External Effects, Separability, and Resource Allocation

by Ernst Baltensperger*

In a well-known article, Ronald Coase [3] argued, in the context of a two-firm externality situation, that a socially optimal resource allocation will be reached through a negotiation process, regardless of whether the externality-causing firm is liable for the damage it causes to the other firm or not. In a recent paper, James Marchand and Keith Russel [5] reexamined the Coase argument, and came to the conclusion that Coase's result regarding the neutrality of liability arrangements on resource allocation is not generally true, but holds only in the restricted case of additively separable cost functions. Marchand and Russel formalize the Coase argument, as suggested by Coase, by assuming that both firms A (the one causing the externality) and B (the damaged firm) include as part of their objective function the compensation which B would pay to A for a reduction of the latter's output, and that this compensation is equal to the cost saving B would experience as a result of the reduction in A's rate of output. That is, the two firms' objective functions are $\Pi^A = P_1q_1 - A(q_1) + C$ and $\Pi^B = P_2q_2 - B(q_1, q_2) - C$, with $C = B(q_1, q_2) - B(q_1, q_2)$, where $\Pi^A$ and $\Pi^B$ denote the two firms' profits, $q_1$ and $q_2$ their outputs, $P_1$ and $P_2$ the (market determined) prices of these outputs, $A(q_1)$ and $B(q_1, q_2)$ the total cost of firms A and B, respectively, and $q_1$ is the quantity of output firm A would produce in the absence of any compensation (the quantity $q_1$ satisfying the condition $P_1 = A_1(q_1)$). Marchand and Russel show that under these conditions the optimal output for firm A (denoted $q_1^*$) exceeds the socially optimal quantity (denoted $q_1^*$), while firm B's optimal output ($q_2^*$) is less than the socially optimal quantity ($q_2^*$), unless B's cost function $B(q_1, q_2)$ is separable into two independent parts $B^1(q_1)$ and $B^2(q_2)$.

Although this model does in fact seem to be an accurate formalization of Coase's verbal presentation, I wish to defend the spirit of Coase's argument and his result against this formulation of the problem. Coase presents his argument in terms of his example of straying cattle which destroys the crop of a neighboring farmer. Assuming that, without compensation, the cattle raiser would maintain a herd of 3 steers, and that a reduction of his herd to 2 steers would save the crop farmer a loss of $3, Coase (p. 6–7) points out that "the farmer would be willing to pay up to $3 if the cattle raiser would reduce his herd to 2 steers," and then goes on:

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1 For related discussions, see James Buchanan and William Stubblebine [1], Otto Davis and Andrew Whinston [4], and Ralph Turvey [11].
The cattle raiser would therefore receive $3 from the farmer if he kept 2 steers instead of 3. But this last statement does not necessarily follow. The cattle raiser may be willing to reduce his herd from 3 to 2 steers for less than $3. If 3 steers is the optimum size of his herd in the absence of compensations, then the rate of change of his profit (not including compensations) with respect to the size of his herd must be zero at this point, and it follows that he would be willing to make a marginal reduction of his herd for a much smaller compensation (for an arbitrarily small compensation, if we can treat his output as a continuous variable). The $3 is an upper limit on what the farmer is willing to pay for the reduction in question. It is not a priori clear what the equilibrium compensation for this reduction will be. The mere fact that B is indifferent between paying $3 and having A's herd reduced to 2 steers on the one hand and the initial situation on the other hand does not imply that A will in fact obtain the full $3 if he reduces his herd to 2. It does not give us any reason to believe that A and B will both optimize on the assumption that A will always be able to extract the full "gains from trade", as they do in the above formulation. (After all, we could just as well turn around this assumption, and assume that B will always get all the gains.) In fact, if he has to pay the full $3, the farmer is not really interested in the transaction, since he makes no net gain. In terms of Marchand and Russel's formulation (their case b.): their objective function for B is obviously completely independent of $q_1$, the output of firm A. It is quite unclear why B would have any interest in "bribing" A to reduce its output under these circumstances. It clearly cannot improve its position by doing so.

I suggest that the problem at issue should be analyzed as a standard price theoretic (exchange) problem. The crop farmer has something to offer to the cattle raiser (compensations), in exchange for something else (reductions in the cattle raiser's herd). We have to determine the equilibrium compensation, as well as the equilibrium reduction of the cattle raiser's herd below what it would be in the absence of compensations. We can think of the crop farmer as having a demand for such reductions, depending on the compensation, or price (per unit of reduction) which he has to pay, and of the cattle farmer as having a supply of such reductions, again as a function of the compensation (per unit of reduction). Viewing the problem in this framework will show that the resulting outcome has little to do with the question of the separability of the cost function. Given the assumption of competitive behavior, which may be quite appropriate in many cases, the Coase result will be shown to hold, regardless of whether the cost function is separable or not. But the framework suggested is suited for the analysis of other behavioral forms as well.

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2 See the discussion of this point at the end of section I.
I. The Model

As suggested by Coase, assume that the two firms include in their objective functions the compensation \( C \) which they would obtain from the other firm in exchange for an output reduction, or would have to pay to the other firm for such a reduction, respectively. That is, the two firms try to maximize

\[
\Pi^A = P_1q_1 - A(q_1) + C \quad \text{and} \quad \Pi^B = P_2q_2 - B(q_1, q_2) - C, \quad \text{respectively.}
\]

Now, let \( C = t(q_1 - q_i) \), \( q_i > q_1 \). That is, \( t \) is the compensation per unit reduction of \( q_i \) below \( q_1 \). We will determine the equilibrium \( t \), as well as the equilibrium reduction of firm A’s output below \( q_1 \), and thus the equilibrium compensation \( C \) in the way suggested by standard exchange theory.

Note that firm A is interested in its own output \( q_1 \) only, while firm B — due to the externality assumption made — is interested not only in its own output \( q_2 \), but also in \( q_1 \). Assuming price taking (competitive) behavior, firm A’s optimal output level is determined by the condition:

\[
P_1 = A_1(q_1) + t. \quad (3)
\]

Taking \( P_1 \) as given, we can express firm A’s optimal output \( q_1 \) as a function of \( t \), the size of the compensation (per unit of reduction). \( \hat{q}_1 \) is, by definition, A’s optimal output for \( t = 0 \). An increase in \( t \) will — assuming increasing marginal cost: \( A_{11} > 0 \) — induce a reduction in A’s optimal output \( q_1 \). Or, to express it differently, it will increase \( \hat{q}_1 - q_1 \), the reduction of A’s optimal output below \( \hat{q}_1 \). Thus (3) determines A’s supply of compensated output reductions, as a function of the size of the (per unit) compensation.

Similarly, firm B is in equilibrium if the following conditions are met:

\[
P_2 = B_2(q_1, q_2) \quad (4)
\]

and

\[
B_1(q_1, q_2) = t. \quad (5)
\]

(4) is the standard condition requiring equality between marginal cost and price for B’s output. But firm B also optimizes with respect to the other firm’s output, \( q_1 \). (5) equates B’s marginal expense of a unit reduction of \( q_1 \) with the corresponding marginal return. For any value of \( t \) (and \( P_2 \)), equations (4) and (5) jointly determine not only firm B’s optimal output \( q_2 \), but also its optimal or “desired” level of \( q_1 \), or, in other words, its demand for reductions of \( q_1 \) below \( \hat{q}_1 \). If B’s demand for compensated reductions is not equal to A’s supply, we have a disequilibrium situation, which presumably will lead to an adjustment in the compensation rate \( t \).
Only if A's supply and B's demand are identical is a full equilibrium situation established. Clearly, if \( t = 0 \), we do not have such an equilibrium, since in this case firm A (by definition) produces \( \hat{q}_1 \), i.e. supplies a zero reduction \( (\hat{q}_1 - q_1) \), while, on the other hand, firm B would desire a zero level of \( q_2 \) (or, at least, a level which brings \( B_1 \) down to zero, if this were possible at positive levels of \( q_1 \)).

The supply curve of firm A and the demand curve of firm B for reductions in \( q_1 \), labeled AA and BB, respectively, are shown in Figure 1. Note that in this figure, the origin represents the point where \( q_1 = \hat{q}_1 \), i.e., where the reduction of \( q_1 \) below \( \hat{q}_1 \) is zero, while the point marked D implies a level of \( q_1 \) equal to zero.

![Figure 1](image)

The slope of the supply curve AA is obtained from (3) as:

\[
\frac{dt}{d(\hat{q}_1 - q_1)} = A_{11} \quad (6)
\]

Increasing marginal cost in the production of \( q_1 \) (a necessary second order condition for a maximum of \( \Pi^A \)) thus implies a positive slope for AA.
The slope of the demand curve BB is obtained similarly from (4) and (5). In drawing this demand curve, we have to take into account the fact that — due to (4) — a change in $q_1$ (i.e., a horizontal movement in Figure 1) implies a change in the optimal $q_2$ (except in the case where $B_{21} = 0$, i.e., where the cost function is separable). Maintaining (4) requires $dq_2 = -(B_{21}/B_{22})dq_1$. Taking this into account, the slope of BB is:

$$\frac{dt}{d(q_1 - q_1)} = -\left[\frac{B_{11} - B_{12}B_{21}}{B_{22}}\right] = -\left[\frac{B_{11}B_{22} - B_{12}B_{21}}{B_{22}}\right]$$

(7)

Note that second order conditions for a maximum of $\Pi^B$ require $B_{11}B_{22} - B_{12}B_{21} > 0$, so that the slope of BB must be negative\(^3\). In the case of a separable cost function ($B_{12} = B_{21} = 0$), this slope is simply given by $B_{11}$. The existence of a nonzero cross effect obviously will make BB flatter, compared to this case\(^4\).

Clearly, the competitive equilibrium point $E$ given by the intersection of AA and BB satisfies the conditions for a social optimum: equations (3), (4) and (5) together imply:

$$P_1 = A_1(q_1) + B_1(q_1, q_2)$$

and

$$P_2 = B_2(q_1, q_2)$$

and thus determine a resource allocation identical with the one obtained in the case of joint optimization (Marchand and Russel's case d.). This is so regardless of whether the cost function is separable ($B_{12} = 0$) or not ($B_{12} \neq 0$).

Of course, in the context of this market for compensated adjustments in $q_1$, the standard problem of stability of equilibrium arises. Will the system converge to the equilibrium point $E$? However, we have no reason to believe that the significance of this problem is any different here than in any other market. It seems reasonable to assume that, as usual, an excess demand for compensated adjustments in $q_1$ would lead to a fall in price (the rate of compensation), and vice versa.

Given the assumption of competitive behavior, there is thus no reason to believe that the social optimum will not be reached, even in the case of a nonseparable cost function, contrary to Marchand and Russel's conclusion.

\(^3\) Note that $\frac{\partial^2 \Pi^B}{\partial q_1^2} = -B_{11}$, $\frac{\partial^2 \Pi^B}{\partial q_2^2} = -B_{22}$, etc.

\(^4\) We could, of course, derive similar curves (BB') from (5) by holding the level of $q_2$ parametrically constant. This would yield a whole family of curves, one for each $q_2$. Each of these curves, however, would represent overall equilibrium for firm B at one point only, namely at the $q_1$ level for which the constant $q_2$ level happens to be optimal (the $q_1$ level which, together with the given $q_2$ level satisfies (4)). These BB' curves would be flatter than the BB curve, and intersect the latter at the one point where the BB' curves represent overall equilibrium. In the case of a separable cost function ($B_{12} = 0$), all the BB' curves and BB coincide, of course.
The assumption of competitive behavior may in some cases of cost interactions be quite appropriate as an approximation for real market behavior. Consider, e.g., the example mentioned by Coase of a confectioner whose machinery inflicts noise and vibrations on his neighbor. If there are many such confectioners and many (potential) neighbors, we may in fact have a situation approximated quite well by the usual competitive market model. The same may be true for the well-known smoke producers, who may have numerous potential locations and neighboring communities, so that again it may be quite reasonable to use a competitive model to describe the "market" for compensated output adjustments. Or, to use Meade's [6] model of apple and honey production as an example for a positive externality: We may think of a situation with many (potential) beekeepers, each of whom has the choice between many (potential) locations and neighboring apple growers. A situation where the competitive model potentially fits very nicely is provided by the borrower-lender relationship in credit markets. Vernon Smith [10] has recently pointed out that the borrower's equity plays the role of an external economy to the lender, since the latter's expected profit or utility depends on it (if default risk is present), with the effect that the (traditional) competitive market equilibrium is not Pareto efficient. If there are numerous (similar) borrowers and lenders, the competitive model suggested above may quite adequately describe a market for compensated equity adjustments (where the compensations in this case presumably would simply take the form of adjustments or "premiums" in the contractual interest rate). These examples depict cases where each A type firm affects the costs of one well identified B type firm (in a well identified way, so that there is no problem of accountability), say, its neighbor, but there are many (potential) A and B type firms, so that there is effective competition between the various A and B firms, and no one can act as a monopolist.

II. Different Forms of Market Behavior

However, the framework suggested in this paper permits an analysis of the problem under different types of behavioral forms as well. For example, suppose that there is just one type A firm, but a large number of potential type B firms (potential neighbors). In this case, firm A can "exploit" the B firm, because the latter has no alternatives to turn to, and the A firm can set the compensation rate such that its marginal revenue (derived from the BB curve it faces) equals the marginal cost of reducing its output (given by $P_1 - A_1(q_1)$, i.e., by AA). Many other constellations are conceivable, among them the case of bilateral monopoly.

For a discussion of these cases, it may be useful to extend the diagram in Figure 1 by adding isoprofitlines for both firms. The isoprofitlines for firm A are obtained from (1) by keeping $\Pi^A$ parametrically constant. Their slope is:
\[
\left[ \frac{dt}{d(q_1 - q_1)} \right] d\Pi^A = 0 = -\left[ \frac{P_1 - A_1(q_1) - t}{q_1 - q_1} \right]
\] (8)

They have the form shown in Figure 2. They are convex from below, have a positive slope below the AA curve (where \(P_1 - A_1(q_1) > t\)) and a negative slope above AA (where \(P_1 - A_1(q_1) < t\)). The higher curves correspond to higher profit levels than the lower ones. A's supply curve AA, of course, is the locus of the minimum points of A's isoprofitlines.

The isoprofitlines for firm B are similarly obtained from (2) by keeping \(\Pi^B\) parametrically constant. Their slope is:

\[
\left[ \frac{dt}{d(q_1 - q_1)} \right] d\Pi^B = 0 = \frac{B_1(q_1, q_2) - t}{q_1 - q_1}
\] (9)

B's isoprofitlines are also shown in Figure 2. They are concave from below, have a positive slope below the BB curve (where \(B_1(q_1, q_2) > t\)) and a negative slope above BB (where \(B_1(q_1, q_2) < t\)). The lower curves correspond to a higher profit level than the higher ones. B's demand curve BB is the locus of the maximum points of B's isoprofitlines. Note that the following discussion is based on the assumption that, in drawing B's isoprofitlines we always adjust \(q_2\) such that condition (4) is maintained, implying \(dq_2 = -(B_{21}/B_{22})dq_1\), exactly as we did in deriving B's demand curve BB\(^5\).

If A acts as a monopolist, and B as a price taker, A will pick his best point on B's demand curve, labeled \(M^A\) in Figure 2. If, on the other hand, B acts as a monopolist, and A as a price taker, B would choose \(M^B\) (his best point on A's supply curve). In the case of bilaterally monopolistic behavior, a variety of outcomes of the negotiation process are possible, depending on the strategies of the two participants. Note that if A had a completely free hand in choosing both \((q_1 - q_1)\) and \(t\), with the only constraint that he must leave B at least as well off as he is in the initial situation (that is, at 0), he would choose \(F^A\), his best point on B's isoprofitline passing through 0. Firm B, in the same position would choose \(F^B\). In these cases, A alone or B alone captures the full gains from trade, respectively. The level of \(q_1\) corresponding to both of these points is the socially optimal one, \(q_1^*\)\(^6\). Points \(F^A\) and \(F^B\), as well as E, all are points of tangency between the

\(^5\) Alternatively, we could keep \(q_2\) parametrically constant in drawing B's isoprofitlines. This would yield a complete family of isoprofitlines for each \(q_2\) level. The loci of maximum points for these curves would give the BB' curves mentioned in footnote 4 above.

\(^6\) Note that in Marchand and Russel's analysis of the Coase argument (their case b.), firm A always obtains the full gains from trade, as pointed out in the introductory section. However, their formulation of the objective functions of the two firms prevents them from reaching the social optimum, so that they do not exhaust the possible gains. Their objective function for B immediately implies that firm B will produce at \(\hat{q}_B\), which is less than the socially optimal quantity (unless \(B_{12} = 0\).
two sets of isoprofitlines. The locus of such tangency points does appear as a vertical line in Figure 2. In one sense it is reasonable to expect that the negotiation process will lead to some point on this locus, because, as long as we are not on it, deals which are in the mutual interest of both parties are always possible. For example, in the case where one of the two firms can act as a monopolist, say A, leading to $M^A$, one could ask why A doesn’t offer extra reductions of $q_1$ beyond $M^A$ at a reduced price (while the “standard” monopoly price given by $M^A$ applies to the corresponding “standard” monopoly quantity). Clearly, this could be in the mutual interest of both parties. (They could exhaust part of the gains from trade possible by moving to $q^*_f$. $M^A$ is the best point for A as long as he can set a single price for his output only.) This argument would in the end lead to the socially optimal quantity $q^*_f$. This, of course, would be the case of a discriminating mo-

7 Equality between the slopes of A’s and B’s isoprofitlines requires $-\left[ P_1 - A_1(q_1) - t \right] = B_1(q_1, q_2) - t$, or $A_1(q_1) - B_1(q_1, q_2) - P_1 = 0$. This equation implicitly defines the locus of such tangency points. Since $t$ does not appear in it, this locus efficient points must appear as a vertical line in Figure 2.

8 This point is made by Guido Calabresi [2], who, however, also emphasizes that the presence of transactions and negotiation costs, which are neglected in the above discussion, may prevent that the efficient locus is reached. On this, see our concluding remarks.
 monopolist. If the monopolist can set different prices for different units of his pro-
duct, he would, presumably, behave as a perfectly discriminating monopolist to
begin with, which is well known to lead to the socially optimal quantity.

III. Imposition of Liability

So fare we have restricted our discussion to the case where the externality caus-
ing firm A is not legally liable for the damage it causes to firm B. We will now
briefly turn to the case where liability is imposed on A by law. In their analysis of
this case (their case c.), Marchand and Rüssel conclude that the resulting resource
allocation, contrary to Coase's conclusion, is not socially optimal, except in the
special case of a separable cost function. They find that firm B's output will ex-
cceed the socially optimal quantity \( q^*_B \), and A's output will be below \( q^*_A \), unless
\( B_{12} = 0 \).

Marchand and Rüssel's analysis is quite correct, as far as it goes. But it neglects
that in this case, precisely as in the case where A is not liable, we can conceive a
compensation scheme which may lead to the social optimum. If A has to always
reimburse B for the extra cost it causes to B (over what B's cost would be if A's
output were zero), B's profit becomes \( P_B q_2 - B(q_1, q_2) + [B(q_1, q_2) - B(0, q_2)]
= P_B q_2 - B(0, q_2) \), which is completely independent of \( q_1 \). Firm B will thus loose
any interest in \( q_1 \). However, firm A now is interested in B's output \( q_2 \), since his
legal liability to B (equal to \( B(q_1, q_2) - B(0, q_2) \)) is dependent on \( q_2 \) (unless \( B_{12} = 0 \)). A will therefore offer B compensations in order to induce it to reduce\(^9\) its
output below \( q^*_2 \) (the level it would be in the absence of such a compensation, i.e.
the level which satisfies \( B(0, q_2) = P_B \)). Similarly, B will offer reductions in its
output, depending on the size of the compensation rate. We can analyze the market
for compensated adjustments in \( q_2 \) in terms perfectly analogous to the case dis-
cussed in the previous parts of the paper.

The objective functions of the two firms now are:

\[
\Pi^A = P_B q_1 - A(q_1) - [B(q_1, q_2) - B(0, q_2)] - \hat{t}(q_2 - q_2) \tag{10}
\]
and

\[
\Pi^B = P_B q_2 - B(0, q_2) + \hat{t}(q_2 - q_2), \; q_2 > q_2^* \tag{11}
\]

where \( \hat{t} \) is the price per unit of reduction in \( q_2 \), and \( q_2^* \) the quantity of \( q_2 \) which
B would produce if \( \hat{t} = 0 \) (i.e., B's optimal output in Marchand and Rüssel's case c.).

\(^9\) This assumes \( B_B(q_1, q_2) > 0 \). Under this assumption, A is interested in a reduction of B's output
below \( q^*_2 \). If \( B_B(q_1, q_2) < 0 \), on the other hand, A would be interested in an increase of \( q_2 \) beyond \( q^*_2 \).

\(^{10}\) See footnote 9 above.
A optimizes with respect to both its own output $q_1$ and B's output $q_2$. Given price taking behavior, A's optimality conditions are:

$$P_1 = A_1(q_1) + B_1(q_1, q_2)$$  \hspace{1cm} (12)

and

$$B_2(q_1, q_2) - B_2(0, q_2) = \hat{t}. \hspace{1cm} (13)$$

These conditions together determine his optimal output $q_1$, as well as the $q_2$ which is optimal from his point of view, and thus his demand for compensated adjustments in $q_2$, as a function of $\hat{t}$.

B's profit (which depends on $q_2$ only) is maximized when:

$$P_2 = B_2(0, q_2) + \hat{t}. \hspace{1cm} (14)$$

This defines B's supply of adjustments in $q_2$ (below $q_2$), as a function $\hat{t}$. Second order conditions for (interior) maxima of $\Pi^A$ and $\Pi^B$ again imply a negatively sloped demand curve (for A) and a positively sloped supply curve (for B). The equilibrium $\hat{t}$ is reached where the optimal $q_2$ levels for both A and B coincide. Inspection of (12), (13) and (14) shows that at such a point the conditions for a socially optimal resource allocation are met, since they imply

$$P_1 = A_1(q_1) + B_1(q_1, q_2)$$

and

$$P_2 = B_2(q_1, q_2)$$

which is identical with the conditions obtained in the case of joint maximization.

As emphasized by Coase, the difference between the case discussed here (liability imposed on A) and the case discussed before is simply a different allocation of property rights. The case discussed first gives A the right to damage B, while the second one does not give him this right and therefore requires him to reimburse B for the damage done to him. Both cases lead to nonoptimal results without compensation schemes (including the one where liability is imposed on A, except in the special case where $B_{12} = 0$, as pointed out by Marchand and Rüssel). But both will result in socially optimal allocations, given an appropriately operating market for compensations.

**IV. Concluding Remarks**

In conclusion, it must be mentioned that the above discussion does neglect certain potentially important factors\(^{11}\). In particular, it disregards the role of the costs

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\(^{11}\) For a critical discussion of the Coase argument, see, e.g., E.J. Mishan [8], and G.A. Mumey [9].
of negotiating and transacting. In fact, of course, such costs do exist and may be quite important, and if so, will change the solution to the problem (as pointed out by Coase in his original paper). In this respect, the presented analysis of the negotiation process is incomplete. But, of course, this is something which does apply to the discussion of market transactions in general, as much as it does to the particular problem under discussion here. Nevertheless, when we want to derive policy conclusions concerning the definition of liability laws and similar matters, these costs, of course, must be taken into account.

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Zusammenfassung

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Sommaire

*Effets extrinsèques, séparabilité et allocation de ressources*

Cet article est consacré à la démonstration de Coase sur le défaut de pertinence en matière d’allocation de ressources. Marchand et Russel ont récemment réexaminé le raisonnement de Coase, et leur conclusion est que ses résultats ne sont pas universellement valables, mais uniquement recevables dans le cas particulier comportant des fonctions de coûts additionnellement séparables. Le présent article démontre que cette conclusion est basée sur une équivoque dans la manière de poser le problème. Aussi longtemps que les coûts procédant des transactions et des négociations ne sont pas pris en ligne de compte, une formulation convenable du problème engendre les résultats obtenus par Coase, ceci indépendamment de la question de savoir si la fonction de coût est séparable ou non.

Summary

*External Effects, Separability, and Resource Allocation*

This paper deals with the Coase Theorem on the irrelevance of liability arrangements with respect to resource allocation. In a recent reexamination of the Coase argument, Marchand and Russel came to the conclusion that Coase's result regarding the neutrality of liability arrangements on resource allocation is not generally true, but holds only in the restricted case of additively separable cost functions. This paper demonstrates that this conclusion is based on a defective formulation of the problem. As long as the costs of transacting and negotiating are disregarded, an appropriate formulation of the problem generates the Coase results, regardless of whether the cost function is separable or not.