Production, Foreign Trade, and Global Curvature Conditions: Switzerland, 1948-1988

ULRICH KOHLI*

1. INTRODUCTION

Our main purpose, in this paper, is to report estimates of a GNP function for Switzerland using annual data covering the last four decades. The question of the price elasticities of imports and exports has long captured the attention of applied economists. The GNP function approach makes it possible to investigate this and related issues within a tight theoretical framework. A secondary objective of this study is to illustrate how required curvature conditions can be imposed globally if needed when estimating a GNP function; this is achieved here by using the Symmetric Normalized Quadratic variable profit function recently proposed by KOHLI (1989).

The GNP function approach to modeling imports and exports (KOHLI, 1978, 1991a; WOODLAND, 1982) is based on production theory, and it can be viewed as an extension of BURGESS’s (1974a, 1974b, 1976) pioneering work on import determination. Imports are treated as an input to the technology, and exports as an output. Imports are used together with domestic labor and capital services to jointly produce exports and goods intended for domestic absorption (e.g. consumption goods and investment goods). This treatment of imports and exports not only accounts for the fact that much of international trade is in raw materials and intermediate products, but it also recognizes that most so-called traded endproducts must still transit through a number of channels (e.g. transportation, insurance, repackaging, storage, and retail) at home (imports) or abroad (exports) before reaching final demand, so that a substantial proportion of the final price-tag is actually accounted for by local value-added. An important advantage of the GNP function approach is that it rests on solid theoretical foundations, and, like standard trade theory, it treats domestic factor endowments and good prices as given. Moreover, it yields a wealth of results regarding the substitution possibilities allowed for by the

* University of Colorado, Boulder, CO, 80309, USA, and University of Geneva, CH-1211 Geneva 4, Switzerland.

An earlier version of this paper was presented at the North-American Meetings of the Atlantic Economic Society, Williamsburg, VA, October 11-14, 1990. I wish to thank ALTON D. LAW for his comments and suggestions. Financial support from the Swiss National Science Foundation, under grant # 12-27750.89, is gratefully acknowledged.
technology which cannot be matched by traditional methods which typically rely on single-equation models. Finally, unlike many models of international trade theory, it can easily be implemented empirically since the data it requires are precisely of the type contained in the National Accounts.

GNP functions have been estimated for a number of countries, but not for Switzerland, although we are aware of three studies applying duality theory and production theory to the determination of Swiss imports. KOHLI (1982) estimated a capital revenue function for Switzerland; however, labor was treated as a variable input, and exports were not explicitly dealt with. MOUNTAIN (1986) has used the same model and the same data base while allowing for variable returns to scale. BACCHETTA (1990), finally, has estimated a number of cost and revenue functions incorporating imports and, in some cases, exports.

The Translog function is probably the flexible functional form most often used in applied production analysis. This is due to its relative simplicity and the ease with which it can be estimated. Yet, use of the Translog functional form is not foolproof. Thus, the GNP function must satisfy a number of regularity conditions; in particular it must be convex in the prices of the variable components, and concave in the quantities of the fixed components. It is well known that the Translog functional form cannot satisfy these conditions globally. Consequently, these regularity conditions are often violated in empirical work, as soon as the number of components exceeds three or four; this has long been a major source of frustration for applied economists. These curvature conditions are implied by economic theory and they must be satisfied for the estimates to be meaningful. In some cases, the required regularity conditions are satisfied at the outset; e.g. KOHLI (1978). In other cases convexity or concavity can be imposed locally using one of the techniques proposed by LAU (1974, 1978), and by WILEY, SCHMIDT, and BRAMBLE (1973); this may then be sufficient for the regularity conditions to be satisfied over the entire range of observed prices and quantities; see KOHLI (1990). In some cases, however, things do not work out that neatly. Thus, BACCHETTA (1990) reports that he has been unable to estimate a two-input, four-output Translog GNP function for Switzerland satisfying all regularity conditions for all observations. Curvature conditions can sometimes be imposed globally, but the flexibility of the functional form is often destroyed in the process. Thus, JORGENSEN and FRAUMENI (1981) impose global concavity in the Translog case, but as shown by DIEWERT and WALES (1987), this seriously restricts the substitution possibilities allowed for by the technology: the function essentially becomes COBB-DOUGLAS over certain areas. Our intention, in this paper, is therefore to use a

---

1. See GOLDSTEIN and KHAN (1985) for a recent survey.
4. Note also that the JORGENSEN and FRAUMENI (1981) technique cannot be used to impose global convexity.
functional form which allows the regularity conditions to be imposed globally, rather than just locally, and this without endangering flexibility.

In several recent papers, DIEWERT and WALES (1987, 1988) have proposed two families of flexible functional forms for cost and expenditure functions: the Normalized Quadratic, and the Symmetric Normalized Quadratic. Global curvature conditions can be imposed in the case of these functions without compromising their flexibility. DIEWERT and OSTENSOE (1988) have defined the Normalized Quadratic variable profit function. This flexible functional form is well suited to model multiple-input multiple-output technologies, and it allows for curvature conditions to be imposed globally at both the fixed and the variable quantity levels without jeopardizing its flexibility. However, the Normalized Quadratic variable profit function suffers from the drawback that it singles out one fixed input and one variable component for asymmetrical treatment. This is not so with the Symmetric Normalized Quadratic variable profit function proposed by KOHLI (1989), and which will be the functional form used in this paper.

The remainder of this paper proceeds as follows. The theoretical model is briefly reviewed in the next section. The functional form which we use is presented in Section 3, and other issues relating to the empirical application are discussed in Section 4. Empirical results are reported in Section 5, and Section 6 concludes; the proofs to three theorems concerning the flexibility and the curvature properties of the Symmetric Normalized Quadratic variable profit function are relegated to the Appendix.

2. THE GNP FUNCTION APPROACH TO MODELING IMPORTS AND EXPORTS

Assume that the technology counts $J$ fixed inputs (factors), and $I$ outputs and variable inputs (we treat variable inputs, including imports, as negative outputs). Let $x=[x_j]$ be the $J$-dimensional vector of fixed input quantities, and $y=[y_i]$ the $I$-dimensional vector of output and variable input quantities.

Output prices are denoted by vector $p=[p_i]$, and the vector of input prices (factor rental prices) by vector $w=[w_j]$. $T$ is the production possibilities set, that is the set of feasible input and output combinations $(y, x)$; we assume that $T$ is a convex cone. The technology can also be represented by a variable profit (or GNP) function defined as follows:

$$\pi = \pi(p, x) = \max_y \{p'y; (y, x) \in T\}.$$ (1)

5. These can also be used in a straightforward way as production functions, revenue functions, factor requirements functions, utility functions, indirect utility functions, reciprocal indirect utility functions, and so on.

6. Throughout this paper, we will use $i$ and $h$ as running indexes for the $I$ outputs, $j$ and $k$ for the $J$ fixed inputs, and $m$ and $n$ for either the outputs or the fixed inputs.
\( \pi(\cdot) \) is well defined for all positive output prices and nonnegative input quantities. Given the assumptions made on \( T, \pi(\cdot) \) is linearly homogeneous and convex in prices; nondecreasing in the prices of outputs and nonincreasing in the prices of variable inputs; and increasing, linearly homogeneous and concave in fixed input quantities.

It follows from HOTELLING's Lemma that differentiation of \( \pi(\cdot) \) with respect to prices yields the GNP maximizing supply of outputs (demand for variable inputs):

\[
y = y(p, x) = \nabla_p \pi(p, x),
\]

where \( \nabla_p \pi(\cdot) = [\partial \pi(\cdot)/\partial p_i] \) is the vector of partial derivatives of \( \pi(\cdot) \) with respect to the components of \( p \): \( \nabla_p \pi(\cdot) \) is the gradient of \( \pi(\cdot) \) with respect to \( p \). The homogeneity properties of \( \pi(\cdot) \) imply that the output supply (variable input demand) functions are linearly homogeneous in the components of \( x \), as well as homogeneous of degree zero in the components of \( p \); furthermore, the convexity of \( \pi(\cdot) \) indicates that all output supply (and variable input demand) functions are necessarily nondecreasing in their own prices.

Similarly, the marginal product conditions imply that the differentiation of \( \pi(\cdot) \) with respect to the fixed input quantities yields the competitive input prices:

\[
w = w(p, x) = \nabla_x \pi(p, x),
\]

where \( \nabla_x \pi(\cdot) = [\partial \pi(\cdot)/\partial x_j] \) is the gradient of \( \pi(\cdot) \) with respect to \( x \). Equations (3) are often called the fixed-input inverse demand functions since they explain fixed input prices as functions of their quantities. The homogeneity properties of \( \pi(\cdot) \) imply that the fixed input inverse demand functions are linearly homogeneous in the components of \( p \), as well as homogeneous of degree zero in the components of \( x \). Furthermore, it follows from the concavity of \( \pi(\cdot) \) with respect to the fixed inputs that the inverse demand functions (3) are nonincreasing in their own quantities.

It is noteworthy that the homogeneity properties of \( \pi(\cdot) \) imply, by EULER's Theorem on homogeneous functions, the following adding up properties:

\[
\pi(p, x) = p' \nabla_p \pi(p, x) = x' \nabla_x \pi(p, x) = p'y = x'w.
\]

That is, variable profits are equal to the total payments to the fixed factors. In the context of the GNP function, (4) simply means that GNP equals national income.

It is often convenient to describe the substitution possibilities allowed for by the technology by a set of elasticities of transformation, intensity, and substitution. Let \( H \) be the Hessian of the GNP function. It is useful to partition \( H \) in the following way:
where $\pi_{pp} = \nabla^2_{pp} \pi (\cdot) \equiv [\partial^2 \pi (\cdot) / \partial p_i \partial p_h]$ is the sub-Hessian of $\pi (\cdot)$ with respect to the components of $p$, and so on. We then define the substitution matrix $\Sigma = [\sigma_{mn}]$ as follows:

$$
\Sigma \equiv \begin{bmatrix}
\Sigma_{pp} & \Sigma_{px} \\
\Sigma_{xp} & \Sigma_{xx}
\end{bmatrix} = \pi \begin{bmatrix}
\pi^{-1}_{pp} & \pi^{-1}_{px} & \pi^{-1}_{xx} \\
\pi^{-1}_{xp} & \pi^{-1}_{xx} & \pi^{-1}_{xx}
\end{bmatrix},
$$

(6)

where $\pi_p = \text{diag} [\nabla_p \pi (\cdot)]$ and $\pi_x = \text{diag} [\nabla_x \pi (\cdot)]$. The elements of $\Sigma_{pp} = [\sigma_{ii}]$ are the elasticities of transformation between variable quantities, while the components of $\Sigma_{xx} = [\sigma_{jk}]$ are the inverse elasticities of substitution between fixed inputs. The elements of $\Sigma_{px} = [\sigma_{ij}]$, finally, are the elasticities of intensity. Symmetry of the Hessian (YOUNG'S Theorem) implies that $\Sigma_{pp}$ and $\Sigma_{xx}$ are symmetric, and that $\Sigma_{pp} = \Sigma_{xx}$. The convexity of $\pi (\cdot)$ with respect to the components of $p$ implies that $\pi_{pp}$, and hence $\Sigma_{pp}$, are positive semi-definite, while it follows from the concavity of $\pi (\cdot)$ with respect to the components of $x$ that $\pi_{xx}$, and hence $\Sigma_{xx}$, are negative semi-definite. Thus, $\sigma_{ii} \geq 0$ and $\sigma_{jj} \leq 0$.

Alternatively we may use more familiar price and quantity elasticities. Let $s_i$ be the share of variable quantity $i$ in GNP ($s_i \equiv p_i y_i / \pi$), and $s_j$ the share of fixed input $j$ ($s_j \equiv w_j x_i / \pi$). We then define $E \equiv [e_{mn}]$, the matrix of the price and quantity elasticities of output supply and input inverse demand, in the following way:

$$
E \equiv \begin{bmatrix}
E_{pp} & E_{px} \\
E_{xp} & E_{xx}
\end{bmatrix} \equiv \begin{bmatrix}
[\partial \ln y_i / \partial \ln p_h] & [\partial \ln y_i / \partial \ln x_j] \\
[\partial \ln w_j / \partial \ln p_i] & [\partial \ln w_j / \partial \ln x_k]
\end{bmatrix}.
$$

$E$ can then be calculated as follows:

$$
E = \begin{bmatrix}
\pi^{-1}_{pp} & \pi^{-1}_{px} & \pi^{-1}_{xx} \\
\pi^{-1}_{xp} & \pi^{-1}_{xx} & \pi^{-1}_{xx}
\end{bmatrix} = \Sigma S,
$$

(7)

7. The inverse elasticities of substitution are sometimes called Hicksian elasticities of complementarity; see SYRQUIN and HOLLENDER (1982), for instance.

8. These definitions conform to DIEWERT's (1974) terminology.
where \( P = \text{diag}(p), X = \text{diag}(x), S_l = \text{diag}\{[s_l]\}, S_J = \text{diag}\{[s_J]\}, \) and \( S = \text{diag}\{[s_l], [s_J]\}; \) 
\( S \) is thus the \((I+J) \times (I+J)\) matrix that contains the GNP shares of the \( I \) outputs and of the \( J \) fixed inputs on its main diagonal, and zeros elsewhere. In interpreting the elements of \( E \), it is important to remember the identity of the corresponding endogenous and exogenous variables. Thus the elements of \( E_{pp} \) are price elasticities of output supply, while the elements of \( E_{xx} \) are the quantity elasticities of the inverse demands for inputs. The elements of \( E_{px} \) indicate the effect of a change in input quantities on output supply, while the elements of \( E_{xp} \) indicate the effect of a change in output prices on input prices. Note that is generally not true that \( e_{ij} = e_{ji} \). However, the homogeneity of (2) and (3) imply the following restrictions:

\[
E_{pp} I_I = 0_I, \quad E_{px} I_J = 1_I, \quad E_{xp} I_J = 1_J, \quad E_{xx} I_J = 0_I, \quad (8)
\]

where \( I_I \) and \( I_J \) are respectively the \( I \)- and \( J \)-dimensional unit vectors, and \( 0_I \) and \( 0_J \) are the \( I \)- and \( J \)-dimensional null vectors.

Upon specification of a functional form for \( \pi(\cdot) \), the GNP function (1) can be estimated. However, assuming profit maximization, it is statistically more efficient to estimate instead the system of output supply functions (2) and inverse input demand functions (3). The estimates can then be used to recover the GNP function, and to calculate estimates of \( \Sigma \) and \( E \). It is important then that the estimated GNP function satisfy the regularity conditions on the variable profit function, i.e. linear homogeneity, symmetry, adding-up, and particularly the curvature conditions. While adding-up usually follows from the construction of the data, and while linear homogeneity and symmetry can easily be imposed, this is not necessarily true for the curvature conditions: one must therefore be particularly careful when selecting a functional form.

3. THE SYMMETRIC NORMALIZED QUADRATIC VARIABLE PROFIT FUNCTION

In this paper we will use the Symmetric Normalized Quadratic variable profit function recently introduced by Kohli \( (1989) \):

\[
\pi = \frac{1}{2} (\beta' x) p' A p / (\alpha' p) + \frac{1}{2} (\alpha' p) x' B x / (\beta' x) + p' C x, \quad (9)
\]

where \( A = [a_{lk}], B = [b_{jk}], \) and \( C = [c_{lj}] \) are unknown matrices of dimensions \( I \times I, J \times J \) and \( I \times J \) respectively; \( \alpha = [\alpha_i], \) and \( \beta = [\beta_j] \) are nonnegative vectors of predetermined parameters; \( \alpha \) is \( I \)-dimensional, and \( \beta \) is \( J \)-dimensional. Furthermore, we require that \( A I_I = 0_I, B I_J = 0_J, \alpha' I_I = 1, \beta' I_J = 1. \) This functional form is a straightforward extension of the family of functional forms proposed by Dievert and Wales \( (1987, 1988) \). It is necessarily linearly homogeneous in prices and quantities, and its Hessian is symmetric provided that \( A \) and \( B \) are symmetric. We show in the Appendix that it is a flexible
functional form at point \((p^*, x^*) = (1/l, 1/l)\); that a necessary and sufficient condition for
global convexity in output prices is that \(A\) be positive semi-definite; and that a necessary
and sufficient condition for the function to be globally concave in fixed input quantities
is that \(B\) be negative semi-definite. If necessary, these curvature conditions can be imposed
by the LAU (1974, 1978), or the WILEY, SCHMIDT and BRAMBLE (1973) techniques. Note
that since the elements of \(\alpha\) and \(\beta\) are to be selected by the researcher, (9) does in fact
define an entire family of functional forms.\(^9\)

The output supply and inverse input demand functions can again be obtained by
differentiation of the variable profit function:

\[
y = (\beta'x)Ap/(\alpha'p) - \frac{1}{2}\alpha(\beta'x)p'Ap/(\alpha'p)^2 + \frac{1}{2}\alpha\alpha'Bx/(\beta'x) + Cx \tag{10}
\]

\[
w = \frac{1}{2}\beta p'Ap/(\alpha'p) + (\alpha'p)Bx/(\beta'x) - \frac{1}{2}\beta (\alpha'p)x'Bx/(\beta'x)^2 + C'p . \tag{11}
\]

These demand and supply functions are linear in the unknown parameters. Moreover,
all demand and supply functions have the same form.

4. EMPIRICAL IMPLEMENTATION

4.1 Technological Change

It is well known that growth in inputs alone is not sufficient to explain all of output growth.
Following BINSWANGER (1974), JORGENSON and FRAUMENI (1981), and MCKAY, LAW­
RENCE, and VLASTUIN (1983), among others, we allow the technology to shift over time,
and we include a time trend \(t\) among our explanatory variables; we rewrite the
Symmetric Normalized Quadratic GNP function as follows:

\[
\pi = \frac{1}{2} (\beta'x)p'Ap/(\alpha'p) + \frac{1}{2} (\alpha'p)x'Bx/(\beta'x) + p'Cx + p'Dxt + \frac{1}{2}dtt (\alpha'p) (\beta'x) t^2 , \tag{12}
\]

where \(D = [d_{ij}]\) is of dimension \(I \times J\) and \(dtt\) is a scalar.\(^10\) This modification means that
the derived demand and supply equations (10) and (11) now become functions of \(t\) and \(t^2\):

9. The Symmetric Normalized Quadratic variable profit function contains as a special case the functional
form proposed by DIEWERT and OSTENSOE (1988). All we need to do is to set \(\alpha_i = \beta_i = 1\) and \(\alpha_i = \beta_i = 0\) \((i \neq l, \ j \neq j)\). However, unlike that functional form, (9) does not single out one output and one fixed input: all outputs
and all fixed inputs are treated symmetrically.

10. Using the terminology of DIEWERT and WALES (1989), we may say that (12) is fully flexible, rather than
just TP (technological progress) flexible. In fact, (12) contains more parameters than the minimum required for
full flexibility with respect to time.
\[ y = (\beta'x)Ap/(\alpha'p) - \frac{1}{2}\alpha (\beta'x)p'Ap/(\alpha'p)^2 + \frac{1}{2}\alpha x'Bx/(\beta'x) + Cx + Dxt + \frac{1}{2}d\alpha (\beta'x)t^2 \]  

(13)

\[ w = \frac{1}{2}\beta p'Ap/(\alpha'p) + (\alpha'p) Bx/(\beta'x) - \frac{1}{2}\beta (\alpha'p) x'Bx/(\beta'x)^2 + C'p + D'pt + \frac{1}{2}d\beta (\alpha'p) t^2. \]  

(14)

4.2 Data

The Symmetric Normalized Quadratic functional form is used to estimate a GNP function for Switzerland using annual data covering the period 1948-1988. We consider two fixed inputs, labor (L) and capital (K), and four variable quantities: imports (M), exports (X), investment goods (I) and consumption goods (C). We require price and quantity series for all six variables. These were obtained by updating the data base of KOHLI and PEYTRIGNET (1975), and BACCHETTA (1990). Fixed input quantities and output (including variable input) prices are normalized to unity for 1980. Fixed input prices and variable input and output quantities are expressed in million 1980 francs; \( t \) is defined as a time trend with unit annual increments and normalized to zero for 1980. Additional details about the construction of the data, and the series themselves are available in KOHLI (1991b).

4.3 Curvature Conditions

As noted above, if the estimated profit function fails to satisfy the convexity or concavity conditions, these can be imposed. To do this, we have selected the technique of WILEY, SCHMIDT, and BRAMBLE (1973). As shown by these authors, a sufficient condition for a matrix to be positive semi-definite is that it can be written as:

\[ A = TT^*, \]

where \( T = [\tau_{ik}] \) is a lower triangular matrix. DIEWERT and WALES (1987), exploiting a result by LAU (1978), have shown that the above condition is also necessary for \( A \) to be positive semi-definite. Thus, with \( I = 4 \), and recalling that \( A_{1} = 0 \), we define:

\[
T = \begin{bmatrix}
\tau_{11} & 0 & 0 \\
\tau_{21} & \tau_{22} & 0 \\
\tau_{31} & \tau_{32} & \tau_{33}
\end{bmatrix}.
\]
We then have:

\[
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{12} & a_{22} & a_{23} \\
a_{13} & a_{23} & a_{33}
\end{bmatrix} = \begin{bmatrix}
\tau_{11} & \tau_{11}\tau_{21} & \tau_{11}\tau_{31} \\
\tau_{11}\tau_{21} & \tau_{21}^2 + \tau_{22}^2 & \tau_{21}\tau_{31} + \tau_{22}\tau_{32} \\
\tau_{11}\tau_{31} & \tau_{21}\tau_{31} + \tau_{22}\tau_{32} & \tau_{31}^2 + \tau_{32}^2 + \tau_{33}^2
\end{bmatrix}.
\]

This reparameterization ensures that the resulting GNP function is convex in output prices. Similarly, a matrix \(B\) is negative semi-definite if it can be written:

\[
B = -TT'.
\]

4.4 Stochastic Specification and Estimation Technique

We assume that the demand and supply functions (13)-(14) are exact, except for errors in optimization. We specify a vector of additive disturbances which we assume to be identically distributed normal random vectors with mean vector zero. The model is estimated by using an iterative version of Zellner’s (1962) method for seemingly unrelated regressor equations; this method is numerically equivalent to maximum likelihood. We have 246 observations (six equations times 41 annual observations) to estimate 24 unknown parameters. All estimations were made on a VAX 8700 computer using SHAZAM, Version 6.2 (White, 1988).

5. EMPIRICAL RESULTS

Estimation of the Symmetric Normalized Quadratic GNP function requires the investigator to choose the components of \(\alpha\) and \(\beta\). We follow Diewerts and Wales (1988) and Kohli (1989) by setting the elements of \(\alpha\) and \(\beta\) equal to \(1/I\) and \(1/J\), respectively. This ensures that \(\alpha'1_I = \beta'1_J = 1\). It proved necessary to impose global convexity, but global concavity obtained without difficulty. The corresponding parameter estimates are reported in Table 1, together with estimates of asymptotic \(t\)-values; these must, however, be interpreted with care since the \(A\) and \(B\) matrices do not have full rank.\(^{11}\) The value of the logarithm of the likelihood function (LL) is also shown.

\(^{11}\) Indeed, as seen in Section 3, \(A\) and \(B\) have the property \(A1_I = 0_I\) and \(B1_J = 0_J\), so they are of less than full rank. Moreover, there is one nonnegativity constraint that is binding (the estimates of \(\tau_{ij}\) is very close to zero). Hence standard nonlinear regression asymptotic theory does not hold; see Gourieroux, Holly, and Monfort (1982).
Table 1
Symmetric Normalized Quadratic GNP Function
Parameter Estimates
(t values in parentheses)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>r MM</td>
<td>169.99</td>
<td>(7.52)</td>
</tr>
<tr>
<td>r MX</td>
<td>-17.514</td>
<td>(-0.57)</td>
</tr>
<tr>
<td>r MI</td>
<td>47.027</td>
<td>(1.49)</td>
</tr>
<tr>
<td>r XX</td>
<td>82.387</td>
<td>(2.62)</td>
</tr>
<tr>
<td>r XI</td>
<td>-124.86</td>
<td>(-3.61)</td>
</tr>
<tr>
<td>r II</td>
<td>-0.00022</td>
<td>(-0.00)</td>
</tr>
<tr>
<td>D LL</td>
<td>-16261</td>
<td>(-7.21)</td>
</tr>
<tr>
<td>D ML</td>
<td>-34054</td>
<td>(-18.32)</td>
</tr>
<tr>
<td>D MK</td>
<td>-30357</td>
<td>(-13.46)</td>
</tr>
<tr>
<td>D XL</td>
<td>738.48</td>
<td>(0.27)</td>
</tr>
<tr>
<td>D XK</td>
<td>58685</td>
<td>(20.58)</td>
</tr>
<tr>
<td>C IL</td>
<td>43324</td>
<td>(19.43)</td>
</tr>
<tr>
<td>C IK</td>
<td>-693.95</td>
<td>(-0.32)</td>
</tr>
<tr>
<td>C CL</td>
<td>88967</td>
<td>(43.38)</td>
</tr>
<tr>
<td>C CK</td>
<td>15495</td>
<td>(6.51)</td>
</tr>
<tr>
<td>C ML</td>
<td>-832.81</td>
<td>(-9.00)</td>
</tr>
<tr>
<td>C MK</td>
<td>-420.96</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>C XL</td>
<td>-376.93</td>
<td>(-4.18)</td>
</tr>
<tr>
<td>C XK</td>
<td>1102.7</td>
<td>(7.04)</td>
</tr>
<tr>
<td>d LL</td>
<td>-693.95</td>
<td>(-0.32)</td>
</tr>
<tr>
<td>d ML</td>
<td>88967</td>
<td>(43.38)</td>
</tr>
<tr>
<td>d MK</td>
<td>15495</td>
<td>(6.51)</td>
</tr>
<tr>
<td>d XL</td>
<td>-832.81</td>
<td>(-9.00)</td>
</tr>
<tr>
<td>d XK</td>
<td>-420.96</td>
<td>(-2.84)</td>
</tr>
<tr>
<td>d tt</td>
<td>-376.93</td>
<td>(-4.18)</td>
</tr>
<tr>
<td>d IK</td>
<td>1102.7</td>
<td>(7.04)</td>
</tr>
<tr>
<td>d CL</td>
<td>922.70</td>
<td>(8.59)</td>
</tr>
<tr>
<td>d IK</td>
<td>-240.62</td>
<td>(-1.28)</td>
</tr>
<tr>
<td>d CL</td>
<td>1269.7</td>
<td>(16.48)</td>
</tr>
<tr>
<td>d CK</td>
<td>864.26</td>
<td>(7.71)</td>
</tr>
<tr>
<td>d tt</td>
<td>-48.928</td>
<td>(-7.53)</td>
</tr>
</tbody>
</table>

LL -2095.20

We next use these estimates to calculate the substitution matrix (Σ) and the matrix of price and quantity elasticities (E). Estimates of the latter elasticities are reported for selected years in Table 2. Of much interest are the own price elasticities of the demand for imports (ε_MM) and of the supply of exports (ε_XX). The first one falls substantially over time in absolute value, from about 2.9 in 1948 to about 0.3 in 1988;12 the latter falls from close to 0.5 in 1948 to about 0.1 in 1988. It thus appears that the supply of exports is less price elastic than the demand for imports. This contrasts with the results of KOHLI (1978, 1990) for Canada and the United States. While the declines in the absolute values of both price elasticities may surprise, it should be emphasized that there is no reason why they should be constant, and one advantage of flexible functional forms is precisely that they allow for such changes over time to be captured. These changes are due to movements in relative output prices and in relative input mixes, and to changes in the technology. It is noteworthy also that the own price elasticities of the supply of investment goods and consumption goods are rather large, more so than in Canada and the United States.

The first part of Table 2 also indicates the effects of cross price changes on imports and exports. One notes that an increase in the price of investment goods has a negative

12. This elasticity tends to be somewhat smaller than the estimates obtained by KOHLI (1982); however, they are not directly comparable since the functional forms and the sample periods are not the same. Moreover, in KOHLI (1982) labor was treated as a variable input, domestic output was not disaggregated, and technological change was assumed to be HARROD-neutral.
Table 2  
Symmetric Normalized Quadratic GNP Function  
Price and Quantity Elasticities for Selected Years

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price elasticities of output supply: ( c_{ih} = \frac{\partial \ln</td>
<td>y_i</td>
<td>}{\partial \ln p_h} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_{MM} )</td>
<td>-2.938</td>
<td>-0.953</td>
<td>-0.610</td>
<td>-0.449</td>
<td>-0.348</td>
</tr>
<tr>
<td>( c_{MX} )</td>
<td>0.799</td>
<td>0.193</td>
<td>0.099</td>
<td>0.046</td>
<td>0.021</td>
</tr>
<tr>
<td>( c_{MI} )</td>
<td>-0.261</td>
<td>-0.145</td>
<td>-0.127</td>
<td>-0.124</td>
<td>-0.128</td>
</tr>
<tr>
<td>( c_{MC} )</td>
<td>2.400</td>
<td>0.905</td>
<td>0.637</td>
<td>0.527</td>
<td>0.454</td>
</tr>
<tr>
<td>( c_{XM} )</td>
<td>-0.575</td>
<td>-0.236</td>
<td>-0.110</td>
<td>-0.050</td>
<td>-0.020</td>
</tr>
<tr>
<td>( c_{XX} )</td>
<td>0.485</td>
<td>0.274</td>
<td>0.180</td>
<td>0.119</td>
<td>0.098</td>
</tr>
<tr>
<td>( c_{XI} )</td>
<td>-0.452</td>
<td>-0.339</td>
<td>-0.255</td>
<td>-0.187</td>
<td>-0.156</td>
</tr>
<tr>
<td>( c_{XC} )</td>
<td>0.542</td>
<td>0.302</td>
<td>0.185</td>
<td>0.118</td>
<td>0.078</td>
</tr>
<tr>
<td>( c_{IM} )</td>
<td>0.310</td>
<td>0.180</td>
<td>0.156</td>
<td>0.188</td>
<td>0.210</td>
</tr>
<tr>
<td>( c_{IX} )</td>
<td>-0.748</td>
<td>-0.343</td>
<td>-0.283</td>
<td>-0.261</td>
<td>-0.267</td>
</tr>
<tr>
<td>( c_{II} )</td>
<td>0.837</td>
<td>0.439</td>
<td>0.401</td>
<td>0.418</td>
<td>0.467</td>
</tr>
<tr>
<td>( c_{IC} )</td>
<td>-0.399</td>
<td>-0.276</td>
<td>-0.275</td>
<td>-0.344</td>
<td>-0.411</td>
</tr>
<tr>
<td>( c_{CM} )</td>
<td>-0.765</td>
<td>-0.448</td>
<td>-0.369</td>
<td>-0.325</td>
<td>-0.281</td>
</tr>
<tr>
<td>( c_{CX} )</td>
<td>0.240</td>
<td>0.122</td>
<td>0.097</td>
<td>0.067</td>
<td>0.050</td>
</tr>
<tr>
<td>( c_{CI} )</td>
<td>-0.107</td>
<td>-0.110</td>
<td>-0.129</td>
<td>-0.141</td>
<td>-0.154</td>
</tr>
<tr>
<td>( c_{CC} )</td>
<td>0.632</td>
<td>0.437</td>
<td>0.402</td>
<td>0.398</td>
<td>0.385</td>
</tr>
</tbody>
</table>

Quantity elasticities of inverse input demand: \( c_{jk} = \frac{\partial \ln w_j}{\partial \ln x_k} \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{LL} )</td>
<td>-0.124</td>
<td>-0.170</td>
<td>-0.188</td>
<td>-0.164</td>
<td>-0.143</td>
</tr>
<tr>
<td>( c_{LK} )</td>
<td>0.124</td>
<td>0.170</td>
<td>0.188</td>
<td>0.164</td>
<td>0.143</td>
</tr>
<tr>
<td>( c_{KL} )</td>
<td>0.267</td>
<td>0.349</td>
<td>0.378</td>
<td>0.377</td>
<td>0.344</td>
</tr>
<tr>
<td>( c_{KK} )</td>
<td>-0.267</td>
<td>-0.349</td>
<td>-0.378</td>
<td>-0.377</td>
<td>-0.344</td>
</tr>
</tbody>
</table>

Quantity elasticities of output supply (RYBCZYSKI Elasticities): \( c_{ij} = \frac{\partial \ln |y_i|}{\partial \ln x_j} \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{ML} )</td>
<td>0.866</td>
<td>0.732</td>
<td>0.600</td>
<td>0.529</td>
<td>0.493</td>
</tr>
<tr>
<td>( c_{MK} )</td>
<td>0.134</td>
<td>0.268</td>
<td>0.400</td>
<td>0.471</td>
<td>0.507</td>
</tr>
<tr>
<td>( c_{XL} )</td>
<td>0.492</td>
<td>0.198</td>
<td>0.068</td>
<td>0.012</td>
<td>-0.019</td>
</tr>
<tr>
<td>( c_{XX} )</td>
<td>0.508</td>
<td>0.802</td>
<td>0.932</td>
<td>0.988</td>
<td>1.019</td>
</tr>
<tr>
<td>( c_{IL} )</td>
<td>0.759</td>
<td>0.874</td>
<td>0.927</td>
<td>1.016</td>
<td>1.115</td>
</tr>
<tr>
<td>( c_{IK} )</td>
<td>0.241</td>
<td>0.126</td>
<td>0.073</td>
<td>-0.016</td>
<td>-0.115</td>
</tr>
<tr>
<td>( c_{CL} )</td>
<td>0.804</td>
<td>0.814</td>
<td>0.821</td>
<td>0.852</td>
<td>0.887</td>
</tr>
<tr>
<td>( c_{CK} )</td>
<td>0.196</td>
<td>0.186</td>
<td>0.179</td>
<td>0.148</td>
<td>0.113</td>
</tr>
</tbody>
</table>


Price elasticities of inverse input demand (STOLPER-SAMUELSON Elasticities): \( c_{lj} = \frac{\partial \ln w_j}{\partial \ln p_l} \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{LM} )</td>
<td>-0.291</td>
<td>-0.412</td>
<td>-0.367</td>
<td>-0.344</td>
<td>-0.308</td>
</tr>
<tr>
<td>( c_{LX} )</td>
<td>0.229</td>
<td>0.091</td>
<td>0.038</td>
<td>0.007</td>
<td>-0.013</td>
</tr>
<tr>
<td>( c_{LI} )</td>
<td>0.214</td>
<td>0.397</td>
<td>0.462</td>
<td>0.438</td>
<td>0.423</td>
</tr>
<tr>
<td>( c_{LC} )</td>
<td>0.847</td>
<td>0.923</td>
<td>0.868</td>
<td>0.899</td>
<td>0.898</td>
</tr>
<tr>
<td>( c_{KM} )</td>
<td>-0.096</td>
<td>-0.310</td>
<td>-0.493</td>
<td>-0.704</td>
<td>-0.764</td>
</tr>
<tr>
<td>( c_{KK} )</td>
<td>0.509</td>
<td>0.757</td>
<td>1.038</td>
<td>1.361</td>
<td>1.595</td>
</tr>
<tr>
<td>( c_{KI} )</td>
<td>0.146</td>
<td>0.118</td>
<td>0.073</td>
<td>-0.016</td>
<td>-0.105</td>
</tr>
<tr>
<td>( c_{KC} )</td>
<td>0.442</td>
<td>0.435</td>
<td>0.382</td>
<td>0.359</td>
<td>0.275</td>
</tr>
</tbody>
</table>
impact on import demand (see the negative sign of \( \varepsilon_{\text{IM}} \)). This can probably best be explained by noting that an increase in investment good prices results in a sharp decrease in the supply of exports and, to a lesser degree, in the supply of consumption goods (see the negative signs of \( \varepsilon_{\text{XI}} \) and \( \varepsilon_{\text{CI}} \)). To the extent that these outputs are "import intensive" one would expect the demand for imports to fall. Indeed, increases in the prices of exports and consumption goods do impact positively on the demand for imports (see the positive signs of \( \varepsilon_{\text{MX}} \) and \( \varepsilon_{\text{MC}} \)). Regarding the three positive output components, it is visible that exports and investment goods, and investment goods and consumption goods are substitutes in production, but there is some evidence of complementarity between exports and consumption goods.

The second part of Table 2 shows the effects of changes in factor endowments on factor rental prices, for given output prices. As expected, an increase in the endowment of any factor leads to a reduction in its rental price, but the effect is weak, particularly for labor.

The elasticities in the third part of Table 2 indicate the effects of changes in factor endowments on the various output components, for given relative output prices. We refer to these elasticities as RYBCZYNSKI elasticities, even though our model differs from the HECKSCHER-OHLIN-SAMUELSON model in some key respects, particularly by not assuming nonjoint production. An increase in labor endowment is found to lead to a substantial increase in the supply of investment goods and in the output of consumption goods. The effect on imports is positive as well, but it has fallen substantially since the beginning of the sample, while the effect on exports has been almost nil in recent years. An increase in the endowment of capital, on the other hand, very heavily favors the production of exports. In recent years, increases in \( \varepsilon_{\text{XK}} \) had a sizable positive effect on imports as well. Thus, capital accumulation in Switzerland seems to have a large positive impact on the volume of Swiss foreign trade.

The last part of Table 2 indicates the effects of output price changes on input prices, for given factor endowments. These elasticities, which we call STOLPER-SAMUELSON elasticities, are the mirror image of the RYBCZYNSKI elasticities which were reported in the third part of the table. It is therefore not surprising to find that, at least in recent years, the income of capital owners is extremely sensitive to movements in the terms of trade: an increase in import prices hurts capital considerably, while an increase in export prices even has a magnifying effect on the return to capital. An improvement in the terms of trade has a much weaker effect on labor income. Finally, an increase in the prices of consumption goods and investment goods favor labor mostly.

13. This too contrasts with available results for Canada and the United States.
14. Note that \( \varepsilon_{\text{CX}} > 1 \) does not necessarily imply \( \varepsilon_{\text{LX}} < 0 \) since production is not assumed to be nonjoint.
6. CONCLUSIONS

The GNP function approach to modeling imports and exports allows for the derivation and estimation of trade functions within a rigorous analytical framework. This contrasts with the standard approach where import and export functions are specified separately, and where the choice of explanatory variables often rests on intuition more than on theory. Furthermore, the GNP function approach yields a multiplicity of other results of interest regarding the substitution possibilities allowed for by the technology that remains unparalleled. It also produces all-important estimates of the elasticities of supply of exports. These are very scarce in the literature, since most studies of export determination focus on foreign demand conditions. Yet, explaining the flow of exports starts with the home country, particularly in a neoclassical context, and in the case of a small open economy.

The functional form which we have used here, the Symmetric Normalized Quadratic variable profit function, allows curvature conditions to be imposed globally, rather than just locally, and without compromising its flexibility. The failure of some data sets to satisfy the curvature conditions might be a reflection of the quality of the data, the level of aggregation, or the model that is used. However, as long as we are interested in abstract concepts such as the aggregate price elasticity of the demand for imports or of the supply of exports, we must have means to estimate these magnitudes in a framework that is consistent with theory.

Some of the results obtained here which are of particular interest are the relatively small own price elasticities of the demand for imports and of the supply of exports, and the many interactions between import and export prices and other output supplies. In particular, our results seem to indicate that the supply of consumption goods is fairly import intensive at the margin, in the sense that an increase in import prices will shift the output mix against consumption goods. On the contrary, an increase in the production of investment goods, brought about by an increase in their prices, is accompanied by a fall in the demand for imports. Moreover exports and investment goods have been determined to be strong substitutes in production. We also found that a rise in export prices greatly favors capital, while increases in the prices of consumption goods and investment goods mostly benefit labor. Increases in the endowment of capital, finally, are found to strongly stimulate foreign trade.

While the results obtained in this study are suggestive and encouraging, there remains much to be done. In particular, the decrease over time in the absolute values of some elasticity estimates is somewhat worrisome. It is not clear, at this stage, whether these movements reflect true underlying trends, or whether they may be due to inadequate modeling of technological change. Other questions which will have to be addressed in future research are the disaggregation of imports and exports, particularly between goods and services, given the weight of the latter in Swiss foreign trade and the current emphasis in the literature on international trade in services; and attention should be devoted to the geographical distribution of Swiss foreign trade since there is little doubt that Switzerland today, not being a member of the EEC but yet clearly affected by increasing European economic integration, finds itself at a major crossroad.
REFERENCES


APPENDIX A: The Symmetric Normalized Quadratic GNP Function

Our first purpose in this Appendix is to demonstrate that (9) is flexible at point \((p^*, x^*)\) such that \(A p^* = 0\) and \(B x^* = 0\) (in the text we have chosen \(p^* = 1\) and \(x^* = 1\)), in the sense that it provides a second-order approximation to an arbitrary GNP function at that point. Second, we want to show that the Symmetric Normalized Quadratic GNP function defined by (9) is convex in prices as long as \(A\) is positive semi-definite, and concave in fixed input quantities as long as \(B\) is negative semi-definite.

A1. Flexibility

Theorem 1: Consider an arbitrary, well-behaved, variable profit function \(\pi^*(p, x)\). The Symmetric Normalized Quadratic variable profit function defined by (9) is flexible in the sense that it provides a second-order approximation to \(\pi^*(\cdot)\) at point \((p^*, x^*)\) such that \(A p^* = 0\) and \(B x^* = 0\).

Proof. \(\pi(\cdot)\) defined by (9) provides a second-order approximation to \(\pi^*(\cdot)\) at point \((p^*, x^*)\) if the following conditions are met:

\[
\begin{align*}
\pi^*(p^*, x^*) &= \pi(p^*, x^*) \quad (A.1) \\
\nabla_p \pi^*(p^*, x^*) &= \nabla_p \pi(p^*, x^*) \quad (A.2) \\
\nabla_x \pi^*(p^*, x^*) &= \nabla_x \pi(p^*, x^*) \quad (A.3) \\
\nabla_{pp} \pi^*(p^*, x^*) &= \nabla_{pp} \pi(p^*, x^*) \quad (A.4) \\
\nabla_{xx} \pi^*(p^*, x^*) &= \nabla_{xx} \pi(p^*, x^*) \quad (A.5) \\
\nabla_{px} \pi^*(p^*, x^*) &= \nabla_{px} \pi(p^*, x^*) \quad (A.6)
\end{align*}
\]

There is one restriction (A.1), \(I\) restrictions (A.2), \(J\) restrictions (A.3), \(I(I+1)/2\) restrictions (A.4), \(J(J+1)/2\) restrictions (A.5), and \(IJ\) restrictions (A.6); this gives a total of \(1+I+J+I(I+1)/2+J(J+1)/2+IJ\) restrictions. However, as shown by DIEWERT and OSTENSOE (1988), only \(I(I-1)/2+J(J-1)/2+IJ\) of these are independent. Indeed, it follows from the homogeneity of \(\pi^*(\cdot)\) and \(\pi(\cdot)\) that only \(I(I-1)/2\) and \(J(J-1)/2\) of the conditions (A.4) and (A.5) are independent, and that (A.4)-(A.6) imply (A.1)-(A.3). Note that \(I(I-1)/2+J(J-1)/2+IJ\) is precisely the number of parameters of (9). Thus, it is sufficient to show that (9) satisfies (A.4)-(A.6) at point \((p^*, x^*)\) for an arbitrary function \(\pi^*(\cdot)\).

Evaluating the Hessian of (9) at \((p^*, x^*)\), substituting into (A.4)-(A.6), and recalling that \(A p^* = 0\) and \(B p^* = 0\), we get:

\[
\begin{align*}
a_{ij} &= [(\alpha' p^*)/(\beta' x^*)] \frac{\partial^2 \pi^*(p^*, x^*)}{\partial p_idp_h} \\
b_{jk} &= [(\beta' x^*)/(\alpha' p^*)] \frac{\partial^2 \pi^*(p^*, x^*)}{\partial x_jdx_k} \\
c_{ij} &= \frac{\partial^2 \pi^*(p^*, x^*)}{\partial p_idx_j}.
\end{align*}
\]

Thus, all parameters of (9) are uniquely determined. QED
A2. Curvature conditions

Theorem 2: The Symmetric Normalized Quadratic variable profit function defined by (9) is globally convex in prices if and only if $A$ is positive semi-definite.

Proof: Diewert and Wales (1987) have shown that the function $g(z) = z'Dz/(\delta'z)$, where $\delta'z > 0$, is globally convex in $z$ if and only if $D$ is positive semi-definite. Hence the first term on the right-hand side of (9) is globally convex in prices if and only if $A$ is positive semi-definite. The second and the third terms on the right-hand side of (9) are linear in prices, and hence they are convex in the components of $p$. The proof is completed by noting that the sum of convex functions is convex. QED

Theorem 3: The Symmetric Normalized Quadratic variable profit function defined by (9) is globally concave in fixed input quantities if and only if $B$ is negative semi-definite.

Proof: Consider function $h(z) = -g(z) = z'Fz/(\delta'z)$ where $F = -D$. It can immediately be seen that $h(z)$ is globally concave if and only if $g(z)$ is globally convex. This requires $D$ to be positive semi-definite, that is $F = -D$ must be negative semi-definite. Hence the second term in (9) is globally concave in the components of $x$ if and only if $B$ is negative semi-definite. The first and third terms of (9) are linear in the components of $x$, and hence they are concave. The sum of concave functions is necessarily concave, which concludes the proof. QED
ABSTRACT

This paper reports estimates of a GNP function for Switzerland using data covering most of the post-war period. The paper provides one of the first empirical applications of a new functional form, the Symmetric Normalized Quadratic variable profit function which allows convexity in prices and concavity in fixed inputs to be imposed globally. Some of our empirical results are as follows: the Swiss demand for imports and supply of exports are relatively price inelastic; investment goods have only a very low import content at the margin; an improvement in the terms of trade favors mostly capital.

RESUME

Cet article présente des résultats économétriques portant sur l’estimation d’une fonction de PNB pour la Suisse et il fourni une des premières applications empiriques d’une nouvelle forme fonctionnelle, la fonction de profit variable quadratique normalisée symétrique; cette fonction permet aux conditions de courbure (convexité en prix et concavité en quantités) d’être imposées globalement. Nos résultats suggèrent notamment que la demande d’importations et l’offre d’exportations sont relativement peu sensibles aux variations de prix; que les biens d’investissement ne contiennent, à la marge, qu’une faible part de produits importés; et qu’une amélioration des termes de l’échange favorise principalement le capital.

ZUSAMMENFASSUNG

Der Artikel präsentiert Schätzungen einer BSP-Funktion für die Schweiz über praktisch den gesamten Nachkriegszeitraum. Der Artikel liefert eine der ersten empirischen Anwendungen einer neuen Funktionsform, der symmetrisch normierten quadratischen variablen Gewinnfunktion, die Konvexität in Preisen und Konkavität in Mengen global als Bedingungen zulässt. Wir haben u.a. folgende empirische Resultate gefunden: Die Nachfrage nach Importgütern und das Angebot an Exportgütern der Schweiz sind relativ preisunelastisch; die Einfuhr der Investitionsgüter hat , marginal betrachtet, wenig Bedeutung; eine Verbesserung der terms of trade bevorzugt meist den Faktor Kapital.