Export Production and Imperfect Hedging

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1. INTRODUCTION

Firms engaged in international operations are highly interested in developing ways to protect themselves from exchange rate risk. Our study shows that an exporting firm can benefit from hedging exchange rate risk even when no perfect hedge is possible. Since in reality, not every currency is traded in a futures/forward market (see BUCKLEY [1992], PAROUSH and WOLF [1992]), the exporting firm uses futures contracts with other underlying assets whose spot prices are highly correlated with the foreign exchange spot rate. In the real world hedging must often be accomplished by using futures contracts on different deliverable instruments. Such hedging may result in an imperfect hedge (see ANDERSON and DANTHINE [1981], BROLL, WAHL and ZILCHA [1995]).

The literature shows that an international firm facing price or exchange rate risk can eliminate this risk altogether if it can use a currency forward market, or another financial asset which is perfectly correlated to the exchange rate (see DANTHINE [1978], KAWAI and ZILCHA [1986], ZILCHA and ELDOR [1991], ZILCHA and BROLL [1992]). The studies of firm behavior under exchange rate uncertainty examine the influence of futures markets on the export production and hedging decision. Two major theorems are derived: One is the «separation theorem» which states that, when futures markets exist, the firm’s export production decision is determined solely by technology, the output price and the forward rate. This result holds if the gain from the futures contract is perfectly correlated with export revenue. The other theorem is the «full hedging theorem» which asserts that with unbiased (zero-risk premium) futures markets, the firm completely avoids exchange rate risk by entering into optimum futures contracts.

Many spot assets, however, are not delivered in any futures market, nor are there bank forward contracts available. Then firms have to cross hedge in a futures contract delivering a different asset (see DUFFIE [1989]). Hence hedging must be accomplished by using existing futures contracts that involve similar price fluctuations with the cash market instrument being hedged. These matches of the futures contract to the cash instrument are known as imperfect hedges. The aim of our study is to examine the role of imperfect hedging on the firm’s export production and risk management policy. Our research provides some insights into the output and welfare implications of imperfect hedging.

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The paper is organized as follows. In section 2, the model of an exporting firm is presented. The main results are derived in section 3, where we examine the impact of imperfect hedging of exchange rate risk on the exporting firm’s decision making. We show that imperfect hedging violates both the separation theorem and the full hedging theorem. Nonetheless, introducing an imperfect hedging devise increases the welfare of the firm though the effect on production is ambiguous. Section 4 offers some discussion and concludes the paper.

2. PRODUCTION AND IMPERFECT HEDGING

Consider a competitive firm with a given production technology which produces a certain commodity and exports all its output to a foreign country. We assume that the firm faces a random exchange rate $\tilde{e}$ and that it is risk-averse. The firm cannot hedge its foreign currency risk perfectly in a given currency futures market, since such a market does not exist. However, there is a forward market for some domestic financial asset correlated with the exchange rate $\tilde{e}$ which can be utilized by this firm; namely, there exists an imperfect hedging devise for this foreign exchange uncertainty.

Let $\tilde{g}$ be the random spot value of this financial asset, and let $g_f$ be the forward price for this asset at the time where the firm makes its production decision. To study the role of imperfect hedging upon the firm’s economic behavior, such as the production level, it is sufficient to consider a forward market with zero-risk premium. Thus we shall assume throughout this paper that this forward market is unbiased, or exhibits no risk premium, i.e., the forward rate is equal to the expected future spot rate. Moreover, we shall assume that for some $\beta > 0$ and some random variables independent of $\tilde{e} \text{ and } \bar{e}$, we have

$$\tilde{g} = \beta \tilde{e} + \bar{e}.$$ 

When $\tilde{e}$ is not a constant we say that the firm hedges imperfectly. If $\tilde{e} = a$, with probability 1, the firm hedges perfectly. We assume that only imperfect hedging is available to this competitive firm and we shall compare the economic implications of imperfect hedging vs. the perfect hedging case.

The firm’s decision problem is to choose its optimal production $x^*$ and optimal forward contracting $z^*$ in a way that maximizes its expected utility of profit, where profit is denominated in domestic currency. Denote by $C(x)$ the firm’s cost function and assume the usual properties: $C(x)$ is an increasing and a convex function of $x$, i.e. $C'(x) > 0$, $C''(x) > 0$; for all $x$. Let $U$ be the firm’s von Neumann-Morgenstern utility function. We assume that it is a strictly concave function, i.e., $U'' > 0$, $U''' < 0$. Thus we assume risk aversion in the sequel.
Let the price of this commodity in the foreign country be \( p \) and assume that it is fixed. The joint distribution of \( \tilde{e} \) and \( \tilde{g} \) is known. The forward contracting in the market for the domestic asset is denoted by \( z \). Without loss of generality we assume that the assets spot price is positively correlated with the foreign exchange spot rate.

The firm’s optimization problem is:

\[
\max_{x,z} EU(\tilde{\Pi})
\]

where the random profit is \( \tilde{\Pi} = \tilde{e} p x - C(x) + z(g_f - \tilde{g}) \). Since the maximand is strictly concave in \( x \) and \( z \) the optimum \((x^*, z^*)\) is the unique solution of the necessary and sufficient conditions:

\[
EU'(\tilde{\Pi}^*) [\tilde{e} p - C'(x^*)] = 0,
\]

\[
EU'(\tilde{\Pi}^*) (g_f - \tilde{g}) = 0,
\]

where \( \tilde{\Pi}^* \) is \( \tilde{\Pi} \) at the optimum values \( x = x^* \) and \( z = z^* \). Although we assume that the futures market is unbiased, i.e., \( g_f = \bar{E}g \), we demonstrate that the firm underhedges.

**Proposition 1:**

Assume that an unbiased futures market exists for imperfect hedging, then:

(a) the separation property does not hold; namely, the optimal export production of this firm depends upon the probability distribution and upon the firm's attitude towards risk.

(b) The firm underhedges, where \( z^* > 0 \) and \( \beta z^* < px^* \).

(c) If there is perfect correlation between \( \tilde{e} \) and \( \tilde{g} \), the firm fully hedges, and, therefore, its profit is riskless.

**Proof:**

(a) Combining equations (2) and (3) we derive

\[
C'(x^*) = \frac{g_p EU'(\tilde{\Pi}^*) \tilde{e}}{EU'(\tilde{\Pi}^*) \tilde{g}},
\]

which clearly proves that the production decision cannot be separated from expectations and risk behavior.

(b) Using the assumption that the hedging market is unbiased, we derive that \( \text{cov}[\tilde{g}, U'(\tilde{\Pi}^*)] = 0 \). Writing \( \tilde{g} = \beta \tilde{e} + \tilde{e} \) we find that

\[
\beta \text{cov} [\tilde{e}, U'(\tilde{\Pi}^*)] + \text{cov} [\tilde{e}, U'(\tilde{\Pi}^*)] = 0.
\]
If \( z^* \leq 0 \) then \( \text{cov} [\tilde{e}, U' (\tilde{\Pi}^*))] \leq 0 \), so that \( \text{cov} [\tilde{e}, U' (\tilde{\Pi}^*)] \geq 0 \) implying \( px^* - \beta z^* \leq 0 \) which is impossible. Therefore \( z^* > 0 \). This indicates that \( \text{cov} [\tilde{e}, U' (\tilde{\Pi}^*)] < 0 \), hence \( px^* - \beta z^* > 0 \).

(c) When hedging occurs in an unbiased market, then \( \text{cov} [\tilde{g}, U' (\tilde{\Pi}^*)] = 0 \). By assumption \( \tilde{g} \) mimics \( \tilde{e} \) without noise, therefore,

\[
\text{cov} [\tilde{e}, U' (\tilde{\Pi}^*)] = 0.
\]

If \( px^* - \beta z^* > 0 \) profit increases in \( \tilde{e} \) while marginal utility is strictly decreasing in contradiction to (5). Equation (5) holds if and only if \( px^* = \beta z^* \). Hence profit is nonrandom.

Comparing the firm's production level under imperfect hedging with the perfect hedging case, we can prove

**Proposition 2:**

*If the forward rate \( g_f = a + \beta E\tilde{\pi} \), then the firm's export production, when using unbiased imperfect hedging, is lower than its production when unbiased perfect hedging is available.*

*Proof:*

When the firm can hedge perfectly on the exchange rate \( \tilde{\pi} \), i.e., \( \tilde{g} = \beta \tilde{\pi} + a \) it can be shown from equations (2) and (3) that its optimal output \( x^{**} \) is given by the equation

\[
C' (x^{**}) = \frac{(g_f - a)p}{\beta}.
\]

(6)

This demonstrates that the separation theorem holds (see, e.g., DANTHINE [1978], BENNINGA, ELDOR and ZILCHA [1984]).

Now assume that only imperfect hedging is possible through \( \tilde{g} = \beta \tilde{\pi} + \tilde{\varepsilon} \), thus \( \tilde{\varepsilon} \) and \( \tilde{g} \) are not perfectly correlated. Using the same argument as in the proof of Proposition 1, we can show that, \( \text{cov} [\tilde{\varepsilon}, U' (\tilde{\Pi}^*)] < 0 \). This implies from equation (2) that

\[
E [\tilde{\varepsilon} p - C' (x^*)] > 0.
\]

(7)

Since \( C(x) \) is convex, comparing (6) with (7) we find that \( x^{**} > x^* \).

*An Example.* Let the firm's export production be \( x_0 \). In order to demonstrate the hedging policy of the firm, suppose the firm has quadratic utility function \( U(\Pi) = \Pi - (A/2)\Pi^2 \)
(A > 0) and assume that the imperfect hedging instrument is unbiased, \( g_f = E\tilde{g} \). In this case the first-order conditions for \( z \) becomes

\[
\text{cov}[\tilde{g}, \tilde{e}(px_o - \beta z^* - az^* - C(x_o) + g\tilde{g}^*)] = 0,
\]

which follows from substituting \( U' = 1 - A\Pi > 0 \) into equation (3). This equation may be solved to give

\[
z^* = px_o \frac{\text{cov}(\tilde{e}, \tilde{g})}{\text{var}(\tilde{g})}.
\]

With our regression assumption the optimal hedge ratio is given by

\[
\frac{\beta z^*}{p x_o} = \frac{\beta^2 \text{var}(\tilde{e})}{\beta^2 \text{var}(\tilde{e}) + \text{var}(\tilde{e})} < 1,
\]

which illustrates the results from Proposition 1.

In our framework the level of hedging depends on \( \beta \). Hedging occurs only if there is a regression between the foreign exchange rate and the domestic asset’s price. An extension of our analysis can be carried out with a more general scheme than our regressibility assumption. Hence, in general, the bivariate probability distribution of \( \tilde{e} \) and \( \tilde{g} \) determines the hedge volume.

### 3. IMPERFECT HEDGING AND WELFARE

Denote by \( \tilde{\Pi}^0 \) the random profit of the firm when no hedging is available (and by \( x^0, z^0 \) the corresponding output and hedging amount). Comparing the firm’s welfare level under unbiased imperfect hedging with the case where no hedging markets exist, we can prove

Proposition 3:

*The firm prefers \( \tilde{\Pi}^* \) to \( \tilde{\Pi}^0 \) regardless whether it produces more or less, i.e., regardless whether \( x^* > x^o \) or \( x^* \leq x^o \).*

*Proof:*

Let us use the strict concavity of the utility function \( U \) and the strict convexity of the cost function \( C \) as follows: Since \( \Pi^* \neq \Pi \) we can write
\[ E[U(\Pi^*) - U(\Pi^\ast)] > EU'(\Pi^*) = EU'(\Pi^*)[\tilde{e}p(x^\ast - x^\ast) + C(x^\ast) - C(x^\ast) + z^\ast(g_f - \tilde{g})]. \]

But \( C(x^\ast) - C(x^\ast) > C'(x^\ast)(x^\ast - x^\ast) \) whenever \( x^\ast \neq x^\ast \), hence

\[ E[U(\Pi^*) - U(\Pi^\ast)] > 0, \]

due to equations (2) and (3).

Note that it is possible that production decreases when unbiased imperfect hedging becomes available since in this case we may observe a lower profit risk \((Var(\Pi^\ast) > Var(\Pi^\ast))\) but also a lower expected profit \((\tilde{E}\Pi^\ast > E\Pi^\ast)\). This follows from the strict concavity of the function \( E\tilde{e}p - C(x) \). Given that \( E\Pi^\ast = E\tilde{e}p x^\ast - C(x^\ast) \) and \( E\Pi^\ast = E\tilde{e}p x^\ast - C(x^\ast) \), if \( x^\ast > x^\ast \) and \( x^\ast < \text{argmax}[E\tilde{e}p - C(x)] \), then \( E\Pi^\ast > E\Pi^\ast \). This shows that in the case where no perfect hedging exists production does not necessarily increase as the imperfect hedging devise becomes available. The fact that the firm will always prefer imperfect hedging devices to none does not exclude the possibility of an adverse effect of such a device on production.

4. CONCLUDING REMARKS AND DISCUSSION

If every financial instrument in the spot market had a futures contract that exactly mirrored its pay-off function, futures markets would provide the opportunity to yield a perfect hedge. However, in the real world hedging must often be accomplished by using futures contracts on different deliverable instruments. Such hedging is called imperfect (or, cross) hedging and generally yields an imperfect hedge.

We know from the literature that an international firm facing exchange rate risk eliminates this risk altogether by using an currency forward market, or another financial asset which is perfectly correlated to the spot rate of foreign exchange. In general, the firm can reduce its income risk by engaging in a hedging activity of assets correlated to the foreign exchange rate. In such case the widely discussed separation theorem and the full hedging theorem do not hold.

Our results are: (1°) With imperfect hedging, the decision making regarding optimal export production and hedging volume cannot be separated. (2°) Although the hedging market is unbiased, imperfect hedging implies an underhedge position. Therefore a forward rate which exhibits no risk premium does not imply a full hedge. (3°) Introducing imperfect hedging always improves the firm's welfare regardless of its impact on the firm's optimal level of export production.

Note the following policy and trade implications: exporting firms benefit when hedging devices are offered by governments, for instance, although the hedging instru-
ment may be imperfect. The effect on international trade, however, is ambiguous because the firm's output for export may decrease or increase.

REFERENCES


SUMMARY

International firms have an incentive for risk management due to the enormous volatility of the floating foreign exchange rates. Often firms must cross hedge since in reality, not every currency is traded in a futures market. That is, the exporting firm uses futures whose value is highly correlated with the foreign exchange spot rate. The aim of our study is to examine the role of such imperfect hedging on the exporting firm’s production and risk management decision.

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RESUME

La vaste incertitude des cours de change sur les marchés financiers n’est pas sans répercussion sur la gestion de l’entreprise ayant des activités internationales. Les firmes d’exportation doivent souvent utiliser une couverture à terme non sur la devise mais sur des actifs financiers en corrélation avec la devise, c’est-à-dire réaliser une couvrance imparfaite. Cet article traite de l’analyse concernant les conséquences d’une couverture à terme imparfaite sur la décision d’exportation et celle de la couvrance optimale dans l’entreprise internationale.