Adoption Decisions and Diffusion:
Implications for Empirical Economics

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1. INTRODUCTION

The purpose of this paper is to investigate the implications of the micro-economics of the adoption decision for the properties of the diffusion curve and to present some guidelines for empirical analysis. The diffusion literature, to date, has followed two separate paths. On the one hand, GRILICHES' (1957) classic article on hybrid corn set the stage for a series of studies, mostly by economists, on the properties of aggregate diffusion curves. In these studies S-shaped, usually logistic, functions are fitted to the percentage of potential adopters who have innovated and (inter-universe) differences in estimated parameters are explained by (inter-universe) differences in the characteristics of the innovations in the given environments, GOTTINGER (1986). On the other hand, there are studies of the adoption behavior of individual units (for extensive reviews see ROGERS, 1962, and ROGERS and SHOEMACKER, 1971). These studies, mostly by sociologists and political scientists, relate a measure of adoption performance, primarily the relative adoption time of different units, to the characteristics of the individual in relationship to the innovation. There has been little work, theoretical or empirical, on combining the two types of studies. As a result several of the factors which determine the characteristics of the diffusion curve have drawn little attention and the implications of the micro studies for the properties of the aggregate diffusion process have not been fully analysed.

In Section 2 a model of the adoption decision is presented. Its basic assumptions are (a) that the adoption decision involves uncertainty; (b) that, since some but not all potential adopters adopt, at least one of the variables determining adoption behavior differs in value between units; and (c) that, since a potential adopter who does not innovate at one point of time may do so at another, some of the decision variables change in value over time. Section 3 shows that these assumptions are sufficient to derive the determinants of the three properties of the aggregate diffusion curve which have received most attention by researchers. Precise functional forms for the diffusion curve are then derived. In order to do this, one needs either to specify or to estimate the distributions of firms by firm-differentiating variables and the time paths of time-differentiating variables. Section 4 briefly summarizes the results in view of pertinent economic interpretations.

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2. ADOPTION DECISION BY FIRMS

The first task in building a micro diffusion model is to describe the firm's decision-making process. For simplicity it is assumed here that each firm, at every point in time t, acts so as to maximize the expected utility from its assets at t+1[1]. The decision problem will be discussed with the aid of the following notation:

$V(\tau)$	the value of the firm's assets in period τ
ρ	an index of the firm's willingness to undertake risky ventures[2]
$U(V;\rho)$	the firm's utility function and it is assumed that $U'_V > 0$, $U''_V < 0$ and that the function displays a decreasing degree of absolute risk aversion in the value of V[3]
$\bar{C}$	the user cost of materials and equipment which must be purchased in order to use the innovation
π_0	1 plus the rate of profit on the inputs used before adoption of the innovation (see Appendix for definition)
$f(\,), \tilde{\pi}_1, \tilde{\pi}$	f (.) is a probability distribution encompassing the firm's subjective beliefs on the incremental rate of profit on the new inputs ($\tilde{\pi}$) and is assumed to be nonzero on some finite interval; $\tilde{\pi}_1$ is 1 plus the random subjective rate of profit on the new inputs, and $\tilde{\pi} = \tilde{\pi}_1 - \pi_0$
h, q	are indexes of families of subjective distributions displaying increasing variance and an increasing mean respectively; that is, if $\bar{\pi} = E[\tilde{\pi}(h, q = 0)]$, where E is the expectations operator, then $\tilde{\pi}(h, q) = \bar{\pi} + q + h[\tilde{\pi}(1, 0) - \bar{\pi}]$.

Note that π_0 is assumed to be known with certainty, that is, the model abstracts from the general market uncertainties associated with all investments. Accordingly h, the

1. Alternatively one could assume that markets exist for distributing risks among individuals and that the maximand is the stock-market value of the firm.
It should also be noted that if either the inputs embodying the innovation or those the innovation is replacing are long-lived, and if there does not exist a perfectly competitive rental market for these inputs, then one should consider the expected value of a utility function defined over the time path of the firm's assets. Most of the crucial variables in the more complicated formulation, however, can be entered into the simpler model used here by appropriate modelling of the time path of the incremental rate of profit on the inputs embodying the innovation.

2. See note 3.

3. Using the Arrow-Pratt index of risk aversion (ARROW, 1979, PRATT, 1964) i.e., $-\partial \log U'_V/\partial V$, ρ is defined as an index such that $\partial^2 \log U'_V/\partial\rho\partial V > 0$, and it is assumed that $\partial^2 \log U'_V/(\partial V)^2 < 0$. It should also be noted that U ( ) is assumed to be both bounded and to possess continuous partial derivatives up to third order.

variance in the subjective distribution of $\tilde{\pi}_1$, becomes an index of the firm's uncertainty with respect to the potentials of the new inputs; and h will be set equal to zero when the economic properties of the new inputs can be assumed to be known with certainty.

The firm's decision problem is written as

$$\max_{C} EU(\tilde{V}_{t+1}; \rho)$$

$$\text{s.t. } \tilde{V}_{t+1} = \pi_0 V_t + \tilde{\pi} C$$

$$C \, e \, \{\bar{C}, 0\}$$

The firm will innovate, be indifferent, or not innovate according as $EU(\tilde{V}_{t+1}; \rho \mid C = \bar{C}) \lesseqgtr EU(\tilde{V}_{t+1}; \rho \mid C = 0)$, or according as;

$$H(\pi_0 V_t, \bar{C}, h, q, \rho) \equiv EU(\pi_0 V_t + \tilde{\pi}\bar{C}; \rho) - U(\pi_0 V; \rho) \lesseqgtr 0 \tag{2}$$

where the identity defines the function H, which will be termed the criterion function.

The relationship between the adoption decision and C, the size of the gamble, is illustrated in Figure 1. Note that $EU(\pi_0 V + \tilde{\pi} C; \rho)$ is strictly concave and is increasing or decreasing at the origin according as $E(\tilde{\pi}) \lesseqgtr 0$. The firm must choose C at one of the corners, $(0, \bar{C})$, and the choice of each of the five representative firms illustrated in the diagram is indicated by the direction of the arrows. That is, firms 1 and 2 do not adopt, firm 3 is indifferent, and firms 4 and 5 do adopt. In fact, firm 5 may purchase more than one unit of the equipment embodying the new technique, but this possibility is ruled out for all but the first adopter in the discussion that follows.

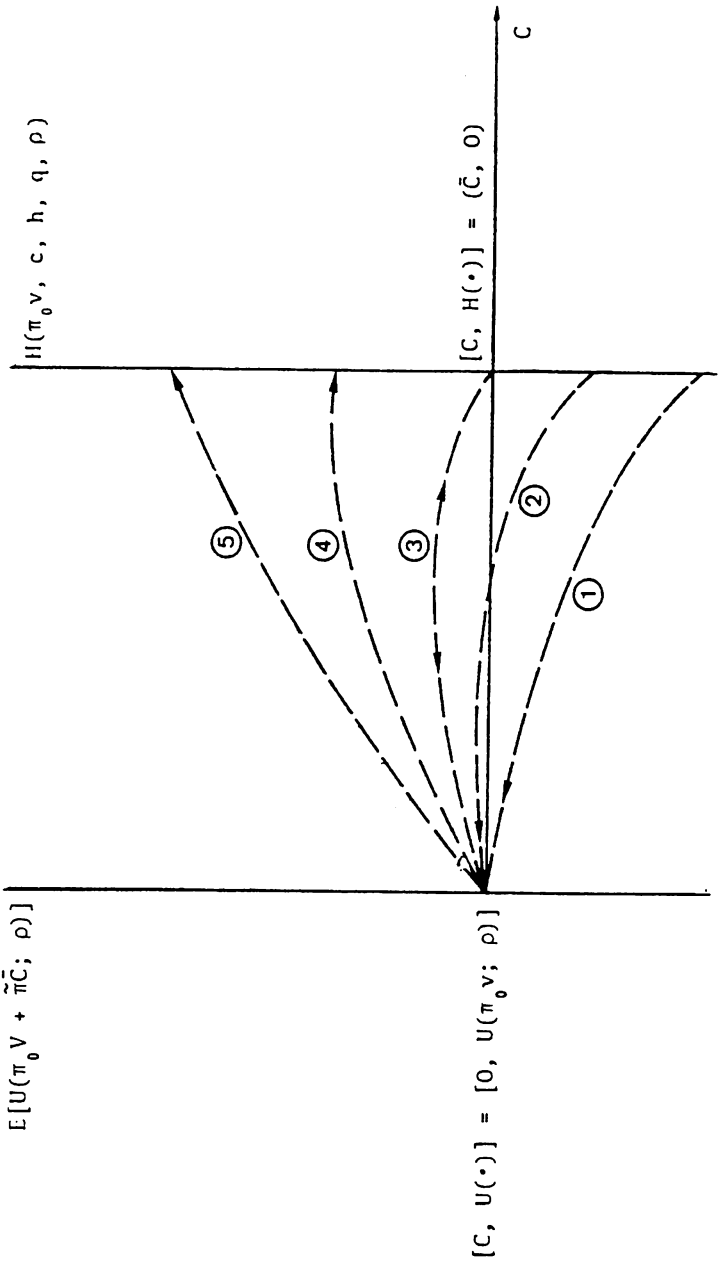
Whether the firm adopts is determined by whether the value of the criterion function is greater than zero. It is easily shown that an increase in the mean (q) or a decrease in the variance (h) of $\tilde{\pi}_1$ increases the value of H[4]. Moreover the incentive to adopt increases with the value of its assets (V) and decreases both as its index of risk aversion (ρ) and the investment requirements of the innovation ($\bar{C}$) rise. To prove these latter points note that there exists a unique Z such that

$$EU(\pi_0 V + \tilde{\pi}\bar{C}; \rho) - U[\pi_0 V - Z(V, \bar{C}, h, q, \rho); \rho] = 0 \tag{3}$$

Subtract (3) from (2) and obtain the criterion function as

4. An increase in h constitutes a mean-preserving spread in the sense defined by ROTHSCHILD and STIGLITZ (1970). From Theorem 2 of that paper (p. 237), it follows that $H'_h < 0$.

Figure 1. The Adoption Decision as a Function of C



$$H(\pi_0 V, \overline{C}, h, q, \rho) = U[\pi_0 V - Z(V, \overline{C}, h, q, \rho); \rho] - U(\pi_0 V; \rho) \qquad (4)$$

Z is the risk premium the firm would be willing to pay (or to receive) in order to be indifferent between the sure bet $\pi_0 V$ and the risky alternative $\pi_0 V + \tilde{\pi}\overline{C}$, hence $H'_z < 0$. This risk premium increases with the firm's degree of risk aversion ($Z'_\rho > 0$) and decreases with the value of its assets ($Z'_v < 0$). It follows that $H'_\rho < 0$, and that $H'_v > 0$ for all potential adopters, that is for all those firms which have not yet adopted[5].

In order to prove that potential adopters are less likely to adopt the greater is the investment requirement ($\overline{C}$), consider two cases. First assume $E(\tilde{\pi}) > 0$. Then $H'_c > 0$ when evaluated at $C = 0$, $H \leq 0$ when evaluated at $C = \overline{C}$, and $H''_c \leq 0$ every where, from which it follows that $H'_c < 0$ when evaluated at $C = \overline{C}$. If $E(\tilde{\pi}) \leq 0$, then $H'_c < 0$ when evaluated at $C = 0$ and negative for any $C > 0$. One can come to the same conclusion for H'_c evaluated at $C = \overline{C}$, that is $H'_c < 0$ for all potential adopters, by examining H for representative firms 1, 2, and 3 in Figure 1. Note that since, when evaluated at $C = \overline{C}$, $H'_c \leq 0$ for all potential adopters ($H \leq 0$), H is smaller when evaluated at $2\overline{C}$ than when evaluated for the same firms at $\overline{C}$. Hence, provided that the variables in the criterion function are continuous functions of time, all firms with the possible exception of the first adopter will purchase only one unit of the innovation upon adoption[6].

Consider now the two types of diffusion studies in the literature – those analysing individual adoption behavior, and those analysing the characteristics of aggregate diffusion curves – in the light of the previous discussion.

First the variables determining the value of the criterion function provide a convenient classification of the factors influencing adoption behavior. Moreover equation (3) indicates that an appropriate method for the empirical analysis of this behavior is a model which introduces a limited dependent variable, taking the value 1 when the firm adopts and 0 otherwise, and continues studying the firm's behavior until after it adopts. In contrast, previous research in this area has focused on one or two, often discipline-specific, factors affecting adoption, and has used relative adoption times (the time between first adoption and adoption by a given unit) as the dependent variable in the empirical analysis. The latter procedure makes it difficult to identify the influence of factors which change the firm's adoption decision over time and of the differential impact of these factors on different potential adopters. In particular, little evidence is available on different potential

5. For proofs of the propositions that $Z'_\rho > 0$ and $Z'_v < 0$ see ROTHSCHILD and STIGLITZ (1970), theorem 3. To show that $H'_v > 0$ for $H \leq 0$ note that $H'_v = [(\pi_0 V - Z; \rho)\, 0 - U'_v(\pi_0 V; \rho)]\, \pi_0 - U'_v Z'_v$, and a sufficient condition for this expression to be positive is that $Z \geq 0$, which from (4) implies that $H \leq 0$.

6. That is, the possibility illustrated as firm 5 in Figure 1 is ruled out for all but the first adopter. This provides some justification for studying the intrafirm and the interfirm diffusion processes separately.

adopters. In particular, little evidence is available on the influence of different information-generating mechanisms on adoption behavior which are likely to be of primary descriptive and policy importance. For example, what is the relative effectiveness of investments in different types of advertising, direct salesmanship, and extension services; how does schooling influence the potential adopter's ability to assimilate this information; and how does information generated through use spread? Since the limited dependent variable technique provides information on the adoption decisions of different units over time, it may provide some insights into these important problems[7].

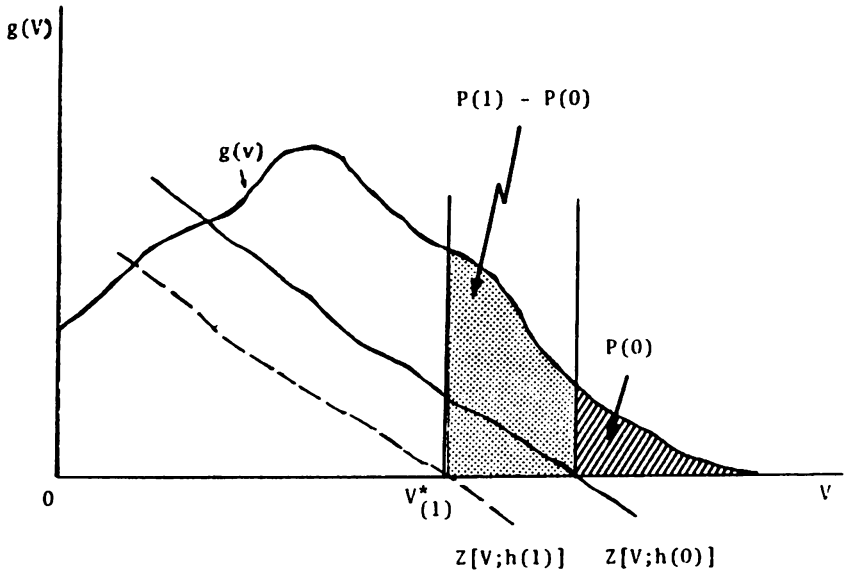
Next, consider the properties of the aggregate diffusion curve. Since at any point in time certain units adopt while others do not, and since a firm that does not adopt in one period may adopt in another, the model underlying the diffusion process must have at least one argument in the criterion function that differs between potential adopters, and one of the discriminating variables must change over time. To illustrate the workings of such a model consider a diffusion process in which $q_{it} = q$, $\rho_{it} = \rho$,$C_{it} = \overline{C}$, $h_{it} = h\ (t)$, and $V_{it} = V_i$ $(i = 1, \ldots, N, t = 1, \ldots, T$, where N is the number of potential innovators in the sample at the beginning of the process). That is, V is the only variable which has different values for different firms (firm-differentiating variable or FDV) and h is the only variable which changes its value over time (time-differentiating variable, or TDV). Let $g(v)$ denote the relative density function of the firms' assets and assume that $h'(t) < 0$; that is, as time passes, information on and experience with the new technique accumulate and the variance in the firm's subjective probability distribution of $\tilde{\pi}$ decreases. From (4) it follows that a firm will innovate, be indifferent, or will not innovate at $t = 0$ according as $Z\ [V_i, h(0), K, \rho, \overline{C}] \lesseqgtr 0$, and as was shown above, $Z'_v < 0$. Figure 2 graphs $g(v)$ and $Z\ [v \mid h\ (0)]$. Let $V^*(0)$ denote the (unique) value of V such that $Z\ [V^* \mid h\ (0)\ ] = 0$. Then the proportion of firms which adopt the innovation at $t = 0$, $P(0)$, is the cumulated density of firms with assets greater than or equal to $V^*(0)$, or mathematically

$$P\ (0) = \int_{V_0}^{\infty} g\ (\varepsilon)\ d\ (\varepsilon) \equiv 1 - G\ [V^*\ (0)],$$

where G is the cumulative density function of V.

7. See ROGERS and SHOEMACKER (1971) for a sociological formulation of these questions. One point should be noted here. There are important cases when the influence of different information generating mechanisms cannot be studied without simultaneous consideration of individual behavior over time towards several related innovations. For example, experience with hybrid seed in developing agriculture has taught development economists that the adoption decision with respect to this input is strongly dependent on adoption decisions with respect to a package of complementary inputs. The results of NERLOVE and PRESS' (1973 part V) study of Phillipine agriculture testify to the importance of this effect. In such cases it is doubtful as to whether single innovation studies could provide consistent parameter estimates and, at any rate, they lose important information on the interrelationships between innovations. It is clear that empirical analysis of the relationship between information generating mechanisms and adoption decisions will be difficult. On the other hand both the data (see the contributions to the NASBETH and RAY 1974 volume) and the techniques (see HECKMAN 1977 and the literature cited there) to do the analysis are now available and related issues are at the heart of most policy and marketing questions related to diffusion.

Figure 2: The Diffusion Process with One FDV and One TDV



As time passes firms learn more about the new technique and h decreases. Setting $H\ [V^*, h(t)] = 0$, and totally differentiating with respect to time, one finds that $dV^*/dt = -(H'_V/H'_h)\ h'_t$ which is less than zero. Figure 2 represents Z [V; h(l)]. Letting V*(l) be the (unique) value of V such that $Z\ [V^*(l), h(l)] = 0$, it follows that $P(l) = 1 - G\ [V^*(l)]$, and $P(l) - P(0) = G\ [V^*(0) - G\ [V^*(l)]$.

At any t firms with assets below a certain critical level (V^*_t) find that adopting would involve them in a greater degree of risk than they are willing to undertake. As time passes information accumulates, uncertainty diminishes, and firms with a lower asset level are willing to chance the new innovation. Previous research has, for the most part, summarized the characteristics of the aggregate diffusion curves in terms of the parameter values of certain generic functions (e.g., logistic) and then related the estimated parameters to certain variables. Neither the function chosen nor the determinants of the parameters have been related to more than the first-order moments of the distribution of

the FDV, and the relationship of these studies to the time path of the TDV, as distinct from its level, has been minimal at best. The next section will analyse the effects of these factors on the diffusion curve, and explicitly derive the aggregate curve for some fairly simple diffusion processes.

Before proceeding, however, it is worth noting one further implication of introducing an explicit model of the decision-making process into diffusion research. Both micro and aggregate diffusion studies must, either explicitly or implicitly, determine the universe of potential user firms studied. It would seem reasonable to include a firm in this universe *iff* the firm would adopt the innovation in a world of certainty. In the present notation a world with certainty is defined by the characteristics $h = 0$ and $\tilde{\pi} = \pi^*$ where π^* is the true rate of profit on the inputs embodying the innovation. That is, in a world of certainty both the new and the old techniques have known rates of profit. Hence a firm should be included in this universe at a given point in time if $H > 0$ at that time, when evaluated at $\tilde{\pi} = \pi^*$ and $h = 0$.However, when evaluated at $\tilde{\pi} = \pi^*$and $H \lesseqgtr 0$, according as $\pi^* \lesseqgtr 0$. That is, whether the firm should be included in the universe depends only on the true incremental rate of profit it would derive were it to use the new inputs. Most aggregate diffusion studies have no way of determining which firms have $\pi^* > 0$ at every t. A common procedure is to study diffusion in a group of firms which includes the potential users as a subset and to define an asymptote to the diffusion process (say K_t) *which equals the percentage of the group who are potential users.* Let $\Gamma^t(\pi^*)$ be the cumulative density of π^* in the group of firms in period t. Then $K_t = 1 - \Gamma^t(0)$. K_t is constant over the diffusion process *iff* $\Gamma^t(0)$ is stable over time. Empirical research should be particularly wary of assuming a constant asymptote if the innovation is undergoing an improvement process aimed at broadening its uses during the period studied. This, in particular, applies to the specific class of market penetration models (SPINRAD, 1980, PETERKA, 1977).

3. PROPERTIES OF THE AGGREGATE DIFFUSION CURVE

Here the framework of Section 2 is used to investigate the properties of the aggregate diffusion curve. Its purpose is both to consider the factors which are likely to affect the properties of the aggregate diffusion curve, and to provide some guidelines for further empirical research. First a set of general results on the determinants of the extent and speed of diffusion are derived, and then the form of diffusion curves in some simple diffusion environments is examined. For concreteness, I shall continue with the example of the last section, that is a diffusion process where V is the only FDV and h is the only TDV. Allowing for more than one FDV or TDV does not change the qualitative results.

The critical value of V at any point in time, that is that value of V which separates those who have adopted from those who have not, is determined by the solution to the equation

$H\,[V^*_t\,,\,h\,(t)\,,\,\rho\,,\,\overline{C}\,,\,q\,]=0$. Throughout I shall use a first-order logarithmic Taylor's approximation to the criterion function and therefore:

$$\log V^*_t = \alpha + \alpha_1 \log h(t) + \alpha_2 \log \rho + \alpha_3 \log \overline{C} - \alpha_4 \log q \tag{5}$$

where $\alpha_i \geq 0$ for $i = 1\,,\ldots,4$

The extent of diffusion at t, P_t, is the proportion of firms who have innovated by t, i.e., all those firms with assets greater than or equal to V^*_t or

$$P_t = \int_{\log V^*_t}^{\infty} g\,(\varepsilon)\,d\varepsilon \equiv 1 - G\,(\log V^*_t) \tag{6}$$

Using (6) and (5) it is easy to show that the level of diffusion at any point of time is an increasing function of the incremental rate of profit on the inputs embodying the innovation, q, and a decreasing function of investment requirements, $\overline{C}$, risk aversion, ρ, and the degree of uncertainty on the rate of profit of the new inputs, h. Now consider families of asset distributions of increasing mean and variance; that is, consider the set of asset distributions constructed so that

$$\log V_i\,(m\,,\theta) = \mu_v + m + \theta\left[\log V_i\,(0\,,1) - \mu_v\right] \text{ for } i = 1\,,\ldots,N \tag{7}$$

where μ_v is the first moment of the distribution of $\log V_i\,(0\,,1)$, and m and θ are indexes of distributions of increasing mean and variance, respectively. Substituting (7) into (5) to determine $\log V^*_t\,(m\,,\theta)$ in terms of $\log V^*_t\,(0\,,1)$, inserting the result into (6), and differentiating with respect to m and then θ, one finds that $\partial P_t/\partial m = g\,(\,)\geq 0$ and $\partial P_t/\partial\theta = g\,(\,)\,\theta^{-2}\,[\log V^*_t\,(0\,,1) - \mu_v\,]$. Provided that $g\,(\,) > 0$, the latter dervative is greater than, equal to, or less than zero according as $P_t \lesseqgtr 1/2$. That is, an increase in the mean of the distribution of asset values will increase the extent of diffusion, and an increase in the variance of the distribution will increase the extent of diffusion in the periods before half of the potential users have adopted, and decrease it thereafter. This reflects the fact than an increase in θ makes the distribution of $\log V$ thicker in both tails. Hence the cumulative distribution to the right of a point $\log V^* > \mu_v$ will increase with θ and the converse will hold for a point $\log V^* < \mu_v$[8]. In short, an increase in an argument of the criterion function will affect the extent of diffusion in the same direction as it affects the value of the criterion function. If the argument is an FDV one considers an increase

8. Figure 4b below illustrates this point for normal distributions of $\log V$.

in the mean of its distribution. An increase in the variance of this distribution will increase or decrease P_t according as $P_t \lesseqgtr 1/2$.

Call any event which reduces the time, γ, required to increase the extent of diffusion from any one level, δ_0, to any other, δ_1, where $\delta_1 > \delta_0$, an event which accelerates the diffusion process. For an asset distribution in which $m = 0, \theta = 1$, it follows from (6) that the critical level of assets required for δ_i $(i = 0, 1)$ is

$$\log V^*_{\delta i}\,(m = 0, \theta = 1) = G^{-1}\,(1 - \delta_i)$$

where $G^{-1}G\,(x) = x$, that is G^{-1} is the inverse function of G, and it is assumed that G^{-1} exists in the relevant interval. Hence the required decrease in the critical value of V for the given increase in P_t is

$$\log V^*_{\delta 0}\,(m = 0, \theta = 1) - \log V^*_{\delta 1}\,(m = 0, \theta = 1) = G^{-1}\,(1 - \delta_0) - G^{-1}\,(1 - \delta_1) \equiv \beta_{\delta_0\delta_1} > 0$$

Noting from (7) that

$$\log V^*_i\,(m, \theta) - \log V^*_j\,(m, \theta) = \theta\,[\log V^*_i\,(0, 1) - \log V^*_j\,(0, 1)]$$

and substituting from (5) the values of $h(\ )$ which will produce the required critical asset values, one has

$$\log h\,(\gamma) = \log h\,(0) - \frac{\theta\beta\,(\delta_0, \delta_1)}{\alpha_1} \tag{8}$$

Since $h' < 0$ (8) can be solved explicitely for γ, and $\delta\gamma/\delta\theta > 0$; that is, an increase in the variance of the FDV or an increase in the heterogeneity of the potential users will slow down diffusion. From (8) it can also be shown that any factor which increases $\log h\,(0) - \log h\,(\gamma)$ for any given γ, that is any factor which increases the rate of spread of information on the innovation, will accelerate diffusion. For example, if one approximates the time path of $h\,(t)$ with an exponential function, i.e., $h\,(t) = h_0 e^{-at}$, then a becomes an index of the rate of spread of information, $\gamma = \theta\beta\,(\delta_0, \delta_1)/\alpha_1 a$ and $\delta\gamma/\delta_a < 0$.

Table 1 provides a summary of the results on the determinants of the extent, speed, and asymptote of the diffusion process. Most diffusion processes are summarized in terms of parameters which are directly interpretable in terms of these three characteristics. The question marks in the second column indicate that the sign of a given relationship is indeterminate without making more stringent assumptions on the shape of the utility function, a topic on which there is little *a priori* information. Only the degree of heterogeneity of the potential user firms (θ or σ_v) and the rate of spread of information on the new inputs (a or $-h'/h$) have clear cut effects on the speed of diffusion process. If

more than one argument of the criterion function changes over time then each TDV's rate of growth would become a determinant of the speed of diffusion, the direction being determined by the sign of the derivative of the criterion function with respect to that variable. The effects of the various variables on the extent of diffusion follow straightforwardly from their influence on the criterion function, while the asymptote changes if and only if the percentage of the universe for which the true incremental rate of profit is greater than zero changes.

To derive precise functional forms for the diffusion curve one must specify or estimate the distribution of the firm-differentiating variable and the time path of the time-differentiating variable. In order to concentrate on the effects of different distributions of the FDV on the slope of the diffusion curve, approximate the time path of the TDV by a simple exponential growth curve, i.e., let

Table 1. Determinants of the Three Characteristics of the Diffusion Curve

Determinant	Extent of diffusion, P_t	Speed of diffusion[a/]	Asymptote,[b/] K_t
θ (σ_v)	$\gtrless 0$ according as $P_t \gtrless \frac{1}{2}$	-	0
μ_v	+	?	0
a $(-h_t'/h)$	+	+	0
$h(t=0)$	-	?	0
$\bar{C}$	-	?	0
ρ	-	?	0
q $E(\bar{\pi})$	+	?	+ [c/]

[a/] Time required to increase diffusion from δ_0 to δ_1 for any $\delta_1 > \delta_0$

[b/] The upper bound of the diffusion curve, at any t.

[c/] Any factor which increases the percentage of the universe for which the true incremental rate of profit on the new inputs is greater than zero $[\Gamma_t(0)]$ will increase K_t.

$$h(t) = h_0 e^{-at} \tag{9}$$

In the present example (9) implies that the degree of uncertainty regarding the new inputs is decreasing at a constant rate and that as t increases the new and old inputs have equally uncertain profit rates. Substituting (9) into (5) and taking derivatives with respect to time, one finds that

$$p'_t = g\,(log\ V^*_t)\ \alpha_1 a \geq 0 \tag{10a}$$

and

$$p''_t = -g'\,(logV^*_t)\ (\alpha_1 a)^2 \tag{10b}$$

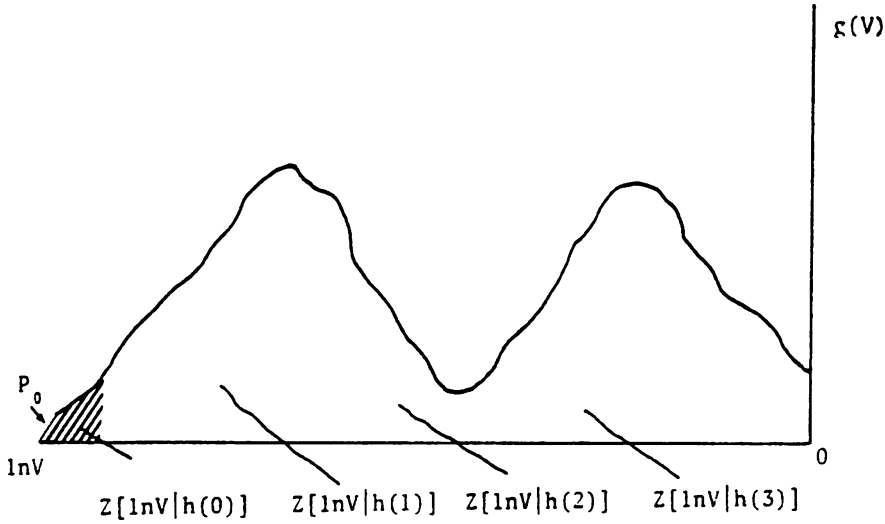
Equation (10) suffices to determine the general form of the diffusion curve for different distributions of the FDV. The diffusion curve resulting from a bimodal distribution of log V is illustrated in Figure 3. The constant rate of growth of the TDV causes the critical value of the FDV to move to the right at a constant rate. As a result, the relative number of adopters in different periods is determined entirely by the relative density of firms in the region about log V^*_t. This density is increasing (decreasing) and decreasing (increasing) respectively, before and after each mode (trough) of the distribution of log V, and is at a local maximum (minimum) at the mode (trough). Hence the diffusion curve will be convex (concave) before each mode (trough), have a point of inflection at the modes (troughs) and be concave (convex) thereafter.

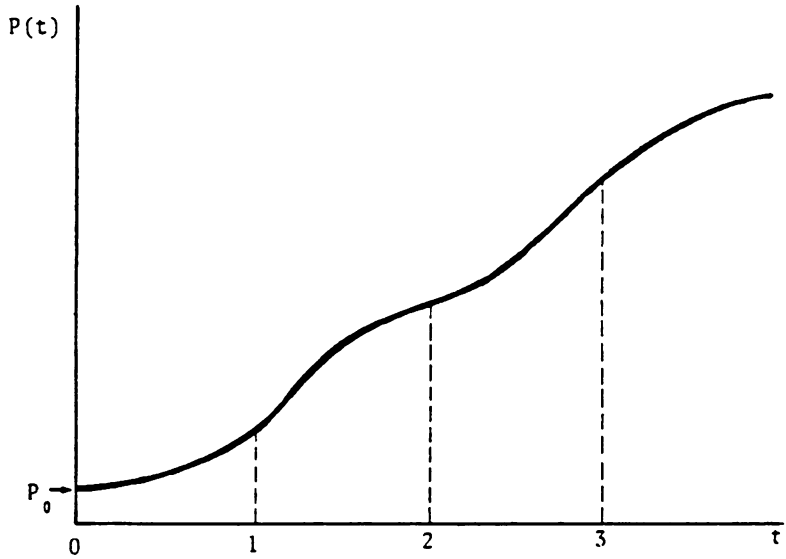
The result is the familiar S-shaped diffusion curve. If the distribution of the FDV is bimodal the diffusion curve will consist of two S, one occurring before log V^*_t passes over the trough of the distribution of log V and one thereafter. In fact if the TDV moves at an approximately constant rate, then every diffusion process[9], will be multi-S-shaped, the number of S being the same as the number of modes in the underlying distribution of the FDV.

The shape of the $h(t)$ function is determined by the way knowledge about the innovation accumulates. Usually some information is passed on by sources outside of the universe of potential users while other information is generated by the use of the innovation. Therefore a useful and more general representation of the $h(t)$ function would be

9. Apart of course from the pathological case of a process occurring in a sample where a mode is at the extreme point of the distribution of the FDV.

Figure 3: The Diffusion Process with a Bimodal Distribution of the FDV





$$h(t) = h_0 \, e^{[-at - b_0 \int^t P(\varepsilon)\, d\varepsilon]} \tag{11}$$

where b represents the percentage decrease in $h(t)$ resulting from a one per cent increase in accumulated firm years of use[10]. The more general learning curve, however, produces only small changes in the form of the diffusion curve. If (11) and not (9) is the correct specification for $h(t)$ then the diffusion curve will be convex until slightly after $\log V^*_t$ passes over a mode of the distribution of the FDV and will tend to turn convex again slightly before reaching a trough.

To illustrate these points consider the familiar case of a universe whose assets distribute log-normally; that is, assume

$$\log V \approx N(\mu_v, \sigma_v^2) \tag{12}$$

If information accumulates at a constant rate then the exact form of the diffusion curve for this universe can be found by substituting (12) and (9) into (10a), changing variables, and integrating over time. The result is

$$P(t) = \Lambda\left[\frac{\alpha_0}{\alpha_1 a}, \left(\frac{\sigma_v}{\alpha_1 a}\right)^2\right]$$

where $\Lambda(\,)$ signifies an accumulative normal curve whose underlying relative density is centered at

$$\alpha_0/\alpha_1 a \;[\text{ where } \alpha_0 = \alpha - \alpha_1 \log h(0) - \alpha_2 \log \rho - \alpha_3 \log \overline{C} + \alpha_4 \log q + \mu_v\,]$$

and which has a variance of $(\sigma_v/\alpha_1 a)^2$. That is, the diffusion curve will be the single-S-shaped cumulative normal with point of inflection occurring at $t = \alpha_0/\alpha_1 a$, $P_t = 1/2$. The curve will shift upwards at every t for increases in $\alpha_0/\alpha_1 a$ (see Figure 4a) and will increase its speed at every P_t in response to decreases in $\sigma_v/\alpha_1 a$ (see Figure 4b).

If, instead of assuming that information accumulates at a constant rate, it was assumed that the growth rate of information was proportional to accumulated use [equation (11) with $a = 0$ instead of equation (9)], the diffusion curve would still be single-S-shaped but

10. The problem of choosing the appropriate form of the learning curve has been discussed before in the context of the learning-by-doing literature (see ARROW, 1962); the question of whether the appropriate index for the growth of information is proportional to time or accumulated use has given rise to some interesting policy debates, see DAVID (1975).

the point of inflection would now be at $P_t = 0.7$. If (11) is assumed with both a and b positive the point of inflection would occur between 0.5 and 0.7.

Other examples are easy to construct but the basic conclusions of the analysis should be clear. First the shape of the aggregate diffusion curve will depend on the characteristics of the universe of potential user firms, a point which has gone relatively unnoticed in the diffusion literature. The empirical researcher should familiarize himself with the distributions of any differentiating variables which exist and examine the actual time path of P_t for evidence of clusters of firms. Second, the time path of the TDV will also influence the properties of the diffusion curve. The amount of information potential adopters have on the new innovation is an obvious candidate for a TDV. Since issues related to the spread of information from different sources are also at the heart of most policy questions regarding diffusion, it would be helpful to have more empirical evidence on different information-generating mechanisms. Perhaps more intensive analysis of individual adoption behavior can provide the required evidence.

This section has summarized the determinants of three major properties of the diffusion curve and exhibited the relationship between its form and the characteristics of the sample of potential users. Further empirical work, especially on the characteristics of information flows, is necessary before the analysis can be fully utilized.

4. CONCLUSIONS

Various attempts have been undertaken, both on the empirical as well as on the theoretical side to describe and prescribe the process of diffusion of new technologies. Thus far, on the empirical side we are left with an elaborate taxonomy of the presumably casual factors involved inducing diffusion; on the theoretical side we are overwhelmed with mathematically sophisticated models on the dynamics of the diffusion process.

What needs to be done is a merging of these approaches for the sake of contributing to better understanding and decision-making in technology policy – both on the firm and aggregate level. However, this requirement alone does not come to grips with the fundamental methodological difficulties that still persist in the economics of technological change. One such difficulty is an appropriate definition and measurement of diffusion.

In the model presented diffusion could be measured by net changes in the size/or number of producers of a new product. While the approach necessarily presents a limitation, net changes in the number of firms provides us with an interesting aspect of diffusion. *It permits us to estimate the evolutionary period*, that is the interval between the time when the first competitor appears in the market and the time when the number of firms in the market reaches its peak, i.e. what is considered to be a "mature" market.

Changes in the number of firms imply entry and exit, therefore, changes in the market structure. Traditional economic theory assigns entry (exit) a prominent role in determining market structure and performance. Yet, very little work has been done on the relation of entry to technological change after SCHUMPETER first proposed his invention, innovation,

imitation process, that is, a successful innovation attracts imitators of technological change into the market.

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APPENDIX

THE DERIVATION OF $\pi(t)$

$$\pi(t) = \frac{\text{Present Value of Net Return}_t - \text{Present Value of Cost}_t}{\text{Present Value of Cost}_t} = \frac{N_t - C_t}{C_t},$$

where

$$N_t = \int_t^{t+T} N'(\tau)\, e^{i_t \tau}\, d\tau$$

$$N'(\tau) = Pr_\tau y_\tau - W_\tau b_\tau ,$$

and τ = lifespan of the machine embodying the innovation
i_t = nominal discount rate
Pr_τ = the price of output at time $\tau = t$
y_τ = the output of a machine of age τ
b_τ = variable input required to run a machine of age τ
W_τ = price index of variable input at time $\tau + t$

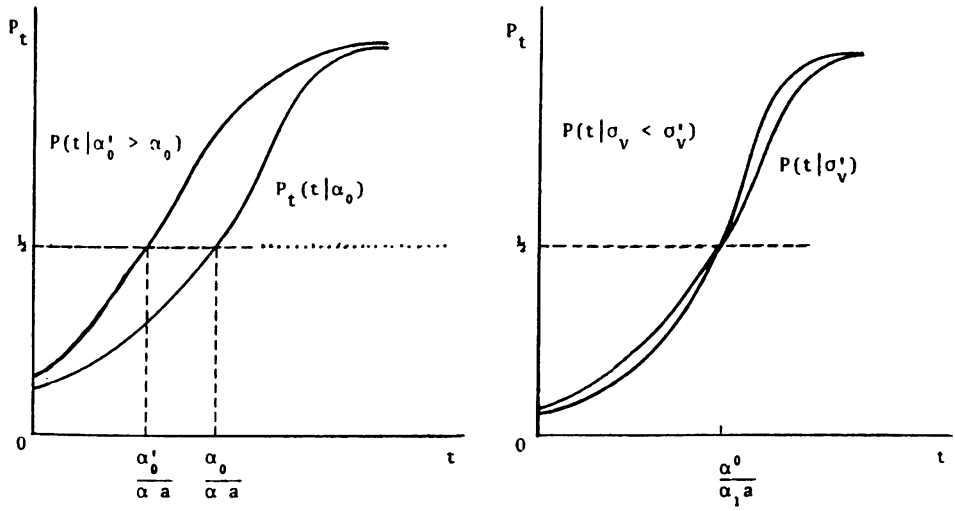
Of the above variables, the sequences $< W_\tau >_t^{t+T}$ and $< Pr_\tau >_t^{t+T}$ are not known to the entrepreneur at the time he innovates, and hence require the formation of expectations. It is assumed that: (a) expectations are formulated in terms of a constant rate of growth of prices, $\hat{g}$; (b) the entrepreneur sets his expectations on the rate of groth of wages = the rate of growth of price of output = the rate of growth of the economy-wide price index; (c) $W_t/P_t = \overline{\omega}$, a constant over the diffusion process; and (d) the *real* discount rate is constant and equal to $\overline{r}$ over the diffusion process.

Given that these expectations and the constancy assumptions are to apply to lifespans of from 15 to 30 years, they do not seem to depart too far from reality; still there will be situations in which they will not be valid and this is the reason for our interest in the errors-in-variable model presented in the text.

It follows that $N'(\tau) = Pr_t e^{g\tau}(y_\tau - \overline{\omega} b_\tau)$ and $N_t = Pr_t \int_t^{t+T} (y_\tau - \overline{\omega} b_\tau)\, e^{-\overline{r}\tau} d\tau$.

Let $\int_t^{t+T} (y_\tau - \overline{\omega} b_\tau)\, e^{-r\tau} d\tau = \theta$. θ is the discounted number of units of net output (output minus the equivalent units of variable input) produced by the machine over its lifespan and given $\overline{r}$ and $\overline{\omega}$, it is completely determined by the physical characteristics of the machinery. On substitution $\pi_t = Pr_t / R_t X_t - 1$.

Figure 4: Accumulated Normal Diffusion Curves



4a. Increases in α_0

4b. Decreases in σ_v

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ABSTRACT

This paper shows the relationship of the adoption decision to the properties of the diffusion curve. A model for the adoption decision among firms is developed involving uncertainty as well as different value and time characteristics of potential adopters. These assumptions are sufficient to determine and explain key properties of the aggregate diffusion curve. The impacts of such model are discussed for empirical innovation and diffusion processors.