

The bar above \( \varepsilon \) attached to the \( \lambda \)-elasticities indicates a total elasticity, i.e., it includes the effect of variations in \( i \) or \( P \) via non-human wealth \( W^n \). The wealth channel reenforces the “direct effect”. It is postulated that \( |\varepsilon (\text{MM}, P) \geq \varepsilon (\text{MM}, i) > 0 \) and most particularly that the interest elasticities of the credit market exceed the interest elasticities of the money market, i.e., \( \varepsilon (\text{MM}, i) < \varepsilon (\text{CM}, i) \). This inequality forms a fundamental order condition of the major results developed in the subsequent analysis. The conditions stated imply that the magnitude of the slope elasticities of \( \text{CM} \) exceed the corresponding slope elasticities of \( \text{MM} \) for any \( i, P \) combinations.

**Graph 1A**

**Graph 1B**
III. The Working of the System

The right side panels in Graph 1 describe the joint determination of \( i \) and \( P \) by the interaction of the two assetmarkets. This interaction is described by the slopes and positions of the two curves. The representation of the two curves as implicit functions defined by the two assetmarket equations implies that the intersection between the two lines in the left side panels occurs necessarily on the horizontal line fixed by the intersection in the right side panel. Once the asset prices are determined, the horizontal line through the intersection point on the right is determined and we require actually only one curve on the left to determine the money stock and the volume of bank credit. The diagram can thus be used to trace the response of the monetary aggregates \( M \) and \( E \), and of the assetprices \( i \) and \( P \), to the behavior of the monetary authorities, or to impulses emitted by the real sector of the economy, or to the behavior of the public.

1. The Response to a Change in the Unborrowed Base

An increase of UB modifies the position of both curves, CM and MM. The change in position is expressed by an elasticity measuring the vertical shift per unit percentage change of UB. The vertical shifts for both curves are noted in equation (20)

\[
a. \quad \varepsilon(i, UB|CM) = - \frac{1}{\varepsilon(CM, i)} < 0
\]

\[
b. \quad \varepsilon(i, UB|MM) = - \frac{1}{\varepsilon(MM, i)} < 0.
\]
An increase in UB thus lowers both curves. The fundamental order conditions imply that the vertical shift of the MM curve induced by a change in UB exceed the vertical shift of CM. This condition assures us that an increase in UB simultaneously lowers i and raises P. An expansion of the base thus conveys via the adjustments of assetmarkets an unambiguously expansive impulse to the output market. The result can be described by means of Graph 2A.

A unit percentage increase in UB lowers CM by a and MM by \((a + b) > a\). The reader notices thus that the intersection point is moved into a new position to the southeast: i falls and P rises\(^\text{10}\).

The associated change in the money stock M can be determined in two ways. The reader is referred to Graph 2B. It can first be considered as a change of the intersection in the left side panel between the horizontal line fixed by the intersection in the right side panel and the \(\lambda\)-curve. The new intersection emerges from two distinct movements: The horizontal line slides downwards along the \(\lambda\)-curve to the left, and secondly, the increase in P noted on the right side panel shifts the \(\lambda\)-curve.

\(^{10}\) It should be noted that the shift elasticities listed in the text are actually approximations. The correct numerators are:

\[
1 + [\varepsilon(\sigma, W^n) - \varepsilon(a, W^n)] \frac{(1 + \omega)UB}{W^n}; \text{ for the CM curve,}
\]

\[
1 + [\varepsilon(\lambda, W^n) - \varepsilon(m, W^n)] \frac{(1 + \omega)UB}{W^n}; \text{ for the MM curve.}
\]

The two second terms of the complete numerator expressions are proportional to the small magnitude \(\frac{(1+\omega)UB}{W^n}\). Moreover, \(\varepsilon(\sigma, W^n) < \varepsilon(\lambda, W^n) < 1\) and \(\varepsilon(m, W^n) < 0 < \varepsilon(a, W^n)\). This ordering reenforces slightly the effect of the fundamental order condition on the relative shift elasticities.
on the left outward by the magnitude $\varepsilon(\lambda, P) \cdot \varepsilon(P, UB)$. This outward shift of $\lambda$ is measured by $d$ in the graph. The horizontal shift $d$ is thus measured by the product of the assetprice elasticity of money demand and the relative change in $P$ induced by the change in $UB$.

This presentation suggests on a first impression the correctness of an assertion that the major issues of monetary theory can be reduced to the question concerning the magnitude of the interest elasticity of money demand. We noted however that the expansive impulse mediated via the response of $i$ and $P$ does not depend on the absolute magnitude of $\varepsilon(\lambda, i)$. This interest elasticity is indeed the dominant component in the total elasticity $\varepsilon(MM, i)$. But the result depends essentially on the relative magnitude of the shifts in $CM$ and $MM$ and this relative magnitude is independent of any particular absolute magnitude of $\varepsilon(\lambda, i)$.

The role of $\lambda$ can apparently be disregarded with a second description of the response of the money stock. This description decomposes the total change in the money stock in a different manner. The first component of the total change results from a horizontal shift of the $M$ curve along the old horizontal line fixed by the initial equilibrium point in the right side panel. This horizontal shift is determined by the expression

$$[1 + \varepsilon(m, P) \cdot \varepsilon(P, UB)]m$$

measuring the displacement of the $M$ curve. The second component describes a movement downwards along the shifted $M$ curve. This second component is expressed by $\varepsilon(m, i) \cdot \varepsilon(i, UB) \cdot m$. This expression and the second term in the bracketed expression above are negative. The first component is denoted by $e$ and the second as $f$ in the graph. The total change is thus $e-f$. The specifications for $e$ and $f$ clearly imply that the total change in $M$ per dollar change in $UB$ is less than $m$. An examination of $\varepsilon(m, i)$ and $\varepsilon(m, P)$ for US conditions establishes however that these elasticities are of comparatively small order. The relative change of the money stock can thus be expressed as $1 + \varepsilon(m, P) \cdot \varepsilon(P, UB) + \varepsilon(m, i) \cdot \varepsilon(i, UB)$. This magnitude is less than unity.

We obtain thus two descriptions of the change in the money stock: One concentrates on slope and position of the $M$ curve and the other on slope and position of the $\lambda$-curve. The two descriptions are, however, necessarily equivalent. The joint interaction of the two assetmarkets determines the following equality

$$1 + \varepsilon(m, P) \cdot \varepsilon(P, UB) + \varepsilon(m, i) \cdot \varepsilon(i, UB) = \varepsilon(\lambda, i) \cdot \varepsilon(i, UB)$$

(21)

The analysis thus assigns no major or central position to the properties of the public's money demand in the determination of either $i$, $P$ or the response of the money stock. The equivalence of the two expressions describing the response of the
money stock also establishes that the elasticity of M with respect to UB is
unambiguously positive. A rearrangement of equation (21) exhibits that a linear
combination of the response elasticities $\varepsilon(i, UB)$ and $\varepsilon(P, UB)$ with coefficients
$[\varepsilon(\lambda, i) - \varepsilon(m, i)] < 0$ and $[\varepsilon(\lambda, P) - \varepsilon(m, P)] > 0$ is unity:

$\varepsilon(i, UB)[\varepsilon(\lambda, i) - \varepsilon(m, i)] + \varepsilon(P, UB)[\varepsilon(\lambda, P) - \varepsilon(m, P)] = 1.$

If one uses the available estimates of the interest elasticities the value $|\varepsilon(\lambda, i) - \varepsilon(m, i)|$ is at most .5. Moreover, homogeneity of first degree in nominal values for
$\lambda$ implies that $[\varepsilon(\lambda, P) - \varepsilon(m, P)] < 1.$ It follows thus that at least one response
elasticity must exceed unity.

2. The Response to a Change in Discount Policy

A change in the discount rate $d$ or the administrator cost of harassment $h$ also
modifies the position of the CM and MM curves. Equation (22) and (23) represent
the respective vertical shifts of the two curves:

a. $\varepsilon(i, d|CM) = -\frac{\varepsilon(a, b)}{\varepsilon(CM, i)} \cdot \varepsilon(b, d+h) \frac{d}{d+h} > 0 \quad (22)$

b. $\varepsilon(i, d|MM) = -\frac{\varepsilon(m, b)}{\varepsilon(MM, i)} \cdot \varepsilon(b, d+h) \frac{d}{d+h} > 0. \quad (23)$

a. $\varepsilon(i, h|CM) = \varepsilon(i, d|CM) \frac{h}{d} > 0$

b. $\varepsilon(i, h|MM) = \varepsilon(i, d|MM) \frac{h}{d} > 0.$

An increase in either component of borrowing costs thus pushes both curves
upwards into new position. The net effect on the assetmarkets depends on the
relative magnitude of the borrowing ratio elasticities of the two multipliers. It is
easily established that

$\varepsilon(a, b) = \varepsilon(m, b) + \frac{b}{1+n-(r-b)} > \varepsilon(m, b) > 0$

with $\varepsilon(m, b) = \frac{b(1+t+d)}{(r+t-b)(1+t+d)+k}.$
indicate that the vertical shifts are proportional to $d(d + h)^{-1}$ and thus increase as $h$ converges to zero. On the other hand, a regime relying on administrative costs of harassment usually constrains the supply of base money via bank borrowing to a comparatively small trickle and thus lowers $b$ to a small fraction of the values associated with the other regime. This implies comparatively small values of $a(a, b)$ and $e(m, b)$ in the US regime. It is moreover unlikely that $h$ exceeds $d$ and thus unlikely that the proportionality factor $h(d + h)^{-1}$ exceeds one half. This compares with a proportionality factor $d(d + h)^{-1}$ of unity under the other regime. The US regime thus produces both low values of $d(d + h)^{-1}$ or $h(d + h)^{-1}$ and also lower values for the $b$-elasticities of the two multipliers. It follows that discount policy has been constrained under a US regime to comparative ineffectiveness. It is also noteworthy that the comparative effectiveness of discount policy under the “classic regime” depends however on a transmission channel essentially neglected by and not subsumable under a Keynesian framework.

The net effect on the money stock is traced in Graph 3 B.

The increase in borrowing costs moves the intersection point of M and $\lambda$ to the northeast from A to B. Once again, this can be described in two non-identical but logically equivalent ways. The increase in the market rate of interest $i$ contracts the money stock according to the movement upwards along the initial $\lambda$-curve to the new horizontal equilibrium line. This component reflects the transmission via the interest channel. The transmission via the P-channel is expressed by the horizontal shift of $\lambda$ to the right along the new equilibrium line. The alternative description is
The (necessary and sufficient) conditions for the shift $e(i, d|CM)$ to be less than the shift $e(i, d|MM)$ can now be stated as follows:

$$\frac{e(CM, i)}{e(MM, i)} > \frac{1+a}{a},$$

(24)

where $a$ is the asset multiplier.

The term on the right side is approximately 1.2 in the USA. It moves nearer to unity as the role of borrowing in the supply of base money increases. The tentative evidence bearing on the comparative interest elasticities in the credit market and the money market supports thus far the inequality (24) with a large margin.

We consider now the effect of $d$ (or $h$) on the asset markets with the aid of Graphs 3A and 3B.

This diagram clearly reveals that inequality (24) is both necessary and sufficient for an increase in $d$ (or $h$) to impart a uniformly deflationary effect on the output market. The new intersection point $I_1$ moves to the northwest of the old intersection $I_0$. The interest rate rises and the asset price falls. The deflationary effect via the P-channel clearly depends on the relative interest elasticities $e(CM, i)$, $e(MM, i)$, and also on the borrowing ratio $b$. The larger $e(CM, i)$ relative to $e(MM, i)$ and also the larger $b$, the further to the northwest the new intersection point will be pushed. The required conditions for a more substantial effect transmitted via the P-channel thus exist in a "classic European" Central Bank regime assigning a large role to bank borrowing in the supply of base money. This result is reinforced in the "classic system" with respect to both transmission channels (via $i$ and $P$) by the absence of administrative costs of harassments (i.e., $h = 0$). The expressions (22)
somewhat more complicated. This may be noted from the equivalence condition of the two descriptions stated in equation (22).

\[
\varepsilon(m, d + h) \frac{d}{d + h} + \varepsilon(m, P) \cdot \varepsilon(P, d) + \varepsilon(m, i) \cdot \varepsilon(i, d) = \varepsilon(\lambda, i) \cdot \varepsilon(i, d) + \varepsilon(\lambda, P) \cdot \varepsilon(P, d).
\]

The description centered on slope and position of the M curve occurs on the left side of (25). The reader should note that the sum of the first two terms describes the horizontal shift of the M curve induced by discount policy action. The sign of this shift is ambiguous, the first term is negative and the second term positive. The last term on the left describes the movement upwards along the M curve to the new equilibrium line and this term is positive. The net result on the left side is however uniquely determined by the unambiguous result on the right. Every term on the right is negative. We obtain thus a contraction of the money stock, which differs in magnitude under the two distinct Central Bank regimes.

3. The Response to a Change in the Ceiling Rate

The vertical shift of CM and MM associated with a change in c is determined by the expression (23)

a. \( \varepsilon(i, c|CM) = - \frac{\varepsilon(a, t)}{\varepsilon(CM, i)} \cdot \varepsilon(t, it) \cdot \varepsilon(it, c) \leq 0 \) (26)

b. \( \varepsilon(i, c|MM) = - \frac{\varepsilon(m, t)}{\varepsilon(MM, i)} \cdot \varepsilon(t, it) \cdot \varepsilon(it, c) \geq 0. \)

The magnitude of the shifts is proportional to \( \varepsilon(t, it) \cdot \varepsilon(it, c) \), i.e., the elasticity of the time deposit ratio with respect to the ceiling rate c. This expression vanishes if i is sufficiently large relative to c and c is not lowered substantially to close the gap between it and c. On the other hand, with the value of it close to c and market rates substantially above c, an increase in c raises it with an elasticity not deviating much from unity. A reduction of c under these circumstances also affects t. The time deposit ratio would fall. Policy directed to the ceiling rate thus affects the monetary system and the economy only in circumstances of sufficiently large i relative to c.

The two curves are moved into opposite directions by an increase (a relaxation) of the ceiling rate c. The CM curve is pushed down and the MM curve moves up. These movements unambiguously lower the assetprice level P. The two shifts reenforce each other in this respect. The net effect on interest rate i depends on the other hand on the relative magnitude of the shifts.
It is useful to consider for this purpose the horizontal shifts

\[ \varepsilon(P, c|CM) = - \frac{\varepsilon(a, t)}{\varepsilon(CM, P)} \cdot \varepsilon(t, it) \cdot \varepsilon(it, c) < 0 \]

\[ \varepsilon(P, c|MM) = - \frac{\varepsilon(m, t)}{\varepsilon(MM, P)} \cdot \varepsilon(t, it) \cdot \varepsilon(it, c) < 0. \]

Both expressions are negative and the order conditions imply that \( \varepsilon(P, c|CM) < \varepsilon(P, c|MM) < 0 \). The interest rate thus unambiguously fall. The outcome is depicted in Graph 4 A.

Both i and P have been lowered with probably a negligible net deflationary effect transmitted to the output market.

The reader should not that the result is quite sensitive with respect to the measure of the money stock used. The previous analysis incorporated the exclusive money stock. The results are substantially reversed by using instead the inclusive money stock. Both CM and MM exhibit under the circumstances negative shift patterns. The reader may easily establish that i is unambiguously (and reenforcingly) lowered by the two shifts associated with an increase in c, whereas the net effect on P becomes dependent on the relative magnitude of the shifts. With an MM shift sufficiently large P actually rises. The analysis of the ceiling rate thus reveals that the choice between an exclusive and an inclusive measure of the money stock involves substantive issues associated with alternative hypotheses concerning underlying behavior patterns. It should also be noted however that this problem emerges essentially only for the case of a ceiling rate policy. This policy
offers thus a good example of arrangements which raise the level of information required in the pursuit of given policy objectives. It follows that in view of the marginal social cost of information acquisition reliance on a ceiling rate policy yields probably a suboptimal policy arrangement for the monetary authorities.

The effect of a ceiling rate increase on the money stock still requires examination. We use for this purpose the equivalence statement of the two distinct descriptions for the response of the money stock.

\[
\epsilon(m, t) \cdot \epsilon(t^*, it) \cdot \epsilon(i^*, t^* c) + \epsilon(m^*, i^*) \cdot \epsilon(i^*, c) + \epsilon(m^*, P^*) \\
\cdot \epsilon(P^*, c) = \epsilon(\lambda^*, i) \cdot \epsilon(\lambda, t^*) + \epsilon(\lambda^*, P^*) \cdot \epsilon(P^*, c).
\]

The reader will notice that both sides contain terms with opposite signs. This is another indication of the information requirement imposed by the use of a ceiling rate. It is probable however that the net effect on the money stock remains quite small with uncertain value. The small order of the net effect emerges from the sum of comparatively small terms with opposite signs. This situation is not radically changed for the inclusive money stock. In the latter case the first term on the left is positive but the last term is negative.

4. The Response to a Change in the Stock Supply of Government Securities \(S^*\)

An increase in \(S^*\) shifts the CM curve upwards according to the expression

\[
\epsilon(i, S^* | CM) = \frac{\epsilon(\sigma, S^*)}{\epsilon(CM, i)} > 0.
\]

The MM curve is affected by very little and is thus approximately left constant. We also note for the USA that \(\epsilon(o, S^*) \sim 1\) and we obtain thus

\[
\epsilon(i, S^* | CM) \sim - \epsilon(i, UB | CM).
\]

Graph 5A depicts the effect on the assetmarkets.

We note that both \(i\) and \(P\) increase. An expansion of \(S^*\) thus emits offsetting impulses on the output market. It is shown at another occasion however that a stable process requires that the \(P\)-effect dominates the \(i\)-effect. It follows that an increase in \(S^*\) yields net expansionary impulses on the economy\(^{11}\). These impulses are comparatively weak, however.

\(^{11}\) The reader may consult "Money, Debt and Economic Activity" by Karl Brunner and Allan Meltzer, Journal of Political Economy, August 1972, and by the same authors, "The Inflation Problem", forthcoming.
The effect on the money stock is traced by Graph 5 B. The M line is shifted to the right by the amount \( \varepsilon(m, P) \cdot \varepsilon(P, S^e) < 0 \) and the \( \lambda \)-curve to the left according to \( \varepsilon(\lambda, P) \cdot \varepsilon(P, S^e) > 0 \). As \(|\varepsilon(m, P)| < \varepsilon(\lambda, P)| \) it follows that the \( \lambda \)-curve moves by a
larger distance than the M curve. It is quite probable therefore that variations in $S^g$ exert negligible effects on the money stock. This consideration is reenforced by the fact that the response of the money stock can be represented as the sum

$$\varepsilon(m, i) \cdot \varepsilon(i, S^g) + \varepsilon(m, P) \cdot \varepsilon(P, S^g)$$

of small terms with opposite sign. It is noteworthy that the comparatively weak effect exerted by $S^g$ on the economy is associated with a negligible or inconclusive effect on the money stock.

5. The Response to Changes in the Public's Currency Behavior

The data of several countries (particularly Germany and England) show variations in the currency ratio $k$ which are difficult to associate with current creditmarket processes or with feedbacks from the real sector. They seem to involve major "autonomous" adjustments in the public's behavior. Variations in $k$ are thus worth our examination. An increase in $k$ modifies both CM and MM according to expression (29)

a. $\varepsilon(i, k|CM) = -\frac{\varepsilon(a, k)}{\varepsilon(CM, i)} > 0$ (29)

b. $\varepsilon(i, k|MM) = -\frac{\varepsilon(m, k)}{\varepsilon(MM, i)} > 0$.

Graph 6 A
A necessary and sufficient condition for the MM shift to exceed the CM shift is offered by the inequality

\[ \frac{\varepsilon(MM, i)}{\varepsilon(CM, i)} < \frac{\varepsilon(m, k)}{\varepsilon(a, k)} \sim \frac{3}{5}. \]

This condition is satisfied with \( \varepsilon(MM, i) \) at most one half of \( \varepsilon(CM, i) \). The result of a comparatively larger MM shift induced by an increase in \( k \) is traced in Graph 6A.

The relative magnitude of the two shifts assures a dislocation of the intersection point to the northwest. Interest rate \( i \) rises and the assetprice level \( P \) falls. An increase in \( k \) unleashes thus unambiguous deflationary impulses on the economy. A comparatively larger shift of the CM curve on the other hand produces conflicting impulses via the assetprices on financial assets and real capital. A relatively steep MM curve converts under these circumstances variations of \( k \) into very large changes of \( i \) accompanied by small changes in the same direction of \( P \).

The response of the money stock is depicted in Graph 6B.

An unambiguous result is obtained by using the description centered on slope and position of the \( \lambda \)-curve. It is shifted to the right according to \( \varepsilon(\lambda, P) \cdot (P, k) < 0 \). It follows that the money stock declines. The alternative description yields a shift of the M curve by the amount

\[ \varepsilon(m, k) + \varepsilon(m, P) \cdot \varepsilon(P, k) \]
with a supplementary motion upwards along the curve represented by $\varepsilon(m, i) \cdot \varepsilon(i, k) > 0$. It follows from the equivalence of the two description that $\varepsilon(m, k)$ dominates the effect of $k$ on the multiplier $m$ mediated via the assetmarket responses. This supports our contention that $\varepsilon(m, P)$ and $\varepsilon(m, i)$ are comparatively small magnitudes yielding a steep $M$ curve and small shifts in $M$ via changes in $P$.

6. Responses to Some Major Arguments in the $\sigma$ and $\lambda$ Function

It is argued on occasion that the public's behavior expressed by money demand $\lambda$ and asset supply $\sigma$ dominate the movements of the money stock. In particular, it is argued that the observable high correlation between the money stock and income reflects essentially, or to a large extent, a reverse causation from income to the money stock. Some major channels of such a reverse causation would operate via the $\lambda$ and $\sigma$ function. It should be noted that within the analysis presented income affects money demand and asset supply via three arguments, the anticipated real net yield on real capital $e$, human wealth $W^h$ and the price-level $p$. All three arguments of $\lambda$ and $\sigma$ depend on real income $y$. The dependence of $p$ on $y$ is governed by a price setting function $p(y, K, w)$ where $K$ is real capital and $w$ efficiency money wages.

The dependence of $W^h$ and $e$ on $y$ is based on the postulate of a stable distribution pattern of income between labor and capital. This distribution implies that $\varepsilon(e, y)$ is large and $\varepsilon(W^h, y)$ small at low levels of activity with the reverse ordering at high activity levels. This pattern is conditioned, however, on previous accelerations and decelerations of activity. The analysis of reverse causation involves thus three distinct arguments of the $\sigma$ and $\lambda$ function which are first examined separately.

We consider first the price-level $p$. An increase in $p$ modifies CM and MM according to expression (30)

\begin{align*}
a. \varepsilon(i, p|CM) = \frac{\varepsilon(\sigma, p)}{\varepsilon(CM, i)} > 0 \\
b. \varepsilon(i, p|MM) = \frac{\varepsilon(\lambda, p)}{\varepsilon(MM, i)} > 0.
\end{align*}

The simultaneous raise in the position of both curves unambiguously increases the market rate $i$. The upward shift of each curve reenforces the effect of the other on $i$. The net effect on $P$ depends, however, crucially on the relative magnitude of the two shifts. It is assumed that $\varepsilon(\lambda, p) \sim \varepsilon(\sigma, p)$. It follows thus that the MM shift exceeds the CM shift. The Graph 7A traces under the circumstances a northwest-
erly shift of the intersection point. An increase of the output price-level thus imparts a deflationary impulse to the output market via the adjustments of the asset market.

*Graph 7 A*

The net effect on the money stock is traced in *Graph 7 B*.
An unambiguous answer is derived in this case by means of the description centered on slope and position of the M curve. The total change in $M$ is expressed by

$$
\varepsilon(m, P) \cdot \varepsilon(P, p) + \varepsilon(m, i) \cdot \varepsilon(i, p) > 0.
$$

The first term describes the outward shift of the curve along the old equilibrium line and the second term measures the effect of moving up along the sloping M curve to the new equilibrium line. We conclude thus that an increase in $p$ raises the money stock. The alternative and equivalent description exhibits the change in the money stock as follows

$$
\varepsilon(\lambda, p) + \varepsilon(\lambda, i) \cdot \varepsilon(i, p) + \varepsilon(\lambda, p) \cdot \varepsilon(P, p).
$$

The components of this description reveal offsetting effects absent in the other descriptions based on the M curve. The latter implies that the first term in the description based on money demand dominates the sum. The reader should note that we derived in this case an **expansion** of the money stock associated with a **deflationary** impulse on the economy. The reader should also note, however, that the increase in $M$ is of small order due to the smallness of $\varepsilon(m, P)$ and $\varepsilon(m, i)$ already noted in previous sections.

Consider now the increase in human wealth $W^h$. We already noted that wealth induces offsetting effects on the public's asset supply $\sigma$. The elasticity $\varepsilon(\sigma, W^h)$ remains thus quite small relative to $\varepsilon(\lambda, W^h)$. We simplify thus by assuming $\varepsilon(\sigma, W^h) = 0$. An increase in $W^h$ affects thus only the MM curve. It's position is shifted by the amount

$$
\frac{\varepsilon(\lambda, W^h)}{\varepsilon(MM, i)} > 0
$$

and the intersection moves to the northwest. It follows that $i$ increases and $P$ falls. The rise of $W^h$ thus unleashes via the adjustments of the assetmarkets a deflationary impulse to the output market. The M description of the effect on the money stock determines also unambiguously that the money stock expands (very) slightly in response to an increase in $W^h$. The magnitude of the response depends decisively on the nature of the Central Bank regime. It will remain small in a US type regime.

We examine lastly the anticipated real net yield $e$ on real capital. A change in $e$ shifts CM and MM in the following manner

\begin{align*}
a. \quad & \varepsilon(i, e|CM) = \frac{\varepsilon(\sigma, e)}{\varepsilon(CM, i)} > 0 \\
b. \quad & \varepsilon(i, e|MM) = \frac{\varepsilon(\lambda, e)}{\varepsilon(MM, i)} < 0.
\end{align*}
CM thus moves up and MM down in response to an increase in e. The asset price level P thus rises unambiguously. Each shift reinforces the other in this respect. The effect on i depends on the other hand on the relative magnitude of the shifts. Additional information may be extracted in this case by means of the horizontal shifts induced by e:

\[ \varepsilon(P, e|CM) = \frac{\varepsilon(\sigma, e)}{\varepsilon(CM, P)} > 0 \]

\[ \varepsilon(P, e|MM) = \frac{\varepsilon(\lambda, e)}{\varepsilon(MM, P)} > 0. \]

With \( \varepsilon(\sigma, e) \sim |\varepsilon(\lambda, e)| \) and \( \varepsilon(CM, P) < |\varepsilon(MM, P)| \) the CM-shift exceeds the MM shift. Graph 8A describes under the circumstances the resulting increase in both i and P.

Whenever the sensitivity of the public's money demand to changes in e exceeds sufficiently the corresponding sensitivity of \( \sigma \), an increase in e lowers i and raises P and thus imparts a pronounced expansionary effect via assetmarkets to the output market. But even with \( \varepsilon(\lambda, e) \) not dominating \( \varepsilon(\sigma, e) \) the rise in i remains comparatively small relative to the increase in P and there remains an expansionary net impulse.

Both descriptions of the effect on the money stock contain offsetting terms. The M description

\[ \varepsilon(m^+, i) \cdot \varepsilon(i^+, e) + \varepsilon(m^-, P) \cdot \varepsilon(P^+, e) + \varepsilon(m^-, t) \cdot \varepsilon(t^-, e) \]
suggests however the occurrence of a negligible effect on $M$. The expression is a difference between terms of small order relative to $\varepsilon (i, e)$ and $\varepsilon (P, e)$ due to the comparative smallness of the multiplier elasticities.

The building blocks necessary for a discussion of reverse causation are available at this stage. An increase in real income $y$ modifies the two curves according to the following expressions

$$
e(i, y | CM) = \frac{\varepsilon (\sigma, y)}{\varepsilon (CM, i)}$$

$$
e(i, y | MM) = \frac{\varepsilon (\lambda, y)}{\varepsilon (MM, y)}$$

where

$$
\varepsilon (\sigma, y) = \varepsilon (\sigma_p^+p) \cdot \varepsilon (p^+_y) + \varepsilon (\sigma_e^+e) \cdot \varepsilon (e^+_y) > 0
$$

$$
\varepsilon (\lambda, y) = \varepsilon (\lambda_p^+p) \cdot \varepsilon (p^+_y) + \varepsilon (\lambda_{Wh}^+Wh^+ y) \cdot \varepsilon (Wh^+_y) + \varepsilon (\lambda^- e) \cdot \varepsilon (e^+_y) \leq 0.
$$

The CM shift is definitely positive. It also rises with increasing utilization rate of real capital. This follows from the fact that $\varepsilon (p, y)$ rises monotonically as utilization rates increase. This pattern combined with the postulate concerning the relative order of $\varepsilon (Wh, y)$ and $\varepsilon (e, y)$ implies that $\varepsilon (\lambda, y)$ is negative at low levels of activity and turns positive at high levels of activity. Eventually, as rates of utilization increase $\varepsilon (\sigma, y)$ and $\varepsilon (\lambda, y)$ converge. Both are dominated in the limit by the first term. We obtain thus the following implication for the assetmarket adjustments. At low levels of activity an increase of $y$ pushes CM up and MM down. The result is actually approximated under the circumstances by the results associated with an increase of $e$. "Reverse causation" thus initiates expansionary feedbacks on the output market via the adjustments imposed on the assetmarkets. Moreover, "reverse causation" induces probably an expansion of the money stock but of the slightest magnitude in a US type Central Bank regime. "Reverse causation" would be more significant in a "classic" regime.

At sufficiently high levels of utilization rates the two shifts are approximated by the first term in the expressions defining $\varepsilon (\sigma, y)$ and $\varepsilon (\lambda, y)$. But this implies that the shifts associated with reverse causation are approximately described by changes in $p$ and the first terms. "Reverse causation" at high rates of utilization thus unleashes a deflationary feedback on the output market. It also induces an expansion of the money stock which remains, however, at a small order in a US type regime. We conclude thus that US type Central Bank regimes preclude a major role of reverse causation in the determination of the money stock. It follows that reverse causation contributes little to the observed correlation between money stock and income. This pattern changes, however, under a "classic" Central Bank regime. This difference between a US and
"European" monetary regime offers thus opportunities to appraise the hypothesis of a substantial role of reverse causation in the observed correlations between money stock and income. A substantial role of reverse causation would imply systematic differences in the correlations observed according to the regime governing the money supply process.

7. The Effect of Inflation

The emergence of inflationary anticipations operates on asset markets and money stock via three distinct channels. Two channels involve impacts of inflation velocity and a third describes the effect of inflation acceleration. The effect of inflation velocity is conveyed by changes in the purchasers anticipated price level \( \alpha_p \) and the suppliers anticipated price level \( \phi \). The latter magnitude affects via the operation of the labor market the money wage \( w \) occurring as an argument in the price setting function explaining \( p \). Changes in \( \phi \) thus modify \( p \), and our previous analysis of \( p \) immediately extends to increases in the suppliers anticipated price level \( \phi \). The adjustments imposed on asset markets convey a deflationary impulse to the output market combined with a negligible change of \( M \) in contexts of US type regimes.

The effect of an increase in the purchasers anticipated price level \( \alpha_p \) still requires a separate analysis. The shifts of the two curves are indicated by expression

\[
a. \, \epsilon(i, \alpha_p | CM) = \frac{\epsilon(\sigma, \alpha_p)}{\epsilon(CM, i)} > 0
\]

\[
b. \, \epsilon(i, \alpha_p | MM) = \frac{\epsilon(\lambda, \alpha_p)}{\epsilon(MM, i)} < 0.
\]

\( CM \) is raised and \( MM \) lowered. The effect on \( P \) is immediately obvious. Each shift reinforces the expansive effect of the other. In order to clarify the effect on \( i \) it is more convenient to work with the horizontal shifts

\[
\epsilon(P, \alpha_p | CM) = \frac{\epsilon(\sigma, \alpha_p)}{\epsilon(CM, P)} > 0
\]

\[
\epsilon(P, \alpha_p | MM) = \frac{\epsilon(\lambda, \alpha_p)}{\epsilon(MM, P)} > 0.
\]
The horizontal shifts are both positive. With $|\varepsilon(\lambda, ap)| \leq \varepsilon(\sigma, ap)$ the CM shift exceeds the MM shift. Graph 9A reveals in this case a simultaneous increase of both $i$ and $P$. The reader will also notice that with $\varepsilon(\lambda, ap)$ sufficiently dominant over $\varepsilon(\sigma, ap)$ $i$ falls. This case is probably excluded by the two components with opposite signs defining the total elasticity $\bar{\varepsilon}(\lambda, ap)$. This elasticity is determined by the expression

$$\bar{\varepsilon}(\lambda, ap) = \varepsilon(\lambda, ap) + \varepsilon(\lambda, W^n) \cdot \varepsilon(W^n, ap) > \varepsilon(\lambda, ap).$$

The net impulse of an increase in $ap$ on the output market conveyed by the assetmarket adjustments remains most probably positive. Moreover, analysis of the effect on the money stock based on the M curve description establishes that $ap$ exerts in a “non-classic” system a vanishing effect on $M$.

The effects of an inflation acceleration are conveyed by the creditmarkets’ anticipated rate of inflation $\pi$. An increase in $\pi$ modifies the CM curve according to the following expression without affecting the MM curve.

$$\varepsilon(i, \pi | CM) = - \varepsilon(i, \pi) \frac{\pi}{i - \pi} > 0.$$  

An increase in $\pi$ thus raises $i$ and $P$ simultaneously and conveys offsetting impulses to the output market. The net effect on the money stock results from two conflicting impulses via $i$ and $P$ on the monetary multiplier. It follows once again that $M$ is little affected in a “non-classic” regime by accelerations in the creditmar-
ket's inflationary anticipations. We conclude thus that neither inflation velocity nor inflation acceleration exerts a significant effect on the money stock in the context of a US type Central Bank regime. Moreover, adjustments of assetmarkets induced by inflation velocity convey dominantly a deflationary or an expansionary impulse to the output market depending on the purchasers and suppliers relative speed in revising anticipated price-levels. Whenever revisions of inflationary anticipations made by purchasers dominate the corresponding revisions made by suppliers the net impulse emitted by inflation velocity is expansionary. The net impulse is contractive with respect to output in the opposite case. Revisions of inflation anticipations dominated by suppliers thus generate the much commented stagflation pattern observed on occasion.

8. The Consequences of an Interest Target Policy

Many Central Banks traditionally guided their adjustments of requirement ratios or portfolios in the light of evolving creditmarket conditions. Interest rates were usually selected as an immediate target controlling the adjustments of available policy instruments. This section of the paper thus explores the consequences of a traditional interest target policy. This exploration requires a specific modification in our previous analysis. The interest rate $i$ enters now as a predetermined variable and the unborrowed base is converted into an endogeneous variable.

We examine first simply the consequences of a change in the target. Suppose specifically that the monetary authorities raise the targeted level of interest rates. Graph 10A demonstrates that both CM and MM must be pushed up sufficiently by suitable reductions of UB until they intersect on the new target line. The new

![Graph 10A](image-url)
intersection point also implies a lower \( P \)-value. This can be derived by sliding first the MM curve leftwards into position \( MM_1 \) until its intersection with the new target line is vertically above the initial intersection with the old target line. At this position the CM line will have moved into position \( CM_1 \), involving a smaller shift according to our fundamental order condition. The new intersection between CM and MM at \( CM_1 \) and \( MM_1 \) is thus to the left and northwest of the old intersection, i.e., \( P \) has a lower value. But this new intersection is still below the target line. Further reductions of UB and additional shifts of CM and MM are required until the lines reach positions \( CM_2 \) and \( MM_2 \). But following our fundamental order conditions the equilibrium point moves necessarily further to the northwest and \( P \) falls still further. An increase of the targeted interest level thus involves also a lower value of the assetprice level.

The effect on the money stock can be traced by means of Graph 10B. The new intersection is immediately determined by shifting the \( \lambda \)-curve to the right according to the amount \( \varepsilon(\lambda, P) \cdot \varepsilon(P, i) \frac{di}{i} < 0 \). The reduction of the money stock can again be partitioned into a component expressing the rightward shift of \( \lambda \) along the old target line and a component expressing the movement along the new \( \lambda \) curve upwards to the new target line. The increase of the targeted interest rate clearly lowers the money stock. But the reader should note that this reduction involves more than interest elasticities of money demand. It depends also substantially on the mechanism centered on the asset price \( P \) of real capital. This channel combined with the effect mediated via the adjustment in UB is reflected by the shift of the M curve to the right.
We examine now the impact of changes in the stock $S^*$ of government securities, the currency ratio $k$, the output price level $p$, the anticipated real net yield $e$ on real capital and also of real income. The case of an increase in $S^*$ is pictured in Graph 11A.

**Graph 11A**

In the absence of an interest target policy expressed by the horizontal target line, interest rate $i$ rises from the intersection of $CM_0$ and $MM_0$ to the intersection of $MM_0$ and $CM_1$. The imposition of the target level forces however an expansion of

**Graph 11B**

In the absence of an interest target policy expressed by the horizontal target line, interest rate $i$ rises from the intersection of $CM_0$ and $MM_0$ to the intersection of $MM_0$ and $CM_1$. The imposition of the target level forces however an expansion of
the unborrowed base UB which lowers both CM and MM to an intersection point on the target line. According to the fundamental order condition the intersection between CM₂ and MM₁ on the target line is to the right of the intersection between CM₁ and MM₀. An "accommodating monetary policy" thus converts an increase of Sₘ into a larger increase in P without any increase in i compared to the effect in the absence of an interest target policy. This policy magnifies thus the impulses emitted by variations in Sₘ. This destabilizing effect of the interest target policy is also mirrored by the response of the money stock shown in Graph IIB.

With free adjustment of interest rates the intersection point is shifted from A to B involving a small change in the money stock. With an interest target policy the larger increase in P shifts λ further to the left along a lower horizontal line and the intersection point settles at C. The money stock thus expands by a substantially larger amount. The larger response in M correlates with the larger impulse produced by variations in Sₘ under an interest target policy.

Variations in the currency ratio generate a radically different pattern. An increase in k pushes both CM and MM upwards. Our previous exploration suggested that CM shifts by less than MM (for US proportions). The intersection B produced by the shifting lines at the positions CM₁ and MM₁ determined by the increase in k is clearly above the target line. The authorities must, therefore, expand UB to move the intersection back to the target line.

The reader should remember that CM moved to CM₁ according to the shift elasticity

\[
\frac{-e(a, k)}{e(MM, i)}
\]

and MM to MM₁ according to the shift

\[
\frac{-e(m, k)}{e(MM, i)}.
\]

Let us denote the first expression by CM(k) and the second by MM(k). The shift elasticities governing the adjustment of the lines to an intersection on the target line in response to an increase in UB are given by

\[
\frac{MM(k)}{-e(m, k)} \text{ for MM₁, } \quad \frac{CM(k)}{-e(a, k)} \text{ for CM.}
\]

Our argument still requires the well established inequality \(|e(a, k)| > |e(m, k)|\). It follows that the ratio of the shifts induced by UB are

\[
\frac{MM(k)}{CM(k)} \cdot \frac{e(a, k)}{e(m, k)} > \frac{MM(k)}{CM(k)}.
\]
The ratio of the MM to the CM shift is thus larger for the backwards motion towards intersection on the target line than for the motion leading to intersection B. A reduction of MM to the initial position would thus be accompanied by a shift in CM leaving this curve still above the old position producing an intersection above the target line. The required further adjustment of UB pushes the intersection eventually to the target line. This whole process implies, however, a relatively larger shift of the MM line. The final intersection point C on the target line is thus to the right of the initial point A. An increase of k under an interest target policy is thus converted into a higher assetprice level P. Such an interest policy effectively prevents the deflationary consequences associated with a rising currency ratio. The reader can easily establish that the \( \lambda \) curve in the left side panel is shifted leftwards along the horizontal target line by the increase in P. The increase in k thus raises actually the money stock under an interest target policy.

It was also shown that an increase in the output price level \( p \) unleashes deflationary impulses on the output market via the adjustments imposed on the assetmarkets. Interest rate \( i \) rises and assetprice level \( P \) falls. This result is also substantially modified by the pursuit of an interest target policy. With price elasticities of \( \sigma \) and \( \lambda \) approximately equal, i.e., \( \varepsilon (\sigma, p) \sim \varepsilon (\lambda, p) \), the required expansion of UB moves the intersection of CM and MM approximately to the initial intersection on the target line. An interest target policy thus screens the output market effectively from the adjustments in the assetmarkets in this case. The money stock, however, increases. This is expressed by a leftward shift of \( \lambda \) along the target line as a result of the increase in \( p \). The increase in \( M \) expresses the stabilizing offset
produced under the circumstances by monetary policy. This stabilization of the output market from impulses generated by price variations via the assetmarkets remains however an essentially short-run phenomenon. In a larger perspective it impairs a stabilizing feedback loop of the inflationary process.

A pattern more similar to the case of $S^*$ emerges again from variations in the anticipated real net yield $e$ on real capital. An increase in $e$ moves the intersection between CM and MM to the northeast. The expansion of UB required to maintain $i$ thus yields an even larger increase in $P$ without any offsetting rise in $i$. An interest target policy thus amplifies the effects of variations in $S^*$ and $e$ and moderates the impact of changes in $k$ and $p$. The interpretation of these results depends on the process generating the variations. Suppose the movements of $e$ and $p$ result to a large extent from previous monetary accelerations. An interest target policy thus associates earlier accelerations with subsequent monetary accelerations. Consider on the other hand a Wicksellian view which attributes the major movements of $p$ to autonomous changes in $e$. An interest target policy amplifies the current Wicksellian impulse and prevents the subsequent retarding feedback on aggregate private real demand via a rising output price level. The total effect of Wicksellian impulses are thus magnified by an interest target policy. It is noteworthy that both monetarist and Wickselian impulse hypothesis yield a similar appraisal of the longer-run destabilizing effect of an interest target policy.

We examine lastly the effects of inflationary anticipation. An increase in the creditmarkets' anticipated rate of inflation $\tau$ pushes CM upwards and the intersection point moves northeast. The effect is thus similar to an increase in $S^*$. The final result with the constraint on interest rates is also similar. Lastly, an increase in the anticipated price level $ap$ pushes CM up and MM down. Our previous analysis established that the intersection point moves in this case also to the northeast. Maintaining the prevailing interest rate yields thus again a result similar to the $S^*$ case. We conclude thus that the expansionary effects of increases in $\pi$ and $ap$ are amplified by an interest target policy. This result is reflected in all cases by a larger response produced by the money stock.

9. Instability of Assetmarkets and Optimal Policy Strategy

It was noted in the introduction to this paper that the analysis of optimal strategy, i.e., of an optimal short-run target guiding the adjustment of the instrument variables available to the monetary authorities was essentially confined to the standard IS–LM paradigm. Two sources of instability confronting the policymaker emerge in a world characterized by the standard paradigm. One source moves the IS line and another reflects the instability of money demand. Poole demonstrated that the choice between a money stock or an interest rate
target policy depends in a specific manner on the relative variances of disturbances associated with money demand and aggregate demand.

The standard paradigm, based on a highly restricted range of substitution relations involving money, equates instability of asset markets with instability of money demand. Asset market instability of sufficient magnitude thus justifies the choice of an interest target policy as a rational device to minimize the impact of this instability on economic activity. The recognition of an extended range of substitution over the whole array of assets modifies the analysis. The state of the asset markets cannot be reduced to the money equation. A credit market equation need be added. Asset market instability may thus emerge not only from λ but also from σ. This section demonstrates that instabilities of σ and λ yield different conclusions with respect to the choice between a monetary aggregate (represented here by UB) and interest rate as a short-run target to guide policy.

The diagrammatic exposition of the problem requires a modification of our previous graph. The λ–M curves in the left side panel are replaced by a single curve d in the (i–y) plane. The d curve represents the aggregate demand for output. Its slope reflects the interest elasticity of aggregate demand and its position depends on the asset price-level P (and of course, also on other variables, particularly fiscal policy variables, the wealth terms, the anticipated net yield e on real capital, and the anticipated price level ap). The dependence of d’s position on P is crucially important for our purposes. Graph 13 A describes the effects of an increase of UB. The reader will note that the effect of UB consists of two components: an increase

Graph 13 A
in y resulting from the movement downwards along d as a result of the decline of i, and secondly a shift to the left of d along the horizontal interest line as a result of the higher level of P\textsuperscript{12}.

It is important to understand that the positions of CM and MM do vary with y. The new intersection point B in Graph 13A thus reflects not only the increase in UB but also the increase in y measured in the left side panel. This increase in y raises both p and e and thus induces further shifts in the CM and MM curves. This dependence of CM and MM on y modifies the shifts induced by UB in the following manner. The Graph 13B shows again the result of an increase of UB on the assetmarket.

The associated increase in y implies that the intersection B in Graph 13B cannot be the final equilibrium point of these markets. The feedback via the output market modifies CM and MM according to the results obtained in section 6. At lower levels of activity the final intersection point is northeast of B and at higher activity levels northwest of B. These secondary modifications in the position of the two curves remain, however, of secondary importance relative to the primary shift associated with UB. The subsequent discussion always assumes that the positions of the assetmarket curves show the final and fully adjusted equilibrium position consistent with the output level y determined in the left side panel.

\textsuperscript{12} The "real balance effect" contributes quite negligibly to the increase in activity. This effect is represented by a small horizontal shift in d expressed by $\varepsilon(d, W^h)\frac{(1 + \sigma)UB}{M}$. It is thus shown that the traditional real balance effect contributes marginally to the transmission of monetary impulses conveyed essentially by adjustments of relative prices.
We consider first the variability of $\lambda$ due to the operation of a random disturbance $x$ occurring in the public's money demand. The variability of this disturbance is expressed in Graph 14A by means of the range of shifts in $\lambda$ induced by $x$.

The extreme rise in $\lambda$ raises MM to the position MM$_2$. At this point the interest rate is $i_2$ and $d$ is moved rightward to $d_2$ as a result of a lower $P$. The resulting extreme intersection point of $d_2$ with the $i_2$ interest line is thus at A, corresponding to the assetmarket intersection A! The other extreme is located at MM$_1$ with an interest rate $i_1$, and with aggregate demand $d_1$ and intersection point B in the left side panel. The instability of $\lambda$ over the range shown in the right side panel thus generates a variability of the activity level ranging between $y_1$ and $y_2$.

The next Graph 14B describes the pattern emerging under an interest target policy. The disturbance in $\lambda$ generates again fluctuations in MM between MM$_2$ and MM$_1$. Interest rate $i$ is held at the given target level in the face of these fluctuations by suitable adjustments of UB. A motion towards MM$_2$ raises UB under the circumstances and the modified extreme position emerge at MM$_2^*$ and CM$_2^*$ intersecting on the predetermined interest line at B and to the left of the central point A. Similarly, a motion towards MM$_1$ yields the modified extreme position at CM$_1^*$ and MM$_1^*$ intersecting at C to the right of the central point A. An interest target policy thus converts the random disturbances of $\lambda$ similar to previous cases examined into variations of $P$ at a constant rate $i$. We obtain thus a variability of $d$ along the target line depending on the assetprice elasticity of $d$ and
the variability of P generated by x. The reader will note here a crucial difference with Poole's analysis executed within the confines of an IS–LM framework. The latter implies that disturbances of λ are effectively blocked from transmission to y under an interest target policy.

This proposition does not extend to our analysis. The interaction between various asset markets in a context of generalized substitution relations yields a transmission channel of asset market impulses beyond the rate of interest on financial assets. This transmission channel converts the λ-disturbances modified by UB adjustments under an interest target policy into variations of d around a central position d₀.

The reader may wonder whether the comparison of the range of variability produced by a UB policy or an i-policy yields a definite result. He may suspect that an interest target policy lowers the range of variability. This suspicion is easily confirmed. The intersection point B in Graph 14B in the right side panel is necessarily southeast of the intersection between MM₂ and CM. Similarly, the intersection C is northwest of the intersection between CM and MM₁. It follows that the range of variation of P is lowered by an interest target policy. The range of variability of y under an interest target policy is thus lowered by two distinct components: One consists of the lower variation of d around its central position due to the lower variation of P, and the second reflects removal of the variability of y due to the movement along the sloping d curve between the extreme interest lines determined by a UB policy.

We turn now to an examination of the role of σ-disturbances. Graph 15A depicts this situation.
The CM line moves between the extreme positions CM₁ and CM₂. An increase towards CM₂ raises UB under an interest target policy and CM and MM are finally moved to CM₂* and MM₂* with intersection D on the interest target line. A reduction towards CM₁ induces a fall in UB until the lines are moved to CM₁* and MM₁* intersecting at C on the target line. A UB policy generates thus variations between intersections A and B on MM₀, whereas an i-policy produces variations between C and D along the target line. We obtain thus under a UB policy a positively correlated change of interest rate i and asset price level P with partially offsetting effects on y. Under an interest target policy we derive a larger variability in P combined with a zero variability in i.

The net effect on y can be examined by means of Graph 15 B.

The UB policy generates variations between y₁ and y₂ and an i-policy between y₃ and y₄. This result depends however crucially on the slope of d and the shifts in d induced by the changes in P. A sufficiently large ϵ₁(d, P) relative to ϵ₁(d, i), i.e., a large horizontal shift of d relative to its slope measure, is a sufficient condition for a smaller range of y under a UB policy. A comparatively flat d curve combined with a small shift in d generates on the other hand a smaller range for the i-policy. An unstable G does not, per se, establish the superiority of an interest target policy. A UB policy is superior to an i-policy whenever the P response to this instability is
sufficiently strong and the "Non-Keynesian" transmission channel adequately large. A detailed analysis of this issue will be presented at another occasion. The last source of instability is usually assigned by this analysis to some disturbances occurring in the aggregate demand function. The argument can be followed with the aid of Graph 16 A.

The disturbances swing the d line between $d_2$ to left of a central position $d_0$ and $d_1$ to the right of $d_0$. These swings are associated at lower levels of activity with variations of the asset market equilibrium point between a point A (for $d_2$) northeast of the central position and a point B (for $d_1$) southwest of the central position. At higher levels of activity the asset market equilibrium swings between a (for $d_2$) and b (for $d_1$). An interest rate target policy replaces the swings between A and B with variations between $A^*$ and $B^*$ along the target line. A high levels of activity  

${\varepsilon(y, Z|UB) = \frac{\varepsilon(d, i) \cdot \varepsilon(i, Z|AM-UB) + \varepsilon(d, P) \cdot \varepsilon(P, Z|AM-UB)}{\varepsilon(d, P) \cdot \varepsilon(P, Z|AM-i)}}$

where $\varepsilon(y, Z|UB)$ is the elasticity of y with respect to the $\sigma$-disturbance Z under a UB policy. The denominator on the right thus indicates the elasticity of y with respect to Z under an i-policy. The expressions $\varepsilon(d, i)$ and $\varepsilon(d, P)$ describe interest and asset price elasticities of aggregate demand d. The remaining three elasticities on the right side are responses of i and P conditioned by the asset markets (AM) and constrained by a UB or an i-policy. The discussion in the text determined that the second term in the numerator on the right side expression is absolutely and algebraically less than the denominator of the same expression. It follows that the ratio of the two Z-elasticities of y has a positive upper boundary less than 1. There is no lower boundary however. With $\varepsilon(d, i)$ sufficiently large in absolute value, the ratio will fall below minus one. This value defines the critical value separating the ranges associated with choices of UB or i-policy. For all elasticity ratios less than minus unity the range of y under a UB policy exceeds the range of y under an i-policy. The range of y under a UB policy is less than the corresponding range under an i-policy on the other hand whenever the elasticity ratio exceeds minus unity. A UB policy is thus superior to an i-policy for all values of the elasticity ratio exceeding minus unity. We conclude thus that the relative superiority of an i-policy in contexts of $\sigma$-disturbances depends on the relative importance of the "Keynesian channel" in the transmission mechanism.
an interest target policy replaces swings between a and b with negligible movements around the central position. Two comparisons are thus necessary, one for higher and one for lower activity levels.

Consider first the case of lower activity levels. Graph 16B represents the situation. An expansion to $d_2$ moves the asset-equilibrium to A and the associated rise in $P$ shifts $d_2$ to $d_2^f$. The resulting extreme point is $C_1$. The opposite extreme point associated with the assetmarket equilibrium B is $C_2$. The range of variability generated under a UB policy by an unstable $d$ thus extends from $y_1$ to $y_2$. An interest target policy moves in this case the assetmarket equilibria to $A^*$ and $B^*$ in Graph 16A on the target line.
Our previous analysis determines that $A^*$ is southeast of $A$ and $B^*$ northwest of $B$. The range of variation of $P$ necessarily increases and the shift of $d$ reaches beyond $d_2^*$ on the left, or $d_1^*$ on the right. The relevant extreme points are thus $D_2$ and $D_1$ involving a larger range of variability for output.

Graph 16C describes the situation at higher levels of activity.

The disturbances of $d$ move the line between $d_1$ and $d_2$. But a positive disturbance lowers $P$ under the conditions stated and thus moves $d_2$ back to $d_2^*$. Similarly, a negative disturbance raises $P$ and moves $d_1$ forward to $d_1^*$. The relevant extreme positions under a UB policy are thus $A$ and $B$ with a range of variation in output extending from $y_1$ to $y_2$.

An interest target policy adjusts UB in such a way that with $\varepsilon(CM, y) \sim \varepsilon(MM, y)$ the asset equilibria are moved back to the central position $CP$ on the target line. In this case an interest target policy removes both variation in $i$ and in $P$. The range of variation in output between $A^*$ and $B^*$ is, therefore, determined completely by the intersection of the extreme positions $d_2$ and $d_1$ with the target line of interest. It is immediately visible by inspection that a UB policy lowers the range of variation below the corresponding range associated with an interest target policy.

The results obtained extend Poole's analysis in a natural manner. Instability of money demand justifies an $i$-policy and instability of aggregate demand a UB policy. Instability of $\sigma$ requires more explicit analysis for a determination of the issue. This complete analysis will be presented at another occasion with necessary and sufficient conditions on the variations of the disturbances of $\lambda$, $\sigma$ and $d$. 
yielding a UB or an i-policy. Our examination is restricted in this paper to an interpretation of these disturbances. The location of instabilities occurring in this analysis is equivalent to a classification of impulses driving the economy. Four major impulses are distinguished: (i) the moving force is centered in the structure of the internal dynamics, (ii) it is seen to emerge from substantial variations in the anticipated real net yield of real capital, (iii) variations in monetary growth, and lastly (iv) variations in money demand. The first three have been pondered for many years in the professional literature. The last has been contributed essentially by the Central Banks. The first two impulses are expressed by simultaneous variations of the d-curve and the assetmarket curves. The third is expressed by simultaneous movements in the opposite directions of the CM and MM line. And the last impulse moves only MM around. The optimal choice of a monetary policy strategy thus depends on the dominance patterns of impulses moving the economic process. The dominant occurrence of a Keynesian-Wicksellian impulse (i), or a Wicksellian impulse (ii) implies the superiority of a UB policy over an i-policy. The same superiority is also established under a dominant monetary impulse. Only the money demand impulse assures a superiority of an i-policy. It is thus remarkable that adherents of the major Wicksellian, Keynesian or monetarist impulse hypothesis should uniformly reject an interest target policy. There emerges thus a purely factual question bearing on the actual state of the dominance patterns. It is fortunately not necessary for this purpose to assess the monetarist hypothesis relative to the Wicksellian and Keynesian impulse hypothesis. The issue is reducible to the relative magnitude of disturbances (or "instabilities") in money demand. It should be noted foremost that no evidence has been adduced thus far corroborating the occurrence of large and persistent "instabilities" in money demand. It is also noteworthy that no econometric model examined thus far in some detail can explain economic fluctuations in terms of a cumulative effect generated by the systems (serially uncorrelated or correlated) disturbances. This implies in particular that these models satisfying the construction rules of a Keynesian system, offer no support for the thesis claiming major disturbances in money demand as the dominant impulse conditioning an economy. Moreover, the rationale offered to justify the occurrence of shifts in money demand is frequently phrased in words suggesting a confusion between the public's money demand \( \lambda \) and its asset supply \( \sigma \). This confusion forms a noble tradition of Central Banking recently codified by the first explicit treatment of the money supply process offered by Central Bank economists. This confusion thus attributes all variations of interest rates to

---


15 The reader may usefully consult the simulation results presented in "Econometric Models of Cyclic Behavior", edited by Bert G. Hickman, Columbia University 1972.

events in the money market MM with a standard formulation of liquidity preference. Changes in interest rate not assignable to UB or income can thus only be assigned to shifts in money demand. But these changes in i may have originated in the credit market and our previous analysis established that assignment of credit-market disturbances to the money market seriously mislead probably the choice of policy target.

The Central Bank’s inclination to assign significance to disturbances in money demand follows from a serious distortion of the time horizon. There is little doubt that within the short-run periods of a few weeks or a month characterizing the policymakers’ usual horizon random influences substantially modify both money demand λ and asset supply σ. It is even possible that these random influences on the asset markets are of similar order over a few weeks’ horizon as the random influences working on the output or labor market. I find it however very difficult to accept the view that random influences in asset market, in particular disturbances of money demand, dominate impulses in output and labor market over horizons including several quarters. I submit that this is quite unlikely. It seems most unreasonable to argue that disturbances of λ generate economic fluctuation of a magnitude similar to the fluctuations attributed to Wicksellian, Keynesian or monetary impulses. The evidence is certainly not conclusive but neither is it negligible. It appears thus that the claim of relevant λ-disturbances resulted probably from an inappropriate transfer of experience within a horizon confined to weeks or a month beyond this shortest horizon. The traditional myopia of Central Bank officials supplemented with their traditional confusion between credit and money thus explains their well established inclination to pursue interest target policies in one form or another.

Concluding Remarks

The discussions presented in the previous sections yield some information about three major issues in monetary analysis: the role and magnitude of “reverse causation”, the indicator quality of monetary growth and the relative desirability of an interest target policy used beyond the guidance for purpose of shortest-run adjustments.

Assertions of possibility or probability of a substantial “reverse causation” have been the standard answer of every Keynesian to practically every regression equation ever presented by monetarists. The possibility certainly exists and so is the possibility of a positive reverse causation from income to government expenditures “which really explains” the Keynesian multiplier coefficient. The existence of possibilities are barely worth the attention frequently invested. The important question bears on actual occurrence and relative magnitude of the phenomenon. It
is noteworthy that the Keynesian countercritique offered little evidence or analysis applying to this issue. Our evidence submitted at other occasions\textsuperscript{17} assigns no substantial role to the operation of reverse causation. The persistence of the observed correlation between money and income over periods and countries exhibiting wide variations in arrangements governing the money supply process reenforces our doubts concerning the role of reverse causation. These doubts are justified by the analysis in this paper. It was shown that in the absence of an interest target policy reverse causation produces little variation in the money stock. An interest target policy introduces however a substantial reverse causation from the price-level to the money stock. Such reverse causation actually operates in an inflationary environment to accelerate the inflationary process.

The second issue pertains to the indicator quality of the money stock. The usefulness of the money stock as an indicator of monetary impulses depends substantially on the association between changes in the money stock and the impulses imparted on economic activity produced by “monetary events”. An inspection of the results assembled immediately establishes that the money stock is a poor index of all possible influences conditioning economic activity. It is sufficient for the reader to notice that changes in inflationary anticipations or human wealth, price-level and the anticipated real net yield on real capital determine in the absence of an interest target policy somewhat poor associations between changes in the money stock and the economic stimulus channeled via interest rate i and asset price-level P. A pronounced reverse causation would actually strengthen the general indicator quality of the money stock with respect to any event. It seems ironic that an extensive reverse causation justifies a supermonetarist thesis concerning the indicator quality of the money stock. Monetarist analysis circumscribed the indicator quality of the money stock much more narrowly. We note in particular that changes in the money stock and economic stimuli emitted by the adjustments of the assetmarkets in response to changes in the base, the stock of securities, the ceiling rate and the currency ratio seem substantially associated; small stimuli are accompanied with small changes in the money stock, and large stimuli with large monetary changes. The “monetary events” indexed by the money stock are thus all events influencing the components of the unborrowed base or simultaneously affect the asset and monetary multiplier. The monetarist proposition asserts that the stimuli produced by the operation of these events via UB, \(a\) or \(m\) are sufficiently associated with changes in the money stock to establish these changes as a useful index of the stimuli received by the economy. The analysis in this paper certainly remains somewhat tentative and offers no sufficient demonstration of this issue. The investigation will be resumed at another occasion.

\textsuperscript{17} The problem was discussed in my paper, “The Role of Money and Monetary Policy”, published in the monthly review of the Federal Reserve Bank of St. Louis, 1968.
It is remarkable that the indicator quality of the money stock seems not essentially affected by the occurrence of an interest target policy. The results actually show an extension of the range beyond the set of "monetary events" defined before. An interest target policy correlates changes in the money stock with stimuli produced by inflationary anticipations. It is noteworthy, moreover, that the effect of changes in the currency ratio is reversed by a persistent interest policy. The paper examined also the relation between the impulse forces and the relative adequacy of an interest target policy. It was concluded that only the central bank thesis of a dominant impulse centered "behind" money demand justifies application of an interest target policy. The major impulse force hypothesis of the Keynesian and monetarist literature uniformly imply that an interest target policy yields poor results for economic stabilization. Lastly, the extension of the range of substitution relations over all assets assigns a special significance to the credit market. Disturbances originating in the credit market affect the economic process quite differently from disturbances originating in the money market and different policy strategies seem appropriate. The operation of the credit markets implies also, that contrary to the results obtained with a Keynesian framework, disturbances originating in the money market cannot be neutralized from affecting the economy by an interest target policy.

Summary

*A Diagrammatic Exposition of the Money Supply Process*

This paper presents the Brunner-Meltzer hypothesis of the money supply process in a closed economy with the aid of a diagrammatic apparatus. It describes the interaction of the credit market and of the money market in the simultaneous determination of money stock, bank credit, interest rate and market value of real capital. The relative magnitude of the interest and asset price elasticities appear as the central structural properties determining the response patterns. The paper also shows that the magnitude of monetary impulses transmitted via changes in interest rate and asset price (i.e. market value of real capital) are generally correlated with the magnitude of the money stock.