International Factor Movements,
Allocation and Prices

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1. Introduction

The world economy increasingly involves capital movements and labour migration: Savings are invested in the country yielding the greatest return, and labour moves to the country which offers the highest wages. This development is especially true of common markets such as the European Community, which are based on free trade and unhampered factor movements. At private level, this process has been advanced by multinational firms. Against this background, the present paper investigates international factor movements.

Take the standard model of international trade as a baseline. Two countries competitively produce two commodities with the help of two factors. Let technology be neoclassical and the same in both countries. There are no transport costs, and factors are trapped in their respective country. Then commodity trade equalizes factor prices (Samuelson 1948). The mirror image economy to the standard model is due to Mundell (1957) who considered mobility of factors, but immobility of commodities. Accordingly, factor movements equalize commodity prices. These papers gave rise to a continuing debate (e.g. Chipman 1971, Nadel 1971, Purvis 1972, Krauss 1974, Rodriguez 1975, Batra and Ramachandran 1980).

Speaking more generally, if the number of commodities equals or exceeds the number of factors, then commodity trade equalizes factor prices (Samuelson 1953). Upon admitting factor mobility, factors still have no incentive to move, and the allocation of mobile factors is indeterminate. There is no compelling reason, however, why there must be more commodities than factors. On the other hand, if the number of commodities falls short of the number of factors, then commodity trade no longer equalizes factor prices. In this situation, factor mobility makes a great deal of difference. Under mobility of some factors, the prices of immobile factors are equalized to a certain extent, and the allocation of mobile factors is determinate.

In discussing international factor movements, the present paper focuses on primary factors of production such as capital, labour, land and natural resources. In the next section, some basic results are derived for a single commodity. After that, more commodities are taken into account. In the last section, the analysis is extended to include an aggregate immobile factor, the balance of trade, differences in technology, and barriers to trade.
2. International Factor Movements

Consider two countries (subscript \( i = 1,2 \)) producing a homogeneous commodity \( X \) by means of capital \( K \) and labour \( N \). Let the production function be neoclassical, that is twice continuously differentiable with diminishing marginal products and constant returns to scale. We assume that the international diffusion of technical knowledge leads in the long run to a uniform technology in both countries. This does not mean, however, that the countries are completely identical. On the contrary, they differ with respect to capital and labour endowment. Thus technology takes the shape \( X_i = F(K_i, N_i) \). It is convenient to carry out the analysis in per capita terms. Then output per head \( x_i := X_i/N_i \) is a well-known increasing function \( f \) of capital per head \( k_i := K_i/N_i \).

Output is shipped to the country paying the best price, so prices coincide (excluding transport costs). Output price is set to unity. We assume that firms employ factors of production in such a way as to maximize profit under perfect competition. The necessary condition is that factor prices correspond to marginal products:

\[
    r_i = \frac{df}{dk_i}, \quad w_i = f - k_i \frac{df}{dk_i}, \tag{2.1}
\]

where \( r_i \) denotes the interest rate and \( w_i \) the wage rate. If capital and labour are trapped in their respective country, then factor prices generally differ (unless the capital-labour ratios happen to be equal).

On the other hand, if capital is allowed to move from country to country, then interest rates are equalized \( r_1 = r_2 \). This implies equal marginal products of capital, equal capital-labour ratios and thus equal marginal products of labour:

\[
    \frac{df}{dk_1} = \frac{df}{dk_2}, \quad k_1 = k_2, \quad f - k_1 \frac{df}{dk_1} = f - k_2 \frac{df}{dk_2}. \tag{2.2}
\]

As a consequence, wage rates are equalized between countries \( w_1 = w_2 \), although labour is immobile and the countries differ with respect to labour endowment. What is more, efficiency increases upon introducing capital mobility. National capital (labour) adds up to world capital (labour) \( K = K_1 + K_2 \) (\( N = N_1 + N_2 \)). The international allocation of capital conforms with the international allocation of labour, which is given exogenously: \( K_i/K = N_i/N = b \). Summing up, the price of the immobile factor is equalized, and the allocation of the mobile factor is determinate.

If both capital and labour move freely between countries, then interest rates and wage rates are equalized. This induces equal marginal products and equal capital-labour ratios. The international allocation of capital and labour is no more fixed. In other words, capital mobility can take the place of labour mobility, and vice versa. Whether capital movements or labour migration actually occur, depends on mobility costs. As a result, factor prices are equalized in a trivial way, and the allocation of factors is indeterminate.
This particular outcome is due to the fact that immobile factors such as land, climate and environment have been ignored. For this reason we take account of a third immobile factor, briefly to be called land A. This involves a different approach; the modified production function is \( X_i = F(K_i, N_i, A_i) \). The analysis will be implemented in terms of land: \( x_i = f(k_i, n_i) \) with \( x_i = X_i/A_i, k_i = K_i/A_i \) and \( n_i = N_i/A_i \). Factor prices correspond to marginal products, see (2.1):

\[
\begin{align*}
    r_i &= \frac{\partial f}{\partial k_i}, \\
    w_i &= \frac{\partial f}{\partial n_i}, \\
    q_i &= f - k_i \frac{\partial f}{\partial k_i} - n_i \frac{\partial f}{\partial n_i},
\end{align*}
\]

where \( q_i \) denotes the land rent. If capital is allowed to move, then interest rates and marginal products of capital are equalized:

\[
    r_1 = r_2, \quad \frac{\partial f}{\partial k_1} = \frac{\partial f}{\partial k_2}.
\]

The international allocation of land \( A_i : = A_i/A \) is given exogenously with \( A : = A_1 + A_2 \). Weighting the capital-land ratios at national level by the international allocation of land gives the fixed capital-land ratio at world level \( k \):

\[
k = a_1 k_1 + a_2 k_2 \text{ with } k := \frac{K}{A}.
\]

This makes two equations (2.4), (2.5) with two variables \( k_i \), hence a solution exists. The capital-land ratios and the labour-land ratios generally differ \( k_1 \neq k_2, n_1 \neq n_2 \). Wage rates and land rents are not equalized \( w_1 \neq w_2, q_1 \neq q_2 \), in contrast to the conclusions drawn from the two-factor model. The international allocation of capital is governed by the fixed allocation of land and by the national capital-land ratios: \( K_i/K = a_i k_i/k \). In summary, the prices of the immobile factors generally differ, and the allocation of the mobile factor is determinate.

If labour is allowed to move, too, then wage rates and marginal products of labour are equalized:

\[
w_1 = w_2, \quad \frac{\partial f}{\partial n_1} = \frac{\partial f}{\partial n_2}.
\]

According to (2.4) and (2.6), the same capital-land ratio, the same labour-land ratio and thus the same marginal product of land obtain in both countries:

\[
k_1 = k_2 = k, \quad n_1 = n_2 = n,
\]

\[
f - k_1 \frac{\partial f}{\partial k_1} - n_1 \frac{\partial f}{\partial n_1} = f - k_2 \frac{\partial f}{\partial k_2} - n_2 \frac{\partial f}{\partial n_2}.
\]

So land rents are equalized \( q_1 = q_2 \), although land is immobile and the countries differ with respect to land endowment. The international allocation of capital and labour agrees with the fixed allocation of land: \( K_i/K = N_i/N = a_i \). As a conse-
sequence, the price of the immobile factor is equalized, and the allocation of the mobile factors is determinate.

If, however, each factor can move freely (let the third factor be, for instance, natural resources), then all factor prices are equalized in a trivial way. The allocation of factors is indeterminate, as long as the same capital-land ratio and labour-land ratio prevails in both countries.

So far, the analysis has been confined to a single commodity. We turn now to the standard model of international trade. Two countries competitively produce two commodities with the help of two factors. Commodities are traded freely, while factors are trapped in their respective country. As a result, the prices of the immobile factors are equalized (Samuelson 1948). If factor mobility is introduced into the standard model, then factors still have no incentive to move, and the allocation of mobile factors is indeterminate.

When a third factor is incorporated into the standard model, commodity trade no longer equalizes factor prices. To see this, consider two countries producing two commodities (superscript $j = 1, 2$) by means of capital, labour and land. The production function is assumed to differ from industry to industry: $X_j = F_j(K_j, N_j, A_j)$ or $x_j = f_j(K_j, N_j)$ with $x_j := X_j/A_j$, $k_j := K_j/A_j$ and $n_j := N_j/A_j$. Commodity $i$ is shipped to the country paying the best price, so commodity prices $p_i$ coincide $p_1 = p_2 = p^i$.

To begin with, we assume international mobility of capital, but immobility of labour and land. Interest rates are equalized, since capital moves freely between countries and industries: $r_1 = r_2$. Labour and land move freely between industries, but are trapped in their respective country. Therefore, wage rates and land rents are equalized only between industries: $w_1 = w_2$, $q_1 = q_2$, $q_j = q^j$. Moreover, it is useful to define $K := \Sigma K_j$, $N := \Sigma N_j$, $A := \Sigma A_j$, $N_i := N_1 + N_2$, $A_j := A_1 + A_2$, $a_j := A_j/A$. The national labour-land ratios $n_i := N_i/A_i$ and the international allocation of land $a_i := A_i/A$ are invariant. Equal factor prices imply equal marginal products:

$$p_1 \frac{\partial f_1}{\partial k_1} = p_2 \frac{\partial f_2}{\partial k_2} = p_1 \frac{\partial f_1}{\partial k_1} = p_2 \frac{\partial f_2}{\partial k_2}. \quad (2.8)$$

$$p_1 \frac{\partial f_1}{\partial n_1} = p_2 \frac{\partial f_2}{\partial n_1}. \quad (2.9)$$

$$p_1 \frac{\partial f_1}{\partial n_2} = p_2 \frac{\partial f_2}{\partial n_2}. \quad (2.10)$$

$$p_1 \left( f_1 - k_1 \frac{\partial f_1}{\partial k_1} - n_1 \frac{\partial f_1}{\partial n_1} \right) = p_2 \left( f_2 - k_2 \frac{\partial f_2}{\partial k_2} - n_2 \frac{\partial f_2}{\partial n_2} \right). \quad (2.11)$$

$$p_1 \left( f_1 - k_2 \frac{\partial f_1}{\partial k_2} - n_2 \frac{\partial f_1}{\partial n_2} \right) = p_2 \left( f_2 - k_2 \frac{\partial f_2}{\partial k_2} - n_2 \frac{\partial f_2}{\partial n_2} \right). \quad (2.12)$$
Weighting the individual capital-land ratios by the allocation of land yields the world capital-land ratio:

\[ k = \sum a_i^j k_i. \]  
\[(2.13)\]

Correspondingly, the individual labour-land ratios combine to give the national labour-land ratio:

\[ a_i n_i = a_1^1 n_1^1 + a_2^2 n_2^2. \]  
\[(2.14)\]

The land shares of industries 1 and 2 add up to the national land share:

\[ a_i = a_1^1 + a_2^2. \]  
\[(2.15)\]

We arrive at 12 equations from (2.8) to (2.15) and 12 variables \( k_i, n_i, a_i \). As a result, the prices of the immobile factors generally differ, and the allocation of the mobile factor is determinate.

If labour is no longer trapped in its respective country, then all wage rates balance \( w_1^1 = w_1^2 = w_2^2 = w_2^3 \). This gives rise to equal marginal products of labour, thus (2.9) equals (2.10). 8 equations from (2.8) to (2.12) and 8 variables \( k_i, n_i \) emerge. Due to symmetry, the same factor intensity prevails in both countries \( k_1^1 = k_1^2, n_1^1 = n_1^2 \). As a consequence, (2.11) equals (2.12). Land rents are equalized \( q_1^1 = q_1^2 = q_2^1 = q_2^2 \), although land is immobile and the countries differ with respect to land endowment. Upon introducing labour mobility, the individual capital-land ratios and labour-land ratios become independent of factor endowment. Is this outcome consistent with a given factor endowment at world level? Land is reallocated appropriately from industry to industry: \( k = \sum a_i^j k_i, n = \sum a_i^j n_i, a_1^1 + a_2^2 = a_i. \) This makes 4 equations and 4 variables \( a_i \). The reallocation of land is accompanied by a corresponding shift in capital and labour. Summarizing, the price of the immobile factor is equalized, and the allocation of mobile factors is determinate.

If the third factor, too, can move freely between countries and industries, then all factor prices are equalized. Hence (2.11) equals (2.12). We arrive at 9 equations from (2.8) to (2.12) and 8 variables \( k_i, n_i \), where 1 equation is redundant. The individual factor intensities are still independent of factor endowment. Accordingly, factors shift between countries and industries: \( k = \sum a_i^j k_i, n = \sum a_i^j n_i, l = \sum a_i^j \). These are 3 equations and 4 variables \( a_i \), so there is a degree of freedom. As a consequence, factor prices are equalized in a trivial way, and the allocation of factors is indeterminate.

Until now, the one-commodity two-factor model has been increasingly disaggregated by regarding more commodities and factors. All these extensions point to the general problem of \( I \) countries producing \( J \) commodities with the help of \( M \) factors. Obviously, the number of countries does not matter, hence it is sufficient to consider
two countries. If \( J \geq M \), then international commodity trade equalizes factor prices. Upon admitting international factor mobility, factors still have no incentive to move, and the allocation of mobile factors is indeterminate. There is no compelling reason, however, why there must be more commodities than factors. On the other hand, if \( J < M \), then international commodity trade no longer equalizes factor prices. Upon introducing international factor movements, everything depends on the number of mobile factors \( M' \). If \( M' < M - 1 \), then the prices of immobile factors generally differ, and the allocation of mobile factors is determinate. If \( M' = M - 1 \), then the price of the immobile factor is equalized, and the allocation of mobile factors is determinate. If \( M' = M \), then factor prices are equalized in a trivial way, and the allocation of factors is indeterminate. The proof can be found in the appendix.

3. Extensions of the Analysis

If there is no immobile factor at all, then factor prices are equalized in a trivial way, and the allocation of factors is indeterminate. In reality, however, there is at least one immobile factor, that is land. If there is exactly one immobile factor, then its price is equalized, and the allocation of mobile factors is determinate. If there are more immobile factors, then the allocation of mobile factors is determinate, but the prices of immobile factors generally differ. It can be established, however, that these prices are equalized on the average. Amalgamate all immobile factors into a single one, then the price of the aggregate immobile factor is equalized.

This process will now be looked into more closely. Consider two countries producing a homogeneous commodity by means of capital, labour and land. Assume, for instance, a *Cobb-Douglas* technology \( X_i = K_i^\alpha N_i^\beta A_i^\gamma \) with \( \alpha + \beta + \gamma = 1 \). Let capital move freely between countries, while labour and land are fixed. Labour and land are merged into the aggregate immobile factor \( Q_i \), with price \( \pi_i \): \( X_i = K_i^\alpha Q_i^{\beta+\gamma} \). Hence \( Q_i \) is a weighted geometric mean of \( N_i \) and \( A_i \): \( Q_i^{\beta+\gamma} = N_i^\beta A_i^\gamma \). Labour and land income add up to the income of the aggregate immobile factor, so the price of the aggregate immobile factor is a "weighted mean" of the wage rate and the land rent: \( \pi_i = w_i N_i/Q_i + q_i A_i/Q_i \). Note that \( \pi_i \) may be greater than \( w_i \) and \( q_i \), since \( N_i/Q_i + A_i/Q_i > 1 \). We arrive at the one-commodity two-factor model which we discussed in the beginning. The price of the aggregate immobile factor is equalized, since capital is mobile. Although wage rates and land rents generally differ, they are equalized on the average. Thus the analysis in the last section gains in importance.

So far, the balance of factor trade has been ignored. This may be justified in the case of labour migration, because labour imports do not have to be paid for (with the exception of guest workers’ remittances). On the other hand, imports of natural resources do have to be paid for. How is the allocation of mobile factors affected by the balance of trade? Consider two countries producing a homogeneous commod-
ity with the help of two natural resources, to be called K and N. Let the production function \( X_i = F(K_i, N_i) \) again be neoclassical. The natural resources move freely from country to country, so their prices are equalized \( r_1 = r_2 = r \), \( w_1 = w_2 = w \). As a result, the allocation of natural resources is indeterminate, as long as the same factor intensity prevails in both countries:

\[
\frac{K_1}{N_1} = \frac{K_2}{N_2}. \tag{3.1}
\]

To avoid the extraction problem, we assume that the initial endowments \( K_i, N_i \) are used up in the current period:

\[
K_1 + K_2 = K_1 + K_2, \quad N_1 + N_2 = N_1 + N_2. \tag{3.2}
\]

Suppose that the balance of commodity trade is in equilibrium. The equilibrium condition for the balance of factor trade is:

\[
(K_1 - K_2) r = (N_1 - N_2) w. \tag{3.3}
\]

We obtain 4 equations (3.1), (3.2), (3.3) and 4 variables \( K_i, N_i \). When the balance of factor trade is in equilibrium, the allocation of natural resources is determined by initial endowments.

In the preceding analysis, the same production functions have been assumed in both countries. But the international diffusion of technical knowledge may be impeded, giving rise to different production functions. In this case, the prices of immobile factors are no longer equalized. However, there is at least a tendency towards factor price equalization, as will be demonstrated now. Consider two countries producing a homogeneous commodity with the aid of capital K and labour N. We assume a neoclassical production function which differs from country to country:

\( X_i = f_i(K_i, N_i) \) or \( x_i = f_i(k_i) \) with \( x_i := X_i/N_i \) and \( k_i := K_i/N_i \). Capital is allowed to move, whereas labour is fixed. In the initial state, imagine \( r_1 < r_2 \), then generally \( w_1 > w_2 \). In response, capital moves to country 2 until the interest rates are equalized. This is associated with a wage increase in country 2, while there is a decline in country 1. As a consequence, there is a tendency towards factor price equalization.

Yet there is an exception to this rule. If in the initial state both \( r_1 < r_2 \) and \( w_1 < w_2 \), then capital movements widen the gap between wage rates. However, this case is unlikely to occur, which can be shown as follows. Figure 1 illustrates the international allocation of capital and labour, where \( f_i \) represents the per capita production function. Now draw a tangent to one of these functions, the slope of which exhibits the marginal product of capital and thus the interest rate. The x-intercept indicates the marginal product of labour which, in turn, is equal to the wage rate. Due to international capital movements, the same interest rate prevails in both countries. Hence
the tangents to both product curves must have the same slope. In addition, the capital-labour ratios at national level must be compatible with the given capital-labour ratio at world level \( k = b_1 k_1 + b_2 k_2 \). Thus the \( k_j \) are determinate. Both \( r_1 < r_2 \) and \( w_1 < w_2 \) in the initial state can only happen if \( k_i(k_2) \) is situated slightly to the right (left) of its equilibrium value. As a result, this case is unlikely to occur.

![Figure 1](image.png)

**Figure 1**

The international allocation of capital and labour

Last but not least, we turn to the mirror image economy. If there are barriers to trade such as transport costs, tariffs or quotas, then commodity prices differ. Factor movements, however, can equalize commodity prices. To see this, consider two countries producing a homogeneous commodity by means of capital and labour. Let technology be neoclassical and the same in both countries: \( X = F(K, N) \) or \( x_i = f(k_i) \) with \( x_i = X_i/N_i \) and \( k_i = K_i/N_i \). Commodity prices are assumed to be different arbitrary constants \( p_i \neq p_2 \). Factor prices correspond to marginal value products:

\[
    r_i = p_i \frac{df}{dk_i}, \quad w_i = p_i \left( f - k_i \frac{df}{dk_i} \right). \tag{3.4}
\]

Now capital is allowed to move, while labour is trapped. Capital rentals are equalized in commodity terms, hence marginal physical products of capital are balanced:

\[
    \frac{r^1}{p_1} \frac{r^2}{p_2}, \quad \frac{df}{dk_1} = \frac{df}{dk_2}. \tag{3.5}
\]
The same capital-labour ratio, the same marginal physical product of labour and the same wage rate in commodity terms prevail in both countries:

\[ k_1 = k_2, \quad f - k_1 \frac{df}{dk_1} = f - k_2 \frac{df}{dk_2}, \quad \frac{w_1}{p_1} = \frac{w_2}{p_2}. \]  
(3.6)

As a consequence, the price of the immobile factor is equalized in commodity terms, and the allocation of the mobile factor is determinate. According to (3.5) and (3.6), commodity prices are equalized in factor terms: \( p_1/r_1 = p_2/r_2, \) \( p_1/w_1 = p_2/w_2. \) Under money illusion, however, capital rentals are equalized in money terms \( r_1 = r_2. \) Then marginal physical products of capital differ, which proves to be inefficient.

Finally, a second commodity is introduced into the picture. Technology is assumed to differ from industry to industry:

\[ X_i = F^i (K_i, N_i) \text{ or } x_j = f^j (k_j) \text{ with } x_j := X_j/N_j \text{ and } k_j := K_j/N_j. \]

Let commodity prices again be distinct \( p_1 \neq p_2. \) Factor prices correspond to marginal value products:

\[ r_i = p_i \frac{df^i}{dk_i}, \quad w_i = p_i \left( f^i - k_i \frac{df^i}{dk_i} \right). \]
(3.7)

Capital rentals are equalized, since capital is allowed to flow from country to country and from industry to industry: \( r_1 = r_2, r_1' = r_2', r_1'/p_1 = r_2'/p_2, r_1'/p_1 = r_2'/p_2. \) Labour moves freely between industries, but is restricted to its respective country. Therefore, wage rates are equalized only between industries: \( w_1 = w_2, w_1' = w_2'. \) Thus the same wage rate in terms of the commodity produced obtains in both countries:

\[ \frac{df^1}{dk_1} = \frac{df^1}{dk_1}, \quad \frac{df^2}{dk_2} = \frac{df^2}{dk_2}. \]  
(3.8)

\[ k_1 = k_2, \quad k_1' = k_2'. \]  
(3.9)

\[ f^1 - k_1 \frac{df^1}{dk_1} = f^1 - k_2 \frac{df^1}{dk_2}, \quad f^2 - k_1 \frac{df^2}{dk_1} = f^2 - k_2 \frac{df^2}{dk_2}. \]  
(3.10)

\[ \frac{w_1}{p_1} = \frac{w_2}{p_2}, \quad \frac{w_1'}{p_1'} = \frac{w_2'}{p_2'}. \]  
(3.11)
On account of \( w_1^1 = w_1^2 \) and \( w_2^1 = w_2^2 \), this holds in terms of the other commodity as well:

\[
\frac{w_1^1}{p_1^1} = \frac{w_2^1}{p_2^1}, \quad \frac{w_1^2}{p_1^1} = \frac{w_2^2}{p_2^1}.
\] (3.12)

As a consequence, the price of the immobile factor is again equalized in commodity terms, and the allocation of the mobile factor is determinate. (3.11) and (3.12) demonstrate that commodity prices are equalized in labour terms. Correspondingly, they are equalized in capital terms. Under factor mobility, \( p_j^1 \) is arbitrary but adjusts to \( p_j^1/p_j^2 = p_1^2/p_2^2 \). In summary, factor movements can equalize commodity prices.

4. Appendix

Consider two countries (subscript \( i = 1,2 \)) producing \( J \) commodities (superscript \( j = 1,\ldots, J \)) by means of \( M \) factors (superscript \( m = 1,\ldots, M \)). \( X_i^j \) denotes output, \( V_i^j \) input. The production functions differ from industry to industry: \( X_i^j = F^j(V_i^{j1}, \ldots, V_i^{jM}) \). Restate this in terms of factor \( M \): \( x_i^j = \bar{F}(v_i^{j1}, \ldots, v_i^{jM-1}) \) with \( x_i^j := X_i^j/V_i^j \) and \( v_i^{jm} := V_i^{jm}/V_i^j \). Let \( J < M \). \( M - 1 \) factors are allowed to move freely between countries, say \( m = 1,\ldots, M - 1 \). Equal factor prices imply equal marginal products:

\[
p_i^1 \frac{\partial f_i^1}{\partial v_i^{j1}} = \cdots = p_j^1 \frac{\partial f_j^j}{\partial v_j^{j1}}, \ldots, \quad p_i^1 \frac{\partial f_i^1}{\partial v_i^{jM-1}} = \cdots = p_j^j \frac{\partial f_j^j}{\partial v_j^{jM-1}}. \] (A.1)

\[
p_i^1 \left( f_i^1 - v_i^{j1} \frac{\partial f_i^1}{\partial v_i^{j1}} - \cdots - v_i^{jM-1} \frac{\partial f_i^1}{\partial v_i^{jM-1}} \right) = \cdots =
\]

\[
p_i^1 \left( f_i^j - v_i^{j1} \frac{\partial f_j^j}{\partial v_j^{j1}} - \cdots - v_i^{jM-1} \frac{\partial f_j^j}{\partial v_i^{jM-1}} \right). \] (A.2)

\[
p_i^1 \left( f_i^j - v_i^{j1} \frac{\partial f_i^1}{\partial v_i^{j1}} - \cdots - v_i^{jM-1} \frac{\partial f_i^1}{\partial v_i^{jM-1}} \right) = \cdots =
\]

\[
p_i^1 \left( f_j^j - v_j^{j1} \frac{\partial f_j^j}{\partial v_j^{j1}} - \cdots - v_j^{jM-1} \frac{\partial f_j^j}{\partial v_j^{jM-1}} \right). \] (A.3)
(A.1), (A.2) and (A.3) comprise $2JM - M - 1$ equations and $2JM - 2J$ variables $v_{jm}$. If $M + 1 > 2J$, then there are less equations than variables; otherwise a corner solution may occur. Due to symmetry, the same factor intensities prevail in both countries:

\[ v_{1m} = v_{2m}, \quad \frac{\partial f^j}{\partial v_{1m}} = \frac{\partial f^j}{\partial v_{2m}}. \]  

(A.4)

Therefore, (A.2) equals (A.3). As a result, the price of the immobile factor is equalized, and the allocation of mobile factors is determinate.

REFERENCES


Summary

*International Factor Movements, Allocation and Prices*

If the number of commodities equals or exceeds the number of factors, commodity trade equalizes factor prices (Samuelson 1953). Upon admitting factor mobility, factors still have no incentive to move, and the allocation of mobile factors is indeterminate. On the other hand, if the number of commodities falls short of the number of factors, commodity trade no longer equalizes factor prices. In this situation, factor mobility makes a great deal of difference. Under mobility of some factors, the prices of immobile factors are equalized to a certain extent, and the allocation of mobile factors is determinate.

Zusammenfassung

*Internationale Faktorbewegungen, Allokation und Preise*

Gibt es ebenso viele oder mehr Produkte als Faktoren, dann führt der Produkthandel zum Ausgleich der Faktorpreise (Samuelson 1953). Wird Faktormobilität zugelassen, so besteht gleichwohl kein Anreiz zu Faktorbewegungen, und die Allokation der mobilen Faktoren ist unbestimmt. Gibt es dagegen weniger Produkte als Faktoren, dann führt der Produkthandel nicht mehr zum Ausgleich der Faktorpreise. In dieser Situation spielt die Faktormobilität eine bedeutende Rolle. Bei Mobilität einiger Faktoren gleichen sich die Preise der immobilen Faktoren in gewisser Weise an, und die Allokation der mobilen Faktoren ist bestimmt.

Résumé

*Mouvements internationaux des facteurs, allocation et prix*

Si le nombre des produits est égal ou supérieur au nombre des facteurs, le commerce des produits se traduit dans une égalisation des prix des facteurs (Samuelson 1953). Etant admise la mobilité des facteurs, il ne demeure cependant aucune impulsion pour le mouvement des facteurs, et l'allocation des facteurs mobiles est indéterminée. Si, par contre, il y a moins de produits que de facteurs, alors le commerce des produits n’implique plus une égalisation des prix des facteurs. Dans cette situation la mobilité des facteurs joue un rôle important. En cas de mobilité de quelques facteurs, les prix des facteurs immobiles s’égalisent d’une certaine manière, et l’allocation des facteurs mobiles est déterminée.