Risk Aversion and the Composition of Wealth in the Demand for Full Insurance Coverage*

By Marc Chesney, Henri Loubergé, Geneva**

1. Introduction

It is a widely-held view in the economics of uncertainty that insurance is an inferior good. This view, which led Hoy/Robson (1981) to investigate the conditions for insurance to be a Giffen good, is based on the plausible hypothesis of decreasing absolute risk aversion (DARA). It originates in the seminal paper of Mossin (1968) who used the DARA assumption to prove two propositions:

1) The maximum acceptable premium for full insurance coverage against a loss of fixed magnitude is a decreasing function of wealth;
2) under partial insurance, the optimal amount deductible is an increasing function of the individual's wealth.

Although Mossin's results are intuitively appealing, they seem to be contradicted by the widespread empirical observation that the income elasticity of insurance spending is greater than unity, an observation linked by Szpiro (1983) to Arrow's (1970) hypothesis of increasing relative risk aversion (IRRA). Some clarification about the effect of wealth on the demand for insurance is thus needed.

In this paper, we keep fundamentally to the well-known framework introduced by Mossin: we consider the demand for full insurance coverage by a risk-averse expected utility maximizer who has a part of his wealth exposed to an insurable risk of complete loss. But as the individual's demand for insurance is obviously not independent from the mixture of his assets, we investigate how his willingness-to-buy full insurance coverage is affected when the amount of wealth and its composition are simultaneously affected by some exogenous factor. It is shown that Mossin's first proposition and Szpiro's result are special cases in a more general model where the effects of risk aversion and of changes in the composition of wealth are considered jointly.

** Department of Political Economy, University of Geneva. The authors are indebted to Karl Borch, Eric Briys, Georges Dionne and Louis Eeckhoudt for their comments on an earlier version of this paper. They remain however responsible for any errors or omissions.
Section 2 presents the model and our main assumptions. Section 3 analyzes the conditions under which the maximum acceptable insurance premium decreases when the amount of total wealth increases. It comes out that the DARA assumption is not sufficient when the structure of wealth is explicitly considered. But the effect of this latter factor is not trivial: indeed the maximum amount spent on insurance may decrease when the value of risky wealth increases. In section 4, it is argued that the willingness-to-insure is better reflected by the maximum acceptable insurance premium rate than by the maximum acceptable insurance premium. The analysis is thus taken up again accordingly. It comes out that the coefficient of relative risk aversion and the wealth elasticity of insurable risky wealth are the main determinants of changes in the willingness-to-insure. In section 5, the main results of the paper and their implications for empirical studies on insurance demand and risk-averse behavior are summarized.

2. The model

Consider an individual whose preferences over risky prospects are represented by a von Neumann Morgenstern utility function $u(\cdot)$, with $u'(\cdot) > 0$. It is assumed throughout the paper that the individual is risk averse and that his absolute risk aversion (in the Arrow-Pratt sense) is a decreasing function of wealth. Thus, $u''(\cdot) < 0$ and $u'''(\cdot) > 0$.

The individual owns total wealth to an amount of $W$. Part of this wealth, to an amount of $L$, is non negotiable (e.g. human capital) and subject to an insurable risk of full loss with probability $\pi$. The rest of this wealth, to an amount of $A$, is invested in non-risky assets. Thus, the insurable loss is the only source of risk.

The maximum insurance premium $P$ the individual would be willing to pay for full coverage is given by

$$\pi u(A) + (1 - \pi) u(A + L) = u(A + L - P). \quad (1)$$

We are interested in the effect that changes in wealth have on $P$. Changes in wealth are assumed to occur from exogenous changes in $A$ and/or $L$. Differentiating (1) we obtain:

---

1 For example, the risk that the hands of a pianist, the face of an actress, the legs of a professional footballer, etc. may be severely injured in an accident. As the risks affecting human capital cannot be diversified, insurance is a particularly well-appropriate risk-transfer mechanism in this instance: see Mayers/Smith (1983).

2 Turnbull (1983) examines the case of multiple sources of risk: risky assets and a random market value of the insurable wealth provide the background to the insurance decision. But he does not consider the impact of changes in the composition of wealth.
\[ dP = -\frac{1}{u'(A + L - P)} \{ N(P) dA + Z(P) dL \}. \]  

Where 

\[ N(P) = \pi u'(A) + (1 - \pi) u'(A + L) - u'(A + L - P) > 0 \]

\[ Z(P) = (1 - \pi) u'(A + L) - u'(A + L - P) < 0. \]

\( Z(P) \) is negative by decreasing marginal utility. \( N(P) \) was studied by Mossin (1968) and shown to be positive under the DAR\( \Delta \)R assumption. Hence, from (2), it is straightforward that:

\[ \frac{dP}{dL} \bigg|_{dA=0} > 0 \quad \text{and} \quad \frac{dP}{dA} \bigg|_{dL=0} < 0. \]

The LHS of (3) indicates that the maximum acceptable insurance premium increases when the increase in wealth originates in risky wealth only. The RHS of (3) corresponds to Mossin's (1968) first results. It is generally interpreted as meaning that insurance is an inferior good: see e.g. Hoyl/Robson (1981). However it must be stressed that this result is based on the assumption that the increase in wealth originates in non-risky wealth only.

Clearly, these are two extreme cases. It may be more interesting to investigate what happens when \( A \) and \( L \) vary simultaneously. This is the purpose of the next section.

3. Changes in the composition of wealth  
and the maximum acceptable insurance premium

Let \( \alpha \) represent the percentage of total wealth which is subject to a loss:

\[ L = \alpha W. \]

When \( A \) and \( L \) change simultaneously, \( \alpha \) may vary or remain constant and, of course, this may have an impact on the maximum acceptable insurance premium.

Using (1) and (4) one gets:

\[ \frac{dP}{dW} = -\frac{H(P)}{u'(W - P)}, \]

where:
$H(P) = N(P) - \pi L_w u'(W - L). \quad (6)$

with $L_w = \frac{dL}{dW}$.

When total wealth increases, the maximum acceptable insurance premium decreases iff $H(P) > 0$. As (6) makes clear, this depends upon the ratio $L_w$ of change in risky wealth to change in total wealth.

From (6) it is also obvious that a sufficient condition for $H(P) > 0$ is $L_w \leq 0$, which means that a decrease (or absence of change) in $L$ is more than compensated by an increase in $A$. But this case, that includes Mossin's case ($L_w = 0$) as an upper bound, is trivial. We will concentrate our attention on the less obvious case where both $A$ and $L$ increase and we will thus assume $L_w > 0$. As, clearly, $L_w \leq 1$, we have:

$0 < L_w \leq 1$.

Starting from (5) we shall prove the following proposition:

**Proposition:** When risky and non-risky wealth increase simultaneously, the maximum acceptable insurance premium for full coverage decreases iff:

1) The ratio $L_w$ of change in risky wealth to change in total wealth is less than a critical level $l$;

2) The maximum acceptable insurance premium $P$ is less than a critical level $\bar{P}$.

The proof employs the method of Mossin (1968).

**Proof:** We want to study the condition for $H(P) > 0$ when $0 < L_w < 1$. Since $\pi \in [0, 1]$, it follows from (1) that $P \in [0, L]$. Thus, using (1) to eliminate $\pi$ from (6), we get:

$H(0) = 0$ and $H(L) = -L_w u'(W - L) < 0$. \quad (7)$

Hence, $H(P)$ cannot be positive for all values of $P$ when $L_w > 0$.

Taking the first derivative of $H(P)$, we get:

$H'(P) = u'(W - P) \left( \frac{(1 - L_w)u'(W - L) - u'(W)}{u(W) - u(W - L)} - R_\alpha(W - P) \right), \quad (8)$

where $R_\alpha(\cdot)$ represents the Arrow-Pratt measure of absolute risk aversion. Evaluating $H'(P)$ at $P = 0$, we note further that this derivative is positive, null, or negative depending on whether $L_w \leq l$, where$^3$

$^3$ It is shown in the Appendix that $0 < l < 1$. 
\[ l = 1 - \frac{u'(W) + R_a(W)[u(W) - u(W - L)]}{u'(W - L)}. \]  

(9)

Since \( R_a(\cdot) \) is monotonically decreasing, it follows from (7), (8) and (9) that \( H(P) \) is uniformly negative for \( P \in [0, L] \) if \( L_w \geq l \). In this case, \( dP/dW > 0 \).

If \( L_w < l \), three cases emerge:

1) If \( P \in [0, \overline{P}] \), where \( \overline{P} \) is defined by \( H(\overline{P}) = 0, H(P) > 0 \);
2) If \( P \in [\overline{P}, L] \), \( H(P) < 0 \);
3) If \( P = 0 \) or \( P = \overline{P} \), \( H(P) = 0 \).

Hence, the maximum acceptable insurance premium decreases only in the first of these cases, i.e. when \( L_w < l \) and \( P < \overline{P} \).

Q.E.D.

This result reflects the combined influences of increasing risk and decreasing absolute risk aversion. If the former effect is strong enough, its influence prevails, and the maximum amount spent on insurance increases; otherwise, the wealth effect prevails and – at the market level – insurance is an inferior good\(^5\). However, the rationale for the existence of \( \overline{P} \) is less obvious.

To illustrate the result, let us consider the example of a logarithmic utility function.

The logarithmic utility function \( u(x) = \ln x \) is acceptable to represent the preferences of our individual, at least for intermediate values of wealth. It is easy to verify that (9) becomes in this case:

\[ l = \alpha + (1 - \alpha) \ln(1 - \alpha), \]

which is an increasing function of \( \alpha \), and strictly less than \( \alpha \) for \( \alpha \in ]0, 1[ \). For example:

\[ \alpha = 0.25 \Rightarrow l = 0.034 \]
\[ \alpha = 0.50 \Rightarrow l = 0.153 \]
\[ \alpha = 0.75 \Rightarrow l = 0.403 \]

\(^4\) \( \overline{P} \) exists by continuity of \( H \) on \([0, L]\), since \( H(P) > 0 \) for \( P \) sufficiently small when \( L_w < l \), and \( H(L) < 0 \); \( P \) is unique because \( R_a(\cdot) \) is monotonically decreasing.

\(^5\) In a mean-variance context, Doherty (1984) derives a comparable result. In his portfolio model, the optimal level of retention on a single asset is negatively related to the weight of that asset in the portfolio (see his Theorem 4).
Thus, if the proportion of risky assets in total wealth is constant, i.e. \( L_w = \alpha \), we obtain \( L_w \geq l \) for \( \alpha \in [0, 1] \). (In this case, it is also easy to verify from (6) that \( \bar{P} = 0 \)). Thus, when the composition of wealth remains constant, the maximum acceptable insurance premium cannot be a decreasing function of wealth for a logarithmic expected utility maximizer. As shown in the next section, this result follows from the constancy of relative risk aversion.

4. Relative risk aversion and the willingness-to-insure

The previous section has shown that the individual’s readiness to spend on insurance depends upon the composition of his wealth, and that changes in this composition influence, in a non-trivial way, the maximum insurance premium he is prepared to pay.

However, it is perhaps not quite satisfactory to focus the attention on the premium amount. The reason is that an increasing amount of risky wealth will exert – other things equal – a positive influence on the premium amount, and this should not be confused with an increase in the willingness-to-insure resulting from the change in the composition of wealth. One may argue that a more appropriate measure of the individual’s willingness-to-insure is given by the maximum premium rate he would accept.

This premium rate, \( \beta \), represents the ratio of the maximum acceptable premium \( P \) to insurable wealth \( L \):

\[
P = \beta L
\]  

(10)

In fact, when \( L \) and total wealth \( W \) increase, \( P \) may increase even though the willingness-to-insure, measured by \( \beta \), decreases. For this reason, in this section, we take up again the analysis of the previous section by concentrating on variations in \( \beta \) brought about by changes in \( W \). This leads us to the following proposition:

**Proposition:** The willingness-to-insure is a decreasing function of wealth if the wealth elasticity of risky wealth is less than a critical value. This value is greater than [equal to, less than] one, if relative risk aversion is decreasing [constant, increasing].

**Proof:** The maximum premium rate \( \beta \) the individual would be willing to accept is given by:

\[
\pi u(W - L) + (1 - \pi) u(W) = u(W - \beta L).
\]  

(1')
From (1') and using (4) we get by total differentiation:

\[
\frac{d\beta}{dW} = \frac{-\pi (1 - L_w) u'(W - L) + (1 - \pi) u'(W) - (1 - \beta L_w) u'(W - \beta L)}{L u'(W - \beta L)} \tag{11}
\]

Using (1') to eliminate \(\pi\) from (11), we get \(d\beta/dW < 0\) iff:

\[
J(\beta) = \frac{u(W) - u(W - \beta L)}{u(W) - u(W - L)} \left[ (1 - L_w) u'(W - L) - u'(W) \right] + \frac{u'(W) - (1 - \beta L_w) u'(W - \beta L)}{W - \beta L} > 0. \tag{12}
\]

As \(\pi \in [0, 1]\), (1') implies that \(\beta \in [0, 1]\). From (12) we verify that \(J(0) = J(1) = 0\). Hence, by Rolle's theorem, \(J(\beta)\) has at least one extreme point over the interval. If this point is unique and \(J(\beta)\) is concave at this point, then \(J(\beta)\) must be positive for \(\beta \in ]0, 1[\).

The derivative of \(J(\beta)\) yields:

\[
J'(\beta) = L u'(W - \beta L) \left( \frac{(1 - L_w) u'(W - L) - u'(W)}{u(W) - u(W - L)} + \frac{L_w}{L} \right) - \frac{(1 - \beta L_w)}{W - \beta L} R_r(W - \beta L), \tag{13}
\]

where \(R_r(W - \beta L) = -(W - \beta L) u''(W - \beta L)/u'(W - \beta L)\) is the Arrow-Pratt coefficient of relative risk aversion.

Extremum points are given by:

\[
(1 - L_w) u'(W - L) - u'(W) + \frac{L_w}{L} (1 - \beta L_w) R_r(W - \beta L) = 0. \tag{14}
\]

There is only one such point and it is a maximum if the derivative of (14) is uniformly negative, i.e. if:

\[
\frac{L_w W - L}{(W - \beta L)^2} R_r(W - \beta L) + L \frac{1 - \beta L_w}{W - \beta L} R'_r(W - \beta L) < 0,
\]

or

\[
\varepsilon < 1 - (1 - \beta L_w) \frac{R'_r(W - \beta L)}{R_\alpha(W - \beta L)}. \tag{15}
\]

where \(\varepsilon\) represents the elasticity of \(L\) with respect to total wealth: \(\varepsilon = L_w/\alpha\).
The expression on the RHS of (15) defines a critical value for $\varepsilon$. This value depends upon the derivative of relative risk aversion. As there is yet no agreement in the literature about the sign of this derivative, we have to consider three cases

1) Relative risk aversion is constant.
   
   In this case, $d\beta/dW < 0$ if $\varepsilon < 1$.

2) Relative risk aversion is increasing.
   
   In this case, $d\beta/dW < 0$ if $\varepsilon < 1 - \lambda_1$, where $\lambda_1$ is a positive term.

3) Relative risk aversion is decreasing.
   
   In this case, $d\beta/dW < 0$ if $\varepsilon < 1 - \lambda_2$, where $\lambda_2$ is a negative term.

Q.E.D.

Stronger statements about the effect of changes in the composition of wealth on the willingness-to-insure would call for more restrictive hypotheses about the set of admissible utility functions. But our proposition brings already some clarifications. For example, it is obvious from (15) that, if $\varepsilon = 1$ (i.e. if the ratio of risky wealth to total wealth remains constant), the willingness-to-insure cannot decrease unless relative risk aversion is decreasing. On the other hand, it also appears that the willingness-to-insure may increase under decreasing relative risk aversion, provided $\varepsilon > 1 - \lambda_2$. Hence, the empirical observation that the wealth elasticity of insurance demand is greater than 1 [see Szpiro (1983)] does not necessarily imply that relative risk aversion is increasing. In our model, the wealth elasticity of (maximum) insurance spending is the sum of the wealth elasticity of $\beta$ and the wealth elasticity of $L$. If the latter exceeds $1 - \lambda_2 > 1$, the wealth elasticity of $P$ exceeds one, even when relative risk aversion decreases. Conversely, the wealth elasticity of insurance spending is less than one when relative risk aversion increases and $\varepsilon < 1 - \lambda_1$. This means that for empirical purposes, cross-country evidence on insurance spending should be supplemented by data on the composition of wealth.

5. Conclusion

This paper has shown that the effect of wealth on the demand for insurance cannot be adequately studied without taking into account the composition of wealth. Although the analysis has been based on a very simplified model of full insurance coverage in a two-state framework, the results might have some relevance for empirical studies on insurance buying behavior and on attitudes towards risk.
Previous models in the literature emphasize the effect of risk aversion and ignore the effect of the composition of wealth. Under the DARA assumption, they predict that individuals with larger portfolios of non-risky assets will be prepared to spend less on insurance for a given risk. Under the IRRA assumption, they predict that the wealth elasticity of insurance will exceed one, if wealth is only composed of risky assets.

The model presented here investigates the combined effects of risk aversion and of the composition of wealth. Two main results emerge from the analysis:

- Firstly, the DARA assumption does not necessarily imply a fall in insurance demand when wealth increases. The final outcome depends, in the first place, upon the relative change in risky wealth and, subsidiarily, upon the level of the maximum acceptable insurance premium.
- Secondly, the observation that the wealth elasticity of insurance spending exceeds one is not an argument in favor of the IRRA hypothesis. The wealth elasticity of insurance spending may exceed one when relative risk aversion decreases; and the IRRA hypothesis does not imply a wealth elasticity of insurance larger than unity.

Finally, this paper inspires a comment on measures of risk aversion. Three measures were defined in the literature dealing with a single source of risk: the Arrow-Pratt measures of absolute and relative risk aversion, and the measure of partial risk aversion (size-of-risk aversion) independently proposed by Menezes/Hanson (1970) and by Zeckhauser/Keeler (1970). The first measure, absolute risk aversion, is appropriate to predict behavior towards risk when wealth increases and the risk remains constant. The third measure, partial risk aversion, pertains to situations where the risk increases and wealth remains constant. The second measure, relative risk aversion, is adequate when wealth and the risk change in the same proportion. Our problem concerns situations where wealth and the risk may change in different proportions. We solved it by making use of the absolute and relative risk aversion measures. But more elegant results would probably have been obtained if we could have used a still undefined measure of "non proportional risk aversion". Further research in this direction is thus needed.
Appendix

Here, we verify that $0 < l < 1$. In the text we obtained:

$$ l = 1 - \frac{u'(W) + R_a(W) [u(W) - u(W - L)]}{u'(W - L)} \tag{9} $$

$l > 0$ iff:

$$ \frac{u'(W - L) - u'(W)}{u(W) - u(W - L)} > R_a(W) $$

or:

$$ \frac{u'(W) - u'(W - L)}{L} \cdot \frac{L}{u(W) - u(W - L)} < \frac{u''(W)}{u'(W)} $$

By decreasing marginal utility,

$$ u'(W) < \frac{u(W) - u(W - L)}{L} $$

By decreasing absolute risk aversion, marginal utility is convex, implying

$$ u''(W) > \frac{u'(W) - u'(W - L)}{L} $$

Hence, $l > 0$.
That $l < 1$ follows directly from the fact that the second term in the RHS of (9) is positive.
References

Abstract

Risk Aversion and the Composition of Wealth in the Demand for Full Insurance Coverage

The economic literature on the demand for non-life insurance has mainly emphasized the influence of risk aversion. Under the assumption of decreasing absolute risk aversion, it has been shown that insurance should be an inferior good. Under the assumption of increasing relative risk aversion, it has been shown that the wealth elasticity of insurance should exceed one. These seemingly contradictory propositions are due to strong implicit assumptions about changes in the composition of wealth. The present paper proposes a more general model where the effects of risk aversion and of the composition of wealth are considered jointly. Necessary and sufficient conditions for a decrease in the willingness-to-insure are defined, and their implications for the interpretation of empirical results on insurance spending and risk-averse behavior are pointed out.

Zusammenfassung

Risiko-Aversion und Zusammensetzung des Vermögens in der Versicherungsnachfrage bei Volledeckung


Résumé

Aversion pour le risque et composition de la richesse dans la demande pour une couverture d’assurance complète

La littérature économique sur la demande d’assurance-domages a surtout mis en avant le rôle de l’aversion pour le risque. Sous l’hypothèse de décroissance de l’aversion absolue pour le risque, il a été démontré que l’assurance devrait être un bien inférieur. Sous l’hypothèse de croissance de l’aversion relative pour le risque, il a été démontré que l’élasticité-richesse de la demande d’assurance devrait être supérieure à un. Ces propositions en apparence contradictoires sont dues à de fortes hypothèses implicites sur les modifications dans la composition de la richesse. Cet article propose un modèle plus général où les effets de l’aversion pour le risque et de la composition de la richesse sont considérés conjointement. Il indique des conditions nécessaires et suffisantes pour une baisse de la propension à s’assurer, et il souligne leur importance pour l’interprétation des résultats empiriques sur la dépense d’assurance et sur le comportement face au risque.