A Two-Class Monetary Growth Model*

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I. Introduction

The neutrality of the monetary sector is a problem that has been widely considered in the recent economic literature. Nevertheless, until now, little has been done to integrate monetary factors in the analysis of the growth process. 'Classical' growth models only consider real variables, such as labour, physical capital and technological progress. However, the analysis of these nonmonetary models is worthwhile if (and only if) we are sure that the monetary structure has no effects on the real variables of the system; in other words money should be neutral.

A first step in this sense was made by Tobin in 1965 and 1967. At the end of his first valuable exposition [10] the author gives the following conclusion: "If money is in any degree a substitute for material wealth in satisfying the thrift propensities of the population, then it is not neutral" [10, p. 72]. Actually, as Tobin demonstrates, equilibrium capital intensity is lower (ceteris paribus) in a monetary growth model than in the corresponding real growth model.

Later Sidrauski [9] pointed out that the non-neutrality of the monetary structure can be proved showing that "The mere existence of outside money, independent of the rate at which the money stock grows or declines through time, will prevent the economy from attaining the Solow-Swan steady state capital stock. The capital stock for which a monetary economy reaches its steady state is always smaller than the one indicated by the Solow-Swan model. It therefore follows that the golden rule saving ratio is not any more the one that maximizes long run consumption. For long run consumption to be maximized the saving ratio has to be greater than the share of profits in national income" [9, p. 796].

To the best of our knowledge an analysis of a two-class monetary growth model has not yet appeared in the economic literature. It is thus natural to ask what happens when the monetary structure is introduced in a two-class growth model; moreover in this context, we shall consider some optimal aspects of the model.

* This paper is partially based on my doctoral dissertation undertaken at the University of Zürich and presented at the University of Fribourg (Switzerland). I wish to express here my indebtedness to Professor Pietro Balestra for his invaluable guidance and assistance. I am also grateful, among others, to Henri Bortis of Churchill College, Cambridge, for helpful discussion. Of course the usual caveat with respect to responsibility holds.

Financial support from the Swiss National Foundation for Scientific Research is gratefully acknowledged.

1 For 'Classical' growth models we mean, in this context, all real growth models (or non-monetary growth models).

2 The 'two-class' growth model we consider here is the two-class growth model with a differentiated interest rate exposed in [1].
The basic assumptions of the model are specified in Section II. In Section III we analyse its equilibrium properties. Section IV will deal with two optimal properties of the model: the total consumption maximizing path and the workers' consumption maximizing path respectively. A particular aspect of the model will be exposed in the last Section, which will also include the main conclusions.

II. The Basic Assumptions of the Model

*Hypothesis of production.* We shall assume a well-behaved production function of the type

\[ Y = f(K, L), \quad (1) \]

where \( Y \) represents total physical output, \( K \) represents the physical capital stock and \( L \) represents labour. The production function is supposed linear-homogeneous, that is to say with constant returns to scale. Labour input is measured in efficiency units so as to allow for neutral technological progress ('labour augmenting'). As usual, the (natural) rate of growth of \( L \) is exogenously given and is constant; it will be denoted by \( n \). We shall also assume (for mathematical convenience, but without any loss of generality) an unitary elasticity of substitution; the constant capital share will be denoted by \( c \).

*The monetary structure.* Beside the physical capital stock there exists a monetary capital stock, represented by the quantity of money existing in the economic system. For sake of simplicity we shall assume that the government does not undertake any public expenditure (this hypothesis is often postulated by macro-monetarists): this means that government deficits are entirely financed by issuing government non-interest bearing debt (fiat money). Let \( M \) be the nominal quantity of money existing in the system and \( P \) the price level: therefore \( M/P \) will be the real value of monetary balances circulating in the economic system. We shall further assume that:

\[ m = \dot{M}/M \]
\[ p = \dot{P}/P \]

is the rate of change of the money supply and the rate of change of the price level (Sidrauski's 'instantaneous expected rate of change in prices').

\[ ^3 \text{Namely: } y(0) = 0 \quad y(k) > 0 \quad y(\infty) = \infty \]
\[ y'(0) = \infty \quad y'(k) > 0 \quad y'(\infty) = 0. \]
\[ y''(k) < 0 \]
Therefore the net increase of the real monetary balances, during one period $t$, will be:

$$(M/P) = (m-p)\cdot M/P. \tag{2}$$

This increase of the real monetary balances represents the real monetary investment of the whole economy. On the other hand it determines the total real disposable income ($Y^d$):

$$Y^d = Y + (m-p)\cdot M/P. \tag{3}$$

This is the major path through which money affects the working of our economy: in real terms savings = investment + government deficit. Hence:

$$S = I + (m-p)\cdot M/P. \tag{4}$$

Hypothesis about the profit rate ($P/K$) and the interest rate ($r$). Pasinetti's Theorem [5] as well as Meade, Samuelson & Modigliani's Dual Theorem [8] are based on the assumption that on the steady-state growth path $P/K$ is equal to $r$. In an earlier model, Kaldor [4] assumed that the workers handed over their savings to the capitalists without getting any sort of compensation in return, i.e. $r = 0$. Our interest rate will fall between these two extremes values. This means that we shall postulate a differentiated interest rate; typically, the rate of interest will be lower than the rate of profits:

$$0 \leq r \leq P/K. \tag{5}$$

Relation (5) shows that we can consider, if necessary, $r$ as exogenously determined (or controlled by the State). We shall retain this hypothesis for most of our developments.

The disposable income of the two classes. As usual, we shall assume a state of full employment so that total physical output ($Y$) may be divided into two broad categories, Wages and Profits ($W$ and $P$). If we denote by $P_c$ the profits which

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4. *I.e.* the real deficit of the government.
5. Mathematically we have:

$$(M/P) = \frac{d(M/P)}{dt} = \frac{MP - MP}{P^2} = \frac{PmM - PpM}{P^2} = (m-p)\cdot M/P.$$

6. By ‘interest rate’ we mean the rate at which the workers place their savings in the hands of the capitalists (or in the hands of the State in a socialist society).
7. The reasons adduced in support of this assumption are exhaustively exposed in [1].
8. These assumptions coincide with those of the corresponding ‘real’ growth models.
accrue to the capitalists and by \( P_w \) the profits which accrue to the workers, we obtain:

\[
Y = W + P_c + P_w. \tag{6}
\]

Since there are two classes, there are two distinct stocks of physical capital, one owned by the capitalists \((K_c)\) and one owned, possibly indirectly, by the workers \((K_w)\). According to our definition of the interest rate, total profits are distributed as follows:

\[
P_w = rK_w \text{ to the workers}
\]

and \( P_c = P/K \cdot K_c + (P/K - r)K_w = P - rK_w \) to the capitalists.

On the other hand relation (3) states that total disposable income equals \( Y + (m-p) \cdot M/P \), the net addition to the stock of real cash balances. Now, how is this share of total disposable income distributed between the two classes? For the sake of generality we shall assume that the workers will receive the quantity \( a(m-p) \cdot M/P \) and the capitalists its complement, i.e. \( (1-a)(m-p) \cdot M/P \), where \( 0 \leq a \leq 1 \). Then, denoting by \( Y^d_c \) and \( Y^d_w \) the respective disposable income of the two classes, we write:

\[
Y^d_c = P - rK_w + (1-a)(m-p) \cdot M/P \tag{7}
\]

and \( Y^d_w = rK_w + W + a(m-p) \cdot M/P, \) where \( 0 \leq a \leq 1 \). \( \tag{8} \)

Between zero and one \( a \) can assume any value whatever. Two particular cases will be analysed later: the first arises when \( a = 0 \) and the second one when \( a = Y_w/Y \).

Saving and investment functions. For each class, a proportional saving function is postulated. Calling \( s_c \) and \( s_w \) the average propensities to save of the two groups we may write:

\[
S_c = s_c \left[ P - rK_w + (1-a)(m-p) \cdot M/P \right] \tag{9}
\]

and \( S_w = s_w \left[ W + rK_w + a(m-p) \cdot M/P \right]. \tag{10} \)

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9 Let us divide the monetary stock in two categories: \((M/P)_c\), the average monetary stock owned by the capitalists and \((M/P)_w\), the average monetary stock owned by the workers. If \( (m-p) \cdot M/P \) is attributed to the two classes proportionally to their monetary stock, the additive income will be \((m-p)(M/P)_c\) and \((m-p)(M/P)_w\) respectively (on the steady-state growth path).

10 As usual we suppose \( 0 \leq s_w \leq s_c \leq 1 \).
The condition under which our system will remain in dynamic equilibrium is

\[ S_c + S_w = \dot{K}_c + \dot{K}_w + (m - p) \cdot M/P, \] (11)

as we know, from (4), that total saving equals total physical investment plus the real increase of the monetary stock.

As we have assumed that the monetary balances of the two classes are \((1 - a) \cdot M/P\) and \(aM/P\) respectively, in dynamic equilibrium the real 'monetary investment' of the capitalists' class will be \((1 - a)(m - p) \cdot M/P\) and the real 'monetary investment' of the workers \(a(m - p) \cdot M/P\). We can now write the investment functions (of the physical capital):

\[ \dot{K}_c = s_c [P - rK_w + (1 - a)(m - p) \cdot M/P] - (1 - a)(m - p) \cdot M/P, \] (12)
\[ \dot{K}_w = s_w [W + rK_w + a(m - p) \cdot M/P] - a(m - p) \cdot M/P. \] (13)

**III. Equilibrium Growth**

*Equilibrium growth of the physical capital.* We shall begin the analysis by computing the rates of growth of the two stocks of physical capital. First, let \(c\) be the (constant) equilibrium profit share \((P/Y)\) and \(B\) the wages/profits ratio \((W/P)\). Then using relations (12) and (13) we get

\[ \frac{\dot{K}_c}{K_c} = s_c \left( \frac{P}{K} + \frac{P}{K} \frac{K_w}{K_c} - r \frac{K_w}{K_c} \right) + (s_c - 1)(1 - a)(m - p)(1 + \frac{K_w}{K_c}) \frac{M/P}{Y} \frac{1}{c} \frac{P}{K}, \] (14)
\[ \frac{\dot{K}_w}{K_w} = s_w \left( B \frac{P}{K} + B \frac{P}{K} \frac{K_c}{K_w} + r \right) + (s_w - 1)a(m - p)(1 + \frac{K_c}{K_w}) \frac{M/P}{Y} \frac{1}{c} \frac{P}{K}. \] (15)

\(^{11}\) \(\dot{K}_{c,w} = I_{c,w}\), the physical investment of the two classes.

\(^{12}\) Note that \(s_c\) and \(s_w\) can be decomposed as follows: let

\[ i_c = \frac{K_c}{Y}, \] the propensity to invest (in physical terms) of the capitalists,
\[ i_w = \frac{K_w}{Y}, \] the propensity to invest (in physical terms) of the workers,
\[ \overline{m}_c = (1 - a)(m - p) \frac{M}{P}, \] idem (in monetary terms) of the capitalists,
\[ \overline{m}_w = a(m - p) \frac{M}{P}, \] idem (in monetary terms) of the workers.

Then we can write: \(s_c = i_c + \overline{m}_c\) and \(s_w = i_w + \overline{m}_w\).

\(^{13}\) Note that:

\[ \frac{M}{P} = \frac{P \cdot M/P}{K} \text{ and } W = B \frac{P}{K}. \]
At a closer scrutiny we note that, given any positive initial value of $K_c$ and $K_w$, the two rates of growth are always positive. For a better stability analysis let us suppose that $K_w/K_c$ is greater than $K_c/K_w$; this clearly means that $K_c/K_w$ will diminish and its inverse will rise. Accordingly, the right-hand-side of (15) will fall and the right-hand-side of (14) will rise, until equality between $K_c/K_c$ and $K_w/K_w$ is reached. We observe the same equilibrating mechanism if we suppose, at the beginning, the inverse situation. Hence, the two stocks of physical capital, in equilibrium, will grow at the same rate. Equilibrium growth is therefore assured if:

$$\dot{K}_c/K_c = \dot{K}_w/K_w = n.$$  \hfill (16)

**Equilibrium growth of the monetary stock.** As specified above, let $(M/P)_c$ be the average monetary stock owned by the capitalists and $(M/P)_w$ the average monetary stock owned by the workers. The rate of growth of the two stocks will be:

$$\frac{\dot{(M/P)_c}}{(M/P)_c} = \frac{(1-a)(m-p)\cdot M/P}{(M/P)_c} = (1-a)(m-p)(1+M_w/M_c),$$ \hfill (17)

$$\frac{\dot{(M/P)_w}}{(M/P)_w} = \frac{a(m-p)\cdot M/P}{(M/P)_w} = a(m-p)(1+M_c/M_w).$$ \hfill (18)

Both rates of growth will be positive if $m > p$, i.e. when the rate of growth of the money supply is greater than the rate of change of the price level.

Again, in dynamic equilibrium, the two stocks will grow at the same rate; i.e.:

$$\frac{(M/P)}{M/P} = \frac{(M/P)_c}{(M/P)_c} = \frac{(M/P)_w}{(M/P)_w} = n.$$ \hfill (19)

For this reason on the steady-state growth path $m - p = n$.

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14 The lower limits of $s_c$ and $s_w$ are slightly different from the limits stated in the corresponding non-monetary models; see footnote 12. In a monetary model $s_c$ and $s_w$ include both propensities to save (which increase the physical and the monetary stocks); then

$$\dot{K}_{c,w} \geq 0 \text{ if } s_c \geq (1-a)(m-p)\cdot M/P\cdot 1/Y_c' \text{ and } s_w \geq a(m-p)\cdot M/P\cdot 1/Y_w'.$$

However, these limits are not restrictive at all; let, for instance, $a = 1/3$, $r = 0.06$, $P/K = 0.10$, $m - p = 0.02$, $M/PY = 1/3$, $c = 1/3$ and $K_c/K_w = 3$. Then $K_c$ and $K_w$ will be positive if $s_c > 0.015$ and $s_w > 0.001$, a very reasonable lower limit for the two propensities to save.

15 We can observe the same equilibrating mechanism pointed out for the two physical capitals.
**Aggregate equilibrium growth.** To begin with, let $b = (M/P)/Y$, the constant 'desired money income ratio'. Now, as $s = S/Y^d$, the well-known Harrod-Domar-Solow equilibrium condition can be rewritten, for our model, as:

$$\frac{Y^d}{K} \cdot \frac{S}{Y^d} = \frac{S}{K}, \quad (20)$$

i.e.

$$\frac{Y^d}{K} \left( s_w + (s_c - s_w) \left[ P - rK_w + n(1-a)\frac{M}{P} \right]/Y^d \right) = \frac{S}{K}. \quad (21)$$

A suitable expression for $K_w$ is easily found solving the following equation:

$$I_c/I = 1 - \frac{K_w}{K} = \frac{s_c[P - rK_w + n(1-a)\cdot M/P] - n(1-a)\cdot M/P}{I}. \quad (22)$$

The value of $K_w$ is substituted into relation (21); solving with respect to $P/K$ we obtain (m stands for 'monetary'):

$$\left( \frac{P}{K} \right)_m = \frac{cn \cdot (n-s_wr)}{n[s_c + s_c nb(1-a) - nb] + s_w n(1-c + nba) - r[s_c s_w (1+nb) - s_w nb (1-a) - s_c nba]} \quad (23)$$

The first conclusion to be drawn is that the equilibrium profits rate depends on all parameters of the model. As for the corresponding non-monetary model, Pasinetti's elegant solution no longer applies. Moreover:

$$(P/K)_m > P/K, \quad (24)$$

where $P/K$ denotes the equilibrium profits rate of the real model.

This result can be explained as follows: as $s_c$ and $s_w$ are equal for both models, the physical investment of the monetary model (ceteris paribus) will be smaller than the physical investment of the real model; hence a lower degree of capitalisation implies a higher rate of profits.

Relation (23) tells us that the equilibrium value of $(P/K)_m$ is a function of all parameters of the model; to study their influence, we shall differentiate $(P/K)_m$ partially with respect to the individual parameters. We immediately note that:

$$\partial (P/K)_m / \partial n, \partial (P/K)_m / \partial r > 0. \quad (25)$$

16 Obviously when $b = 0$ (i.e. $M/P = 0$) we are back to the equilibrium profits rate of the corresponding non-monetary growth model; cf. [1].
This last result can be interpreted as follows. As \( r \) rises, more profits are transferred from the capitalists to the workers; hence, as \( s_c > s_w \), investment will be reduced and \( (P/K)_m \) will rise. We can further establish that:

\[
\partial (P/K)_m / \partial a > 0 \text{ if } r > n, \tag{26}
\]

and

\[
\partial (P/K)_m / \partial s_w < 0 \text{ if } s_c \leq n/r. \tag{27}
\]

Relation (26) states that the partial derivatives of \((P/K)_m\) with respect to \(a\) is positive if \(r > n\). For \(r\) lower than \(n\) it is negative: this is important, as \(r\) is assumed to be exogeneous. Finally, for econometrically reasonable values of \(s_c\) and \(s_w\) we can write:

\[
\partial (P/K)_m / \partial s_c < 0, \tag{28}
\]

and

\[
\partial (P/K)_m / \partial c > 0. \tag{29}
\]

These last results are slightly different from those obtained in [1]; this is due to the fact that \(s_c\) and \(s_w\) do include a 'monetary propensity to save' (cf. note 12). Nevertheless, within certain limits, (27) and (28) show that as \(s_c\) or \(s_w\) increases the capital intensity will 'rise and, consequently, \((P/K)_m\) will fall. On the other hand, in a more general way, an increase in \(c\) (the equilibrium profits share) will result in an increase in the marginal productivity of capital.

IV. Two Optimal Aspects of the Model

The consumption maximizing path. At this stage of the analysis we shall try to answer the question whether there exists an interest rate which brings our monetary model on the total consumption maximizing path. This is particularly appealing as we are considering an exogenous interest rate \(r\). If such a rate does exist, it may be called the 'Golden Rule Monetary Interest Rate'. We know that on the long run steady-state growth path consumption per capita is maximized when the rate of profits is equal to the natural rate of growth. Hence, from (23):

\[
r^*_m = \frac{c - s_w(1 - c + nba) - s_c[c + nb(1 - a)] + nb}{s_c nba - s_w[s_c(1 + nb) - nb(1 - a) - c]}. \tag{30}
\]

Once again, our result differs from the analogous obtained in [1]\(^{18}\). In a similar way can be determined \(b^*\), the money income ratio.

\(^{17}\) A more detailed analysis is given in the original exposition, cf. footnote, page 1.

\(^{18}\) A star denotes optimal values.
A first particular case of special interest arises when \( s_c = 1 \); we get:

\[
\begin{align*}
r^*_m &= n \quad (31) \\
b^*_m &= s_w(1 - c)/na \cdot (1 - s_w). \quad (32)
\end{align*}
\]

Relation (31) is particularly attracting, as it states that, if \( s_c = 1 \), (see footnote 12), per capita consumption is maximized when the interest rate equals the natural rate of growth (and the rate of profits). A different way to look at the problem could be done by supposing, in a very particular case, \( n = r = P/K_m \); this happens when:

\[
\begin{align*}
(s_c)_m^* &= 1 \quad (33) \\
or \quad (s_w)_m^* &= \frac{c + nb}{1 + nb} \quad (34)
\end{align*}
\]

Again, this result do not coincide with those obtained in [1]; (where \( s_c^* = 1 \) or \( s_w^* = c \)), but (33) and (34) state that total consumption is maximized for \( \max(s_c) \) or \( \max(s_w) \). Of course, these results only apply in the case of a non-differentiated interest rate; if we retain our basic assumption we can deduce from (30) that there always exists a propensity to save of the capitalists (workers) of less than unity ((c + nb)(1 + nb)) which maximizes consumption per capita. Hence the stringent condition \( s_c = 1 \) of Pasinetti is, once again (see [1]), removed.

**Maximizing workers' consumption.** Instead of considering the path which maximizes total consumption, in a two-class model it could be more interesting to seek the path which maximizes the consumption of the workers, whose disposable income is determined, inter alia, by the interest rate, exogenously given.

More precisely, we will analyse the following aspect of the optimal path\(^{19}\): given the values of the propensities to save (and a given technology), what value of the interest rate maximizes workers' consumption?

As we know, an increase in the interest rate has two distinct effects on \( C_w \). First, \( Y_w^d \) (workers' disposable income) will increase, so as their total consumption will also automatically increase. Secondly, as \( s_c > s_w \), an increase in the interest rate, ceteris paribus, means a decrease in over-all savings. Investment will therefore decrease: consequently total disposable income will fall: this will of course reduce the disposable income of the workers' class and therefore their consumption. Since these two effects act in opposite directions, they will offset each other at some point: this will be the maximum we are looking for.

To start with, let us write total workers' consumption:

\[
C_w = (1 - s_w) \left[ (1 - c)Y + rK_w + an \cdot M/P \right]. \quad (35)
\]

\(^{19}\) For a more exhaustive exposition see [1], pp.252-3.
On the steady-state growth path, all consumption paths are parallel. At the origin, for instance, the path which maximizes workers' consumption is the one with the highest value. We shall therefore compute $C_w$ at the origin and maximize it with respect to $r$.

From (23), after the necessary manipulations, we can write:

$$K_w = Y \left[ s_w(1-c) + nba(s_w-1) \right] \cdot (n-s wr)^{-1};$$  

hence, introducing the latter into (35) we get:

$$C_w = (1-s_w)nY(1-c-rba+nba)\cdot (n-s wr)^{-1}.$$  

If we assume a Cobb-Douglas production function we may write:

$$Y/K = K^{c-1},$$  

where, without loss of generality, we have assumed at the origin $L = 1$. As we know that $Y/K = (P/K)^{c-1}$, we may now write, in dynamic equilibrium:

$$Y = (P/K)^{c/c-1} \cdot c^{c/c-1}.$$  

Substituting relation (23) into (39) we get:

$$C_w = (1-s_w)n^{2c-1} \cdot (n-s wr)^{-1} \cdot (1-c-rba+nba) \cdot \left\{ n \left[ s_c c + s_c nb(1-a) - nb \right] + s_w n(1-c+nba) - r \left[ s_c s_w(1+nb) - s_w nb(1-a) - s_c nba \right] \right\}^{c/c-1}.$$  

If we want to maximise $C_w$ with respect to $r$ we write $(C_w/r) = 0$ and we obtain the value of $r$ which maximizes the consumption of the workers (note that the second order condition is satisfied):

$$r_m^{**} = \left\{ s_w(1-c) \left[ s_w + nb(s_c - 1) \right] + s_w nba(2s_w - 1 - s_c) \right\} \cdot \left\{ s_c na^2 b^2 / (1-c) + s_w b \left[ (1-s_c) \cdot s_w(c-1-anb) + (1+s_c)s_w a + (1-c)s_c s_w / nb - a [2s_c + nba(1+s_c-s_w)/1-c] \right] \right\}^{-1}.$$  

We note immediately that for $b = 0$ we obtain $r_m^{**} = n/s_c$, the solution obtained in [1]. Obviously, in a monetary model $b(= M/PY)$ will never be equal to zero; we can then write\(^{20}\):

$$(P/K)_m^{**} > r_m^{**}, \text{ if } a \neq Y_w/Y,$$  

\(^{20}\) If $a = (W+rK_w)/Y = Y_w/Y$ we obtain $(P/K)_m^{**} = r_m^{**}$. Obviously, this is a very particular case.
which means that in our monetary model the rate of profits which maximizes workers' consumption is always greater (for \( a \neq Y_w/Y \)) than the corresponding optimal rate of interest. Clearly, this means that the monetary structure, in this context, is not neutral, as we know that in the corresponding real model (cf. [1]) workers' consumption is maximized when the rate of interest equals the rate of profits, i.e. \( (P/K)^** = r^{**} \).

V. A Particular Case

A special case arises when we consider the following assumption. The workers do not own any monetary stock, i.e. \((M/P)_w = 0\); this clearly means that the capitalists' class own the entire stock: \((M/P)_c = M/P\). As the workers receive their wages and profits in currency, they will have a 'real disposable income' lower than \(W + rK_w\), i.e.:

\[
Y^d_w = W + rK_w - p \frac{M/P}{Y} (W + rK_w),
\]

where \( p \) is the (instantaneous) rate of inflation. Hence:

\[
Y^d_c = P - rK_w + (m - p) \frac{M}{P} + p \frac{M/P}{Y} (W + rK_w).
\]

Relation (43) states that the workers are not exempt from the inflationary process (proportionally to their income); on the contrary they do not receive any share of \( M/P \) (the nominal increase of the monetary stock) which is connected with a typical capitalist procedure. In this particular case the rate of profits \((P/K)^**_{m+} \) and the rate of interest \( r^{**}_{m} \) which maximize workers' consumption turn out to be:

\[
(P/K)^**_{m+} = \frac{n}{s_c + nb(s_c - 1)},
\]

\[
(r)^**_{m} = n \frac{s_w(1 - bp) + s_c bm - nb}{s_w(1 - bp)[s_c + nb(s_c - 1)]}.
\]

It is interesting to point out that the optimal rate of profits stated in (45) does not depend on \( s_w \); this denotes a certain analogy with Pasinetti's Theorem. At a closer scrutiny, it turns out that:

\[
(P/K)^**_{m+} > (r)^**_{m} \text{ if } s_c < n/m.
\]
This means that the optimal rate of profits will be greater than the optimal rate of interest if $s_c$ is lower than the ratio $n/m$. On the contrary, if $s_c > n/m$, the opposite result applies; however this would be economically unsound. Note that this upper limit of $s_c$ is strictly connected with this particular assumption.

**Conclusion**

As it has been pointed out at the beginning, the non-neutrality of the monetary structure in growth models has been proved by showing that:

1. it leads to a lower capital intensity (cf. Tobin, [10]);
2. "For long run consumption to be maximized the saving ratio has to be greater than the share of profits in national income" (cf. Sidrauski, [9]).

These results have both been confirmed by our analysis. In addition to this we have been able to show that:

3. The equilibrium rate of profits (ceteris paribus) turns out to be greater than the rate of profits of the corresponding real model; this is easy to understand, as a lower capital intensity will result in an increase in the marginal productivity of capital.
4. The rate of interest which maximizes workers' consumption is not equal to the rate of profits. Relation (42) states that the rate of profits, under this assumption, turns out to be higher. In the corresponding real model they are equal.

All these results prove, once again, the non-neutrality of the monetary structure in neo-classical (and more general) models of economic growth. This is the main conclusion to be drawn from our analysis.

**References**

Zusammenfassung

Ein monetäres Zwei-Klassen-Wachstumsmodell


Résumé

Un essai de modèle monétaire de croissance à deux classes

Dans cet article on introduit une structure monétaire dans un modèle réel de croissance à deux classes et à taux d’intérêt différencié. La non-neutralité de la structure monétaire dans un modèle de croissance implique un taux de capitalisation physique qui est, dans un modèle monétaire, inférieur à celui du modèle non monétaire correspondant. Notre analyse confirme ce résultat et prouve, en outre, qu’en équilibre le taux de profit monétaire est supérieur au taux de profit réel et que le taux d’intérêt qui maximise la consommation des travailleurs est inférieur au taux de profit correspondant, tandis que pour le modèle non monétaire les deux taux sont égaux.

Summary

A Two-Class Monetary Growth Model

This paper integrates the monetary factors in a two-class economic growth model with a differentiated interest rate. The non-neutrality of the monetary structure in growth models has already been proved showing that it leads to a lower capital intensity, so affecting the Golden Rule Path. Our analysis confirms this result; moreover it shows that the equilibrium profit rate turns out to be greater than the profit rate of the corresponding real model and that the interest rate which maximizes workers’ consumption is lower than the analogous profit rate, while for the corresponding real model they are equal.