Macroeconomic Models with Quantity Rationing
A Graphical Approach

URS MÜLLER*

1. INTRODUCTION

Inflexible or sticky prices do in practice not mean that all prices are just fixed, but rather that some prices do not completely adjust to their equilibrium value. A multimarket treatment thus has to take into consideration price effects as well as quantity spillovers. Using a simple general equilibrium model, this paper presents a graphical method to compare the Walrasian general equilibrium solution to a quantity constrained general equilibrium (or general disequilibrium or temporary general equilibrium with quantity rationing) solution. But unlike most studies of macroeconomic models with quantity rationing [see e.g. HELLER and STARR (1979), BACKHOUSE (1981) and KÄHKÖNEN (1982)], we will not concentrate on policy effects with and without binding quantity constraints, but rather on the comparison between the Walras solution and different disequilibrium solutions. To this end, we will decompose the adjustment process into four phases and present a new graphical apparatus.

The paper stands in the tradition of CLOWER (1965), BARRO and GROSSMAN (1971), and MUELLBAUER and PORTES (1978). We assume that prices in some markets are sticky and do not instantaneously adjust to their new Walrasian general equilibrium levels. The resulting excess supplies or demands lead to quantity rationing. Agents who cannot realize their initially planned (notional) quantities will revise their plans in other markets which results in effective supplies and demands different from their notional levels. The remaining non-sticky (flexible) prices adjust to their new constrained equilibrium values.

The remainder of the paper is organized as follows: First we present a small two-sector general equilibrium model with three traded goods. Then we add the optimization behavior of the agents in the presence of quantity constraints and develop a graphical method to compare quantity constrained equilibrium solutions to Walrasian general equilibrium. This apparatus is then used to analyze two specific cases: too high wages (strong unions) and too low interest rates (credit rationing).

* Basle Business Cycle Research Group, P.O.Box 112, CH-4003 Basel.
I wish to thank BEN FRIEDMAN for helpful comments on an earlier draft of this paper. Financial support from the Swiss National Science Foundation is gratefully acknowledged.
2. OUTLINE OF THE MODEL

We first develop a small general equilibrium model. Unlike most economic models, such a model does not only analyze one or two markets ceteris paribus, i.e. omitting the effects on and feedbacks from all other markets, but it considers all markets in an economy simultaneously. Some general equilibrium models are very large and rich in details, others assume for simplicity and tractability that the economy consists of only a few important markets and different market participants\(^1\). Our small general equilibrium model consists of two sectors, households and firms. There is no government sector, no external sector, no inventories and no idle cash. The households have net money balances with the firms, which use these deposits to finance the production of investment goods in order to accumulate physical capital. There are three traded goods, consumption \(C\) with price \(p\), labor \(L\) with wage rate \(w\), and end of period deposits \(D\) with interest rate \(r\). Households demand the consumption good and supply labor and deposits, firms supply consumption and demand labor and deposits (loans). The model is best characterized by the following equations. Time subscripts for the current period are omitted.

\[
U = U(C, L, D), \quad (1)
\]

\[
w L + (1 + r - 1) \cdot D_{-1} = p C + D, \quad (2)
\]

\[
\pi = p C - w L - r D_{-1}, \quad (3)
\]

\[
X(L, K) = Q(C, I), \quad (4)
\]

\[
K = (1 - d) \cdot K_{-1} + L_{-1}, \quad (5)
\]

Households are utility maximizers; in (1) they draw utility \(U\) from consumption \(C\) and the stock of deposits \(D\) and disutility from labor \(L\). The budget constraint (2) indicates that interest payments are received between periods. Households have savings (deposits

\(^1\) See e.g. HANSEN (1970) or SCARF and SHOVEN (1984).

\(^2\) There are two reasons for money in the utility function: (1) Being rich is a good feeling. (2) Having money will allow for future consumption. The latter has the advantage of rescaling a multi-period optimisation problem to a one-period optimisation.
D-1), interest payments and labor income and use these funds for consumption purposes. The rest is the deposit stock at the end of the period. Equation (3) is the profit (or better cash flow) definition of the corporate sector. Cash flow is the surplus of sales revenues over labor compensation and interest payments. A negative cash flow induces the need for further credits. From (2) and (3) it can be easily seen that \( \pi \) is equal to the negative change in \( D \). Thus, the credit requirement of the firm sector reflects the households’ budget constraint (2) and the one-period cash flow \( \pi \) in (3): profits decrease the debt balance of the firms. Equation (4) describes the production technology of the profit maximizing firms: the physical capital stock \( K \) and labor \( L \) are used to produce consumption \( C \) and investment goods \( I \). The current capital stock in (5) consists of accumulated past investment, where \( d \) is the depreciation rate; the gestation lag defines the period length. It is assumed that the factor market for \( K \) always clears within the corporate sector; thus it need not be treated explicitly. As for (1) and (4), the usual curvature assumptions apply. The model is primarily static. However, there are a few dynamic features: capital accumulation and the associated shift of the production possibility frontier on the one hand (originating from long term profit maximization), and interest payments and the accumulation of nominal capital to finance future consumption on the other hand.

**Table 1: Qualitative Market Functions for Households (H) and Firms (F)**

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C^d )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( C^s )</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( L^s )</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( L^d )</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( D^s )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( D^d )</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>


The description of the basic general equilibrium model has to be completed by the derivation of the corresponding demand and supply functions for the three traded goods. Omitting endowments (D and K), table 1 shows the signs of the first partial derivatives of the six market functions with respect to the prices, where the superscripts d and s stand for demand and supply, respectively. All own price effects have the standard textbook signs. An increase in the price of the consumer good p lowers demand for C. Falling real wages lower households' labor supply and assuming that the price elasticity of consumption is close to unity, savings will be reduced consequently. An increase in the wage rate w heightens labor supply L of the households. Additional compensation is used both for additional consumption and savings. An increase in the interest rate r induces additional savings D. This is realized by both lowering consumer expenditures and rising labor supply. The price reactions of the firms are primarily determined by the production technology (4). An increase in the price of the consumer good has two consequences: First a shift from investment to consumer goods, thus reducing the demand for loans D, and second a boost of total output, thus raising labor demand. An increase in the wage rate reduces labor demand, total production and the output of both consumer and investment goods (thus lowering the demand for loans). An increase in the interest rate makes investment less attractive. The consequence is a reduction of loan demand, total output and labor demand, and a shift towards the production of consumer goods. The six market functions of table 1 simultaneously determine the prices and quantities of these three markets. The production technology (4) then determines also total output and the production of the investment good I. The Walrasian solution to the general equilibrium model can (again omitting endowments and using asterisks to denote general equilibrium values) be written as follows:

\[ C^d (p^*, w^*, r^*) = C^s (p^*, w^*, r^*) , \]

\[ L^d (p^*, w^*, r^*) = L^s (p^*, w^*, r^*) , \]

\[ D^d (p^*, w^*, r^*) = D^s (p^*, w^*, r^*) . \]

In a Walrasian world, the market functions depend only on prices and endowments. But when some prices are sticky, at least one agent will be constrained in his optimization

3. Some of these sign assumptions are arbitrary. Eventually they depend on the functional form and parameters of the basic model.
behavior. Table 2 shows the qualitative spillover effects on the remaining markets when the initially planned (n for notional) quantities are greater than the quantity the other market side is willing to trade. The household reactions can be easily derived from the budget constraint (1). When a household perceives excess demand for consumption, he either reduces labor supply or increases the supply of deposits. We assume that he will do both. Excess supply of labor (unemployment) reduces both consumer expenditures and savings. In the case of excess supply of funds, households will increase consumer expenditures and reduce labor supply. Thus, the reaction functions with respect to quantity constraints in the quantity spaces as illustrated in a graph introduced by SVENSSON (1980) will never coincide. This results graphically in so-called wedges. In the case of substitutive goods (like C and D in our model), the wedge is open and has a negative shape. Complementary goods (like C and L or L and D) result in a sharp wedge with a positive slope. The corresponding reasoning for the firm sector operates mainly through the production technology (4). Given an excess supply of consumption goods, the firms can either switch to investment goods and thus increase loan demand or they can reduce output by lowering labor demand. Again we assume that they will do both. Labor shortages reduce output of both consumer and investment goods, thus lowering demand for loans D. Excess demand for funds reduces the production of the investment good. There is both a reduction in total output and a shift towards the production of the consumer good. The

Table 2: Qualitative Reactions to Quantity Constraints

<table>
<thead>
<tr>
<th>constraint</th>
<th>households</th>
<th>firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L^s -</td>
<td>L^d -</td>
</tr>
<tr>
<td></td>
<td>D^s +</td>
<td>D^d +</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>C^d -</td>
<td>C^s -</td>
</tr>
<tr>
<td></td>
<td>D^s -</td>
<td>D^d -</td>
</tr>
<tr>
<td>D</td>
<td>C^d +</td>
<td>C^s +</td>
</tr>
<tr>
<td></td>
<td>L^s -</td>
<td>L^d -</td>
</tr>
</tbody>
</table>
results from table 2 will be used in figures 1 and 2 to construct double wedge diagrams as presented by MUELLBAUER and PORTES (1978).

The adjustment process to a temporary general equilibrium when some prices are sticky and some are flexible can be decomposed into four phases (not necessarily in a temporal sense). Starting with the Walrasian general equilibrium solution, we disturb the price vector by setting some prices at an arbitrarily fixed value. The four phases then are:

a) Notional demands and supplies given the prices
b) The flexible prices (and the quantities) adjust: notional equilibrium
c) Effective demands and supplies at given prices and quantity constraints
d) The flexible prices (and the quantities) adjust: effective equilibrium

This concept shall now be illustrated by two applications to our model outlined above: The case of too high wages ("Strong Unions") and the case of too low interest rates ("Credit Rationing").

3. EXAMPLE 1: STRONG UNIONS

The Walrasian general equilibrium (6) to (8) is depicted in the upper part of figure 1. For simplicity, the substitutive wedges (between C and D) and all complementary wedges (between L and D, and C and L) look identical. The southeast quadrant with the 45 degree line is only necessary to close the system. Now we disturb the equilibrium price vector and set a fixed wage rate $w^f > w^*$. From table 1 it is easily seen that this results in excess demand for consumption and excess supply for labor and deposits:

$$C^d (p^*, w^f, r^*) > C^s (p^*, w^f, r^*)$$  \hspace{1cm} (6a)

$$L^d (p^*, w^f, r^*) < L^s (p^*, w^f, r^*)$$  \hspace{1cm} (7a)

$$D^d (p^*, w^f, r^*) < D^s (p^*, w^f, r^*)$$  \hspace{1cm} (8a)

These notional quantity plans are shown in panel a) of figure 1. The letters H, F and W denote households, firms and Walras equilibrium, respectively. In the second phase, the inequalities in (6a) and (8a) result in price adjustments. The price of the consumption good increases and the interest rate falls to their respective constrained equilibrium values (denoted n) in the markets for C and D. This situation, which may be called notional equilibrium, is shown in panel b) of figure 1 and can be written as follows:
Figure 1: Strong Unions: $w' > w^*$
In the third phase, the households feel rationed in the labor market and reduce their demand for C and their supply of D. Given the price vector and the excess supply in notional equilibrium, the effective households' plan for the consumption-deposits space can be found following the dashed lines in panel c) of figure 1. Compared to the first phase, the inequalities in the flexible markets have changed:

\[ C^d (p^n, w^f, r^n) = C^s (p^n, w^f, r^n), \]  
\[ (6b) \]

\[ L^d (p^n, w^f, r^n) < L^s (p^n, w^f, r^n), \]  
\[ (7b) \]

\[ D^d (p^n, w^f, r^n) = D^s (p^n, w^f, r^n). \]  
\[ (8b) \]

The fourth phase allows the two flexible prices and the quantities to adjust to new effective equilibrium values (denoted e): p decreases and r increases slightly, partly offsetting the price movements of the second phase. In panel d) of figure 1 all superfluous lines have been dropped, but for ease of comparison, the Walras solution is added. The effective equilibrium (or general equilibrium with quantity constraints or general disequilibrium) can be written as:

\[ C^d (p^e, L^e, r^e) = C^s (p^e, w^f, r^e) \]  
\[ (6d) \]

\[ L^d (p^e, w^f, r^e) < L^s (p^e, w^f, r^e) \]  
\[ (7d) \]

4. Note that the functional form of \( C^d \) and \( D^d \) change when prices are replaced by the respective quantities.
$D^d (p^e, w^f, r^e) = D^s (p^e, L^e, r^e)$  \hspace{2cm} (8d)

Within the framework of our small general equilibrium model we may state the following relations between the disequilibrium solution due to too high wages and the Walras solution:

$$w^f > w^*: \quad p^e > p^*, \quad r^e < r^* \quad C^e < C^*, \quad L^e < L^*, \quad D^e < D^*.$$  \hspace{2cm} (9)

High wages decrease labor demand, thus reducing output of both consumption and investment goods. Households are rationed with respect to labor, and their effective consumption function becomes Keynesian: the left-hand side of (6d) contains the effective quantity of labor and thus, given the fixed wage rate $w^f$, labor income (cf BARRO and GROSSMAN 1971). The suboptimal values of savings and investment lower the long-run welfare through a reduction of the capital stock and a negative shift of the production possibility frontier.

4. EXAMPLE 2: CREDIT RATIONING

The second example works in exactly the same way as the first. We assume that the Walrasian equilibrium price vector is disturbed by a too low interest rate $r^f < r^*$. The results which are shown in figure 2 can be summarized as follows: An interest rate below the Walras equilibrium value does not only lead to excess demand for deposits, but according to table 1 also to excess demand for consumption and labor. These notional quantity plans are shown in panel a) of figure 2. Rising consumer prices and wage rates equilibrate the markets for C and L in panel b), but the excess demand for funds persists. Thus, the corporate sector feels rationed in the credit market D and revises its investment plans downwards accordingly. Total production is reduced, lowering labor demand (cf table 2), and there is a shift towards the substitutive output raising consumption supply as depicted in panel c). The resulting excess supplies on the markets for consumption and labor are eventually equilibrated by falling consumer prices and wage rates. The effective equilibrium, given a too low interest rate $w^f$, is shown in panel d) of figure 2 and can be written as:

$$C^d (p^e, w^f, r^f) = C^s (p^e, w^f, D^e)$$  \hspace{2cm} (10d)

$$L^d (p^e, w^f, D^e) = L^s (p^e, w^f, r^f)$$  \hspace{2cm} (11d)
Figure 2: Credit Rationing: $r^f < r^*$
\[ D^d (p^e, w^e, r^f) > D^s (p^e, w^e, r^f) \] (12d)

\( D^e \) is the effective amount of credits the corporate sector can obtain. It is the limiting factor in the economic behavior of the corporate sector. This constraint spills over onto the other two markets, as can be seen from the market functions for \( C^s \) in (10d) and \( L^d \) in (11d). Summarizing, we may state the following relations between the disequilibrium and the Walras solution:

\[
\begin{align*}
  r^f < r^* : & \quad p^e > p^*, \quad w^e > w^*, \\
  C^e > C^*, \quad L^e < L^* & \quad D^e < D^*. \quad (13)
\end{align*}
\]

Low interest rates decrease the supply of deposits. Firms are rationed in their demand for loans (credit rationing) and the available quantity of deposits enters the effective consumption supply function with a positive and the effective labor demand function with a negative sign. Since the firms cannot finance their investment plans, they reduce aggregate output (and employment), but also increase the production of the consumer good. Instantaneous utility of the households rises (more consumption and leisure), but long-run welfare decreases for the same reasons as in example 1.

5. CONCLUSIONS

In this paper we have presented a two-sector economy with three traded goods in which some prices are sticky and some are flexible. In order to compare a constrained with the Walrasian general equilibrium solution, the adjustment process was decomposed into four phases which were presented graphically. This apparatus was then applied to two specific cases: too high wages (strong unions) and too low interest rates (credit rationing). The policy relevance of this study is quite straightforward: There are social cost of sticky prices. Too high wages do not only result in unemployment, they also lower investment activities of the corporate sector. A lower productive capital stock with an average older vintages leads to a slower growth path of the overall economy. Thus, a high-wage policy is only a short-term optimization for the (employed) members of a union. The second example of too low interest rates causes dynamic credit rationing, a reduction of the investment activities and a lower capital stock, again shifting the production possibility frontier and dampening the pace of general growth. If the interest rate \( r \) is the mortgage rate, a value below its Walras level reduces residential construction. The consequence is lower housing supply, thus aggravating the disequilibrium (shortage) in the housing
market. Wrong prices result in a misallocation of resources with negative long-term welfare effects. The quintessence is quite obvious (and well known): Let markets, not policy, govern prices.

Extensions to this model can point to different directions. The addition of further explicit markets will not qualitatively alter the treatment of the model. Also, the graph can be extended by adding a new row and column for each additional good. An extension to more than two sectors, however, cannot be presented graphically unless there are only two participating sectors in each market. This is the case when we introduce banks as financial intermediaries demanding deposits from the households and supplying loans to the firms. Further, there are some dynamic elements in the model, but the explicit optimization behavior is only static. Finally, we may be interested in the numerical value of welfare losses of out-of-equilibrium prices. Such a numerical implementation will lead to an applied general disequilibrium model.

REFERENCES


5. This phenomenon can be observed in Switzerland (see MUllER 1988). Moreover, if there is an institutional link between the mortgage rate and the rents, it is a subsidy of those who have a flat or live in their own house at the cost of those looking for new housing.

SUMMARY

Macroeconomic Models with Quantity Rationing: A Graphical Approach

This paper presents a two-sector general equilibrium model with three traded goods in which some prices are sticky and some are flexible. In order to compare a constrained with the Walrasian general equilibrium solution, the adjustment process is decomposed into four phases which are presented graphically. This apparatus is applied to two specific cases: too high wages (strong unions) and too low interest rates (credit rationing).

ZUSAMMENFASSUNG

Makroökonomische Modelle mit Mengenrationierung: ein graphischer Ansatz


RESUME

Modèles macroéconomiques avec rationnement quantitatif: une approche graphique

Cette étude présente un modèle d’équilibre général avec deux secteurs et trois biens échangés, dans lequel quelques prix sont rigides et d’autres sont flexibles. Pour comparer la solution restreinte avec la solution d’équilibre de Walras, le processus d’ajustement est décomposé en quatre phases, qui sont présentées graphiquement. Cette méthode est appliquée à deux cas spécifiques: Taux de salaire trop élevés (syndicats forts) et taux d’intérêt trop bas (rationnement de crédit).