International Trade Policy Reforms and their Simulation*

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1. INTRODUCTION

In recent years there has been considerable interest in international trade policy reform at both the theoretical level and at a more practical level, as witnessed by the recent academic literature and by the considerable effort put into the Uruguay Round negotiations organised under the auspices of the General Agreement on Tariffs and Trade (GATT). While there appears to be general agreement that reductions in tariffs, quotas and other distortions to international trade are desirable, there is nevertheless reluctance by national governments and their constituents to wholeheartedly embrace the concept of free trade.

The actual process of tariff and quota reductions is always slow and the reasons for this are not difficult to perceive. Firstly, the trade policy reform problem is a special case of the general problem of the second best as expounded by MEADE (1955) and LIPSEY and LANCASTER (1956). As is well known, the reduction of a subset of distortions towards the first best is not necessarily welfare improving due to the spillover effects of the policy change in other markets. Hence, a country that attempts tariff and quota reductions cannot be guaranteed a welfare gain unless the direction of the reform is carefully chosen. The goal of the quota and tariff reform literature is to construct reform formulae that are guaranteed to yield welfare improvements. Secondly, in the context of a trading world (as opposed to a small open economy), any trade policy reform will have international repercussions and the problem of quota reform therefore becomes inherently multilateral: the world prices of traded goods will change in order to re-establish market equilibrium. As a consequence, there are additional terms of trade effects that must be taken into account in order to properly evaluate the desirability of trade policy reform for each individual country. Thirdly, if international negotiations are undertaken to solve the interdependency problem of trade policy reform in a many country world, there is the implicit requirement that all participants should gain from any chosen policy alternative. Hence, the reform directions should be strictly Pareto improving and it is not necessarily obvious how the actual policy reform should be chosen to accommodate the welfare goals of each of the participating countries.

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Given that we are dealing with a trading world, the analysis of tariff and quota reforms becomes complicated by the need to take account of the effects of reforms upon the world prices for all goods and the subsequent effects upon the real incomes of countries and upon tariff revenues and quota rents. Each of these effects of quota reform are included in definitions of shadow prices of traded goods that are applied in the ensuing analysis to evaluate the welfare implications of quota reform. As in the case of multilateral tariff reform analysed by TURUNEN-RED and WOODLAND (1991, 1993, 1995), it turns out that the shadow prices relevant for the evaluation of tariff and quota reforms differ from world market prices and are complex functions of substitution matrices (ANDERSON and NEARY (1992), NEARY (1993), TURUNEN-RED and WOODLAND (1995)). Their use, however, allows us to determine whether Pareto improving trade policy reforms exist and whether particular directions of reforms are welfare improving.

The present paper is concerned with the development and analysis of specific formulae for tariff and quota reforms on a multilateral basis in a situation where the trade policy reforms are not accompanied by international transfers of income. Elsewhere, in TURUNEN-RED and WOODLAND (1993), we have examined tariff policy reform without any international income transfers in a model that only contains tariff distortions. In the present paper we extend this model to include quota as well as tariff distortions to trade and extend the policy reform formulae to take account of quotas. We consider three different policy reforms, one involving reforms to both tariffs and quotas and two that deal only with tariff reform leaving quotas unchanged. In each case we prove that the policy reform is Pareto improving in welfare under certain assumptions. An assumption common to the three theorems is that all income effects in the neighbourhood of the initial distorted equilibrium are concentrated on the numeraire, which we assume to be free of tariffs and quotas. This assumption on preferences is very restrictive and is not likely to be observed in practice. However, it is a sufficient condition for yielding a Pareto-improvement in welfare and not a necessary condition. It is expected, therefore, that mild violations of the assumption would not negate the conclusions of our theorems, while significant violations would do so.

To evaluate the robustness of our results concerning the Pareto improving welfare effects of the proposed trade policy reforms we undertake a numerical simulation analysis. By specifying the particular nature of hypothetical world economies, we can consider a large number of possible tariff and quota distortions, calculate the equilibrium, and so determine the comparative statics effects of each trade policy reform proposal upon welfare. We can then evaluate the success of each policy reform at each of several preference parameter settings that reflect different degrees of violation of the crucial assumption on preferences. Overall, our results conform to our expectations and suggest that for at least two of our three policy reforms the Pareto improving outcome is reasonably robust to violations of the assumption on preferences.
2. A MODEL OF WORLD TRADE WITH TARIFF AND QUOTA DISTORTIONS

Our model is of a world consisting of \( K \) countries trading in \( N \) goods. All agents are perfectly competitive in behaviour, but there are distortions to the competitive equilibrium caused by taxes and quotas imposed on international trade by the governments of countries. The model distinguishes between two groups of countries and two groups of goods. One group of countries (denoted by subset \( K^1 \)) is allowed to impose tariffs on all goods but does not impose any quotas; the other group of countries (denoted by subset \( K^2 \)) can impose both tariffs and quotas, but on separate groups of goods. Thus, one group of goods, \( N^r \), is subject only to tariff distortions, while the other group, \( N^s \), is subject to quota distortions by one group of countries \( (K^2) \) and tariff distortions by the other group \( (K^1) \). Beginning from an equilibrium subject to these distortions, we wish to characterise piecemeal policy reforms that are welfare improving for all countries.¹

We use the symbol \( K = K^1 \cup K^2 \) to simultaneously denote the set of separate nations and the number of nations, while the number and set of internationally traded commodities are both denoted \( N \). The international prices of the \( N \) tradeables are denoted by \( p^T = (p^T_t, p^T_r) \), where the subscripts \( t \) and \( r \) denote the sub-vectors of prices of goods subject to tariffs and goods subject to quota restrictions respectively.² The first tariffed commodity serves as the numeraire, and hence we also use the notation \( p^T = (1, q^T_t, p^T_r) = (1, q^T) \) where \( q \) denotes the sub-vector of prices of all goods except the numeraire, which takes explicit account of the price normalisation \( p_1 = 1 \). The symbol \( \tau^k \) refers to (specific) trade taxes and subsidies, called «tariffs», imposed by each country \( k \in K \) on goods in group \( N^* \). In countries \( k \in K^1 \) the vector \( \tau^k_t \) is the explicit tariff vector for the commodities in the set \( N^r \), while in countries \( k \in K^2 \), the sub-vector \( \tau^k_r \) refers to the quota premium or implicit tariff (price increase due to the quota on imports) on goods in set \( N^r \). Given tariffs \( \tau^k \), the domestic prices of internationally tradeable commodities in country \( k \) are \( p^k = p + \tau^k \).

Each of the \( K \) nations consists of an aggregate production sector and an aggregate consumption sector with one household. The maximal net revenue function, \( S^k \), defined by

\[
S^k (p, \tau^k_t, \tau^k_r, u^k) \equiv S^k (p^k, u^k) \equiv G^k (p^k) - E^k (p^k, u^k), \quad k \in K^1,
\]

characterises the behaviour of the nations that do not have quota restrictions, where \( G^k \) is the \( k \)th nation’s GNP (variable profit) function, \( E^k \) is the expenditure function of its

¹. The model formulation corresponds to that recently developed and analysed by Turunen-Red and Woodland (1995).

². Our notation conventions are as follows. All vectors are column vectors with the superscript \( T \) denoting transposition. Vector inequalities are \( x \geq 0 \) (every element of \( x \) is nonnegative), \( x > 0 \) (every element of \( x \) is positive) and \( x \gg 0 \) (every element of \( x \) is nonnegative and at least one is positive). Finally, subscripts of functions denote derivatives.
household, and the variable $u^k$ is the welfare level of its household. Assuming that the functions $S^k$ are twice continuously differentiable, the gradients of $S^k$ with respect to $p$, denoted by $S^k_p (p^k, u^k)$, give the differentiable (compensated) net exports of all countries, while the matrices $S^k_{pp}$, consisting of the second order derivatives of functions $S^k$ with respect to prices $p$, are the derivatives of the $k$th nation’s net exports with respect to the international prices of tradeable commodities, keeping the level of utility fixed. These matrices therefore describe the pure substitution effects of changes in international prices and are referred to as the net substitution matrices for countries $k \in K^1$. It is well known that the functions $S^k$ are convex and linearly homogenous in domestic prices $p^k = p + \tau^k$. Therefore, the Hessian matrices $S^k_{pp}$ are positive semidefinite and satisfy the identities $p^{KT} S^k_{pp} (p^k, u^k) = 0$.

While nations that have quota restrictions on some goods could be modelled as above with an additional set of equations of the form $x^k_t = S^k_t (p^k, u^k) \equiv \partial S^k / \partial p^k_t$, where $x^k_t$ is the vector of net export quotas, it is convenient to follow the approach of ANDERSON and NEARY (1992) and define the variable net revenue function

$$\hat{S}^k_t (p_t + \tau^k_t, x^k_t, u^k) \equiv \min_{p^k_t} \left[ S^k_t (p^k_t, p^k_t, u^k) - p^k_t T x^k_t \right], \ k \in K^2. \tag{2.2}$$

This function yields the value of net exports of goods $N^t$ at domestic prices, given the restricted net export vector, $x^k_t$, of goods $N^t$. Like $S^k$, this function is convex and linearly homogenous in domestic prices for tariffed goods $p^t = p_t + \tau^t$. Additionally, the net export vector for tariffed goods $N^t$ is given by $x^t_t = \partial S^k / \partial p^k_t \equiv S^k_t$ and the second derivative matrix $\partial S^k_t / \partial p^k_t \partial p^k_t \equiv S^k_{tt}$ is positive semidefinite such that $p^{KT} S^k_{tt} = 0$.

We now define the general equilibrium model that we shall employ to analyse international trade policy reform. Given competitive behaviour of all producers and consumers, we can state the general equilibrium model of the $K$ country world as

$$\sum_{k \in K^1} S^k_t (p_t + \tau^k_t, p_r + \tau^k_r, u^k) + \sum_{k \in K^2} S^k_t (p_t + \tau^k_t, x^k_t, u^k) = 0, \tag{2.3}$$

$$\sum_{k \in K^1} S^k_t (p_t + \tau^k_t, p_r + \tau^k_r, u^k) + \sum_{k \in K^2} x^k_t = 0. \tag{2.4}$$

3. The net revenue function and its properties are discussed in WOODLAND (1982, 537-553) and was used extensively in TURUNEN-RED and WOODLAND’S (1991) analysis of tariff reform.

4. This function is a generalisation of the net revenue function used by WOODLAND (1982, 173) to deal with nontraded goods. The generalisation is to allow the net exports of the quota restricted goods to be arbitrary rather than zero, reflecting the observation by FALVEY (1988) that quota restricted goods are effectively locally nontraded.
Equations (2.3) are the world market equilibrium conditions for the tariffed commodities, while equations (2.4) are the world market equilibrium conditions for the goods with quantity restrictions. Equations (2.5) state that each nation’s balance of trade (value of net exports at world prices) is equal to exogenously given values \( b_k \) which, in general, can be either zero or nonzero. If all \( b_k \) are zero, then our model specifies a world without international transfers of income. However, if any of the trade balances are nonzero, they are interpreted as international transfers of income and as such they must satisfy the world budget constraint

\[
\sum_{k \in K} b_k = 0. \tag{2.6}
\]

The exogenous variables of the model consist of the policy variables: tariffs on goods \( N^i \) in all countries, \( \tau^i \); quotas on trade of goods \( N^r \) in nations \( K^2 \), \( x^r \); and the international transfers of income or trade balances, \( b_k \). Given these exogenous variables, equations (2.3) – (2.5) determine the endogenous variables consisting of the world price vector \( p^T = (p^T_1, p^T_2) \) and the national utility levels, \( u^k \), for all nations \( k \in K \). From this equilibrium, the quota premia or implicit tariffs for quota restricted goods in nations \( K^2 \) can be obtained as

\[
\tau^k_r = p^T + \frac{\partial x^r}{\partial T^k},
\]

where \( S^k = \frac{\partial S^k}{\partial x^r} \).

A piecemeal multilateral tariff/quota reform comprises small (differential) changes in the policy variables and the reform problem is to determine changes that will increase utility levels in every nation. That is, the problem is to obtain a strict Pareto improvement in welfare by an appropriate choice of trade policy reform. With this objective in mind, system (2.3) – (2.5) may be totally differentiated to yield the differential comparative static system

\[
\begin{align*}
&\begin{bmatrix}
P^T S_{p_i}^k & dP_i + \sum_{k \in K^1} [S_{p_i}^k + S_{p_i}^T] d\tau^i_k
\end{bmatrix} + \sum_{k \in K^1} \begin{bmatrix}
0 & 1
\end{bmatrix} \begin{bmatrix}
\frac{\partial x^r}{\partial u^k}
\end{bmatrix} + \sum_{k \in K^1} \begin{bmatrix}
[S_{p_i}^k]
\end{bmatrix} du^k + \sum_{k \in K^2} \begin{bmatrix}
\frac{\partial x^r}{\partial u^k}
\end{bmatrix} \begin{bmatrix}
0
\end{bmatrix} du^k = \begin{bmatrix}
0
\end{bmatrix}, \\
&\begin{bmatrix}
\frac{\partial x^r}{\partial \tau^i}
\end{bmatrix} = \begin{bmatrix}
0
\end{bmatrix},
\end{align*}
\tag{2.7}
\]

\[
\begin{align*}
p^T S_{p_r}^k du^k + (p^T S_{p_r}^k + S_p^T) dp + p^T S_{p_r}^k d\tau^k = 0, & \quad k \in K^1 \tag{2.8} \\
p^T S_{u}^k du^k + (p^T S_{u}^k + S_u^T) dp + x^T dp + p^T S_{u}^k d\tau^k + (p^T S_{u}^r + p^T) dx^r = 0, & \quad k \in K^2
\end{align*}
\]
is the world substitution matrix, indicating the responses of world net exports of goods arising from a unit increase in their prices, given the levels of utility in each nation.

The differential system (2.7) – (2.8) may be expressed more compactly in matrix form as

$$Adu + Bdp + C_d\tau_r + C_d\tau_r + Ddx_r = 0,$$

(2.10)

where the matrices and vectors are implicitly defined by (2.7) – (2.9). We seek conditions under which system (2.10) has a solution with $du \gg 0$ under various restrictions on the policy reform vectors $d\tau_r$, $d\tau_r$, and $dx_r$.

3. MULTILATERAL POLICY REFORM WITHOUT INTERNATIONAL TRANSFERS OF INCOME

Almost all results in the tariff reform literature are based upon the assumption that a tariff reform may be accompanied by intercountry income transfers. Given such lump sum compensation, the tariff reform may be directed toward attaining efficiency gains in production and consumption, and the efficiency gains may then be redistributed across nations applying the international income transfers. However, if no international income transfers are permitted, as is assumed throughout this paper, tariff and quota reforms must achieve two crucial objectives – efficiency gains and welfare gains. The efficiency gains are relatively easy to obtain by reducing the distortions in the world economy, as has been demonstrated in TURUNEN-RED and WOODLAND (1991, 1995). However, if the tariff and quota reforms must also perform the task of distributing the efficiency gains so that all nations benefit, the task of selecting the direction of a tariff reform becomes much more difficult.

The policy reforms that we have in mind are small multilateral changes in the trade taxes and in trade quotas. These policy reforms may be specified as

$$d\tau^k = a^k d\alpha \quad k \in K^1$$
$$d\tau^k = a^k d\alpha \quad k \in K^2$$
$$dx^k = a^k d\alpha \quad k \in K^2,$$

(3.1)
where the vectors $a^k$ are interpreted as directions of policy reform and the scalar $d\alpha > 0$ is the step size of the reform. The following theorem establishes necessary and sufficient conditions under which a specific policy reform defined by (3.1) will increase the welfare of every country, that is, create a Pareto improvement in welfare.

**Theorem 1:** Assume that the world net substitution matrix $S_{qq}$ has an inverse. Then, the tariff and quota reform (3.1) is strict Pareto improving if and only if there does not exist a $\lambda \in R^K$ such that

$$[\overline{\beta}^1, ..., \overline{\beta}^K, -\theta] > 0,$$

where the $\overline{\beta}^k$ are sequentially defined by

$$\overline{\beta}^k = \begin{cases} -p^k T S_{pq}^k, & k \in K^1, \\ -p^k T S_{qq}^k, & k \in K^2, \end{cases}$$

$$\overline{p}^k T = \lambda_k p^T + \sum_{j \in K} \lambda_j \overline{v}^j T,$$

$$\overline{v}^j = \begin{cases} [0, -(p^T S_{pq}^j + S_{qq}^j) S_{pq}^{-1}] , & k \in K^1, \\ [0, -(p^T S_{iq}^j + S_{qq}^j) S_{iq}^{-1}] , & k \in K^2, \end{cases}$$

and

$$\theta = \sum_{k \in K^1} \overline{p}^k T S_{pp}^k a^k + \sum_{k \in K^2} \overline{p}^k T S_{\theta} a^T + \sum_{k \in K^2} (\overline{p}^k T S_{\theta} + \overline{p}^k T) a^T .$$

This theorem provides necessary and sufficient conditions for the existence of a Pareto improving tariff and quota reform. If there is no $\lambda$ that will enable (3.2) to be satisfied, a strict Pareto improving tariff and quota reform exists; if there is a $\lambda$ that enables (3.2) to be satisfied, it is not possible to devise a tariff and quota reform that will increase the welfare of each country.

Unfortunately, the conditions in Theorem 1 are extremely complicated and do not lend themselves to informative analysis without some additional simplifying assumption. Elsewhere (TURUNEN-RED and WOODLAND [1995]) we employ a rank condition on the matrix of net exports to establish further results. Here we employ a different line of attack by making an assumption about preferences, an assumption that leads to considerable simplification and to several clear-cut results concerning specific tariff and quota reforms.

This assumption may be expressed as
Assumption A

\[ S_{ju}^k < 0, S_{ju}^r = 0 \quad j \neq 1, \quad j \in N \quad k \in K^1 \]
\[ \hat{S}_{ju}^k < 0, \hat{S}_{ju}^r = 0 \quad j \neq 1, \quad j \in N \quad k \in K^2 \]

where \( S_{ju}^k \equiv \partial S_j^k / \partial u_k \) for \( j \in N \) and \( k \in K^1 \) and \( \hat{S}_{ju}^k \equiv \partial \hat{S}_j^k / \partial u_k \) for \( j \in N \) and \( k \in K^2 \).

It is well known that the terms \(-S_{ju}^k / S_j^k\) and \(-\hat{S}_{ju}^k / \hat{S}_j^k\) coincide with the effect upon the demand for good \( j \) of a unit increase in the consumer’s income and that \( S_j^k \) and \( \hat{S}_j^k \) may be assumed negative. Accordingly, Assumption A is that the income effects on the demands for non-numeraire commodities are all zero and that the numeraire commodity is normal. This is a rather extreme assumption that relegates all income effects to the numeraire commodity. It has been used elsewhere in the analysis of tax reform by, for example, KEEN (1989) and in the analysis of tariff reform by TURUNEN-RED and WOODLAND (1993).

An important implication of Assumption A is that \( \lambda > 0 \) if (3.2) is to have a solution. We use this restriction on the signs of the \( \lambda \) parameters in order to derive the following corollary of Theorem 1 and the subsequent results concerning specific tariff reform recommendations.

**Theorem 2:** Suppose that Assumption A holds, the numeraire good is not taxed, and the world substitution matrix \( S_{qq} \) has an inverse. Then a tariff and quota reform of the form (3.1) is strict Pareto improving if the country-specific numbers

\[ t^k \equiv p^T S_{pp} a^k - (p^T S_{pq} + S_{qq}^{-1}) S_{qq} f_q, \quad k \in K^1 \]
\[ p^T \begin{bmatrix} \hat{S}_{uu} & \hat{S}_{ur} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a^k \\ a^r \end{bmatrix} - \left[ p^T \hat{S}_{uu} + \hat{S}_{qr} x_T^T \right] S_{qq}^{-1} f_q, \quad k \in K^2 \]

are all positive, where

\[ f_q = \sum_{j \in K^1} S_{jq} a^j + \sum_{j \in K^2} \begin{bmatrix} \hat{S}_{qj} \\ 0 \end{bmatrix} \begin{bmatrix} a^j \\ a^r \end{bmatrix}. \]

Theorem 2 is useful in that the numbers \( t^k \) can be easily calculated for any proposed tariff and quota reform, given that the countries’ net exports and net substitution matrices are known. The first term in each expression for \( t^k \) indicates the net substitution effects of trade policy reform in each country. These effects are the changes in the countries’ net tariff and quota rental revenues due to the trade policy reform when the world prices of all goods and utilities are kept constant. The vector \( f_q \) is common to all countries and indicates the aggregate changes in the world net outputs of non-numeraire goods that arise from the trade policy reform. The vectors premultiplying \( f_q \) in (3.7) convert the
aggregate output effects into a terms of trade perturbation that results in additional changes in the countries' tariff and quota rental revenue, indicated by the second terms of the country-specific expressions for $t^k$.

We now focus attention upon particular formulae for multilateral tariff reforms and use Theorem 1 to determine whether these reforms are effective in raising the welfare of all nations. Throughout this subsection we maintain the simplifying Assumption A that the only income effects upon demand are on the numeraire commodity and that this good is normal in consumption. The proofs of the subsequent theorems proceed by demonstrating that the country-specific numbers $t^k$ are all positive for the specific tariff and quota reforms under consideration, and then applying Theorem 2 to establish the implication that the reforms are strict Pareto improving in welfare.

### A Specific Tariff and Quota Reform

The specific policy recommendation to be considered here is similar in spirit to a commodity tax policy suggested by Keen (1989; 7) and adapted to the case of tariff policy reform by Turunen-Red and Woodland (1993). Here we further adapt the idea to the situation where the trade distortions include both tariffs and quotas. Consider, therefore, the policy reform with the directional vectors, $a^k$, defined so that they satisfy the following equations:

$$
\begin{bmatrix}
S_{pp}^k & a^k \\
S_{tr}^k & 0
\end{bmatrix}
= 
\begin{bmatrix}
S_{t}^k \\
x^k
\end{bmatrix},
$$

$\forall k \in K^1$

$$
\begin{bmatrix}
S_{pp}^k & a^k \\
S_{tr}^k & 0
\end{bmatrix}
= 
\begin{bmatrix}
S_{t}^k \\
x^k
\end{bmatrix},
$$

$\forall k \in K^2$.

(3.9)

The first of these, for example, can be satisfied by choosing $a^{kT} = (0, a^k_T)^{-1}$, where $a^k_T = (S_{pp}^k)^{-1} S_{q}^k$. Similarly, we can derive an explicit formula for the directional vector $a^k$ for $k \in K^2$ with first element zero. For this policy reform it is assumed that there is no tariff (and therefore no change of tariff) on the numeraire commodity and that the inverses exist.

While this trade reform policy is defined implicitly by (3.9), it can be given a rather simple interpretation. In particular, if world prices and utility levels were to remain constant, this trade reform policy would cause equal percentage changes in the net exports of goods for all countries. To see this we note that, for $k \in K^1$, the net exports of goods are given by $S_{p}^k$ and hence $dS_{p}^k = S_{pp}^k a^k = S_{pp}^k d\alpha = S_{p}^k d\alpha$ if utility and prices are constant, thus indicating that net exports are all changed by the common proportion, $d\alpha$. For $k \in K^2$ this same proportional change in net exports can similarly be shown to hold for tariffed goods, while the proportional change in net exports of quota restricted goods is specified directly. This trade policy reform can therefore be interpreted as one of trade liberalisation, assuming of course that $d\alpha > 0$. 

The welfare implications of the trade policy reform specified by (3.9) are given by the following theorem.

**Theorem 3:** Suppose that Assumption A holds, the numeraire good is not taxed, and the world substitution matrix $S_{qq}$ has an inverse. Then a multilateral tariff and quota reform described by directional vectors $a^k$ in (3.9) yields a strict Pareto improvement in welfare if tariff revenue plus quota rents are positive in every country.

The assumption that tariff revenue plus quota rents are positive in each nation does not seem unrealistic. It seems reasonable to expect that all nations imposing trade taxes and quotas would be protectionist, either in an attempt to exploit their market power or to accommodate domestic pressures for industry support, and, hence, to expect that tariff revenue plus quota rents would be positive. Furthermore, it seems reasonable to assume that all countries have some tariff or quota distortions. Under this assumption and our admittedly rather restrictive Assumption A regarding income effects, Theorem 3 shows that the tariff reform policy described by (3.9) is welfare improving for all nations. This tariff and quota reform policy can be effected simply by knowing the initial trade vectors and the national substitution matrices at that initial equilibrium.

It should be noted that the policy reform in Theorem 3 is truly multilateral, requiring all nations to participate to ensure its success. This is evident from the proof of the theorem. This proof proceeds by showing that the crucial term $\theta$ in (3.6) may be written as $\theta = \sum_{k \in K} \lambda_k t^k$ where $t^k$ is given by (3.7) and then showing that each $t^k$ is positive, thus making $\theta > 0$. Under the proposed policy reform the common term $f_q$ vanishes (using the world market equilibrium conditions) and as a result it turns out that $t^k > 0$ for all $k \in K$. If the policy is enacted unilaterally or by a subset of countries, the world market equilibrium condition cannot be used to make $f_q$ vanish. Thus, the policy reform must be truly multilateral. This is in contrast to some of the results established by TURUNEN-RED and WOODLAND (1995), whereby unilateral policy reforms created efficiency gains that could be distributed among all relevant countries by a system of income transfers to attain Pareto improvements in welfare.

It should further be noted that the reforms involve both tariff and quota reforms. For nations $k \in K^2$ the policy reform involves altering tariffs on $N^r$ goods in a fairly complex manner, depending upon the adjustment of quotas on $N^r$ goods, so as to make the percentage change, at constant world prices and utility level, in net exports of $N^r$ goods equal. For $N^r$ goods the quota reform is very simple, namely an equi-proportional relaxation of the quotas. Thus, we see that any attempt to proportionally relax quotas in $N^r$ goods is to be accompanied by alterations in tariffs on $N^r$ goods in the $K^1$ countries and by tariff reform on all goods in $K^1$ countries if the reform is to be assured of success. Just as FALVEY (1988) found that quota reform by a small open economy in the presence of tariffs created complications, so quota reform alone in the multi-country context is complicated by tariffs and terms of trade effects. Moreover, unlike the small country open economy case, the current context does not seem to reduce to a simple quota reform policy even when all tariffs are zero and the only trade distortions are quotas.
While the proof that the trade policy reform specified by (3.9) yields a strict Pareto improvement in welfare was based upon the rather restrictive behavioural Assumption A, that all income effects (at the margin) operate through the non-taxed numeraire good, it is important to remember that this assumption is a sufficient and not a necessary condition. Thus, it is to be expected that small deviations from this assumption will not negate the strict Pareto improvement outcome. Some simulation results presented further below suggest that the result in Theorem 3 may be reasonably robust with respect to Assumption A.

Specific Tariff Reforms in the Presence of Quotas

Attention is now focused upon pure multilateral tariff reforms, without any changes in quotas on international trades. It is shown that two different tariff reform proposals will be Pareto improving in welfare under certain conditions. These results extend those obtained by TURUNEN-RED and WOODLAND (1993), who deal with a model containing tariffs only, to a context in which there are quota distortions to the competitive equilibrium.

First, consider the pure tariff reform defined by

\[
\begin{align*}
    d\tau^k &= -\varepsilon \tau^k \, d\alpha & k \in K^1 \\
    d\tau^2 &= -\varepsilon \tau^2 \, d\alpha & k \in K^2 \\
    d\tau^2 &= 0 & k \in K^2
\end{align*}
\]

(3.10)

where \( \varepsilon > 0 \) and \( d\alpha > 0 \). This reform involves an equi-proportional reduction in all tariffs. Any reform of this type attempts to move the domestic price ratios of tariffed goods in every country towards the world price ratios. Such an attempt is successful under certain conditions as specified in the following theorem.

**Theorem 4:** Suppose that Assumption A holds, the numeraire good is not taxed, and the world substitution matrix \( S_{qq} \) has an inverse. Then a multilateral tariff and quota reform described by directional vectors \( a^k \) in (3.10) yields a strict Pareto improvement in welfare if

\[
\begin{align*}
    f^k &= [p^T S_{pp}^k + S_p^T] \, \bar{p} > 0 & k \in K^1 \\
    f^k &= [(p^T S_{pp}^k, 0) + (S_{pp}^T, x_p^T)] \, \bar{p} > 0 & k \in K^2
\end{align*}
\]

(3.11)

where

\[
\bar{p} = p + (0, -p^T S_{pq} S_{qq}^{-1})
\]

(3.12)
and the initial trade balances, \( b^k \), are zero for all countries \( k \).

The price vector \( \bar{p} \) defined by (3.12) may be interpreted as the world shadow price vector for traded goods. It takes into account the tariff revenue implications of an increase in the world supply vector for traded goods and, for policy reform issues, it is these world shadow prices that are relevant and not the world market prices. The world shadow and market price vectors differ by a vector \( \bar{\tau} = \bar{p} - p = (0, -p^T S_{pq} S_{qq}^{-1}) \) that will only be zero if there are no explicit tariff distortions (in which case \( p^T S_{pq} = 0 \)). Accordingly, the vector \( \bar{\tau} \) may be interpreted as the world shadow tariff vector, being the difference between world market and shadow prices. Specifically, \( \bar{\tau} \) measures the marginal effects (keeping utility levels constant) upon tariff revenue of exogenous unit increases in the world supplies of traded goods. The latter cause world prices for non-numeraire goods to change by amounts given by the matrix \( S_{qq}^{-1} \). These price changes cause world net exports to change by \( -p^T S_{pq} S_{qq}^{-1} \) and hence world tariff revenue to change by \( -p^T S_{pq} S_{qq}^{-1} \). The world shadow price vector for traded goods, \( \bar{p} \), therefore takes into account the tariff revenue effects of an increase in the amount of each good available.\(^5\)

The world shadow tariff vector, \( \bar{\tau} \), may be further interpreted by noting that it can be written as a matrix weighted average of all national tariff vectors. In particular, if it is assumed (without loss of generality due to the homogeneity of the price system) that the numeraire good is not subject to trade taxes in any country then we can demonstrate that \( \bar{\tau} \) satisfies the equation system

\[
S_{pp} \bar{\tau} = \sum_{k \in K^1} S_{pp}^k \bar{\tau}^k + \sum_{k \in K^0 \setminus K^1} \left( \begin{array}{c} \bar{\delta}_k \\ 0 \end{array} \right) \tau^k.
\]

(3.13)

Using the definition of the matrix \( S_{pp} \) in (2.9) it is readily shown that the matrices pre-multiplying the national tariff vectors on the right hand side of (3.13) sum to the matrix \( S_{pp} \), so that \( \bar{\tau} \) may indeed be interpreted as a matrix weighted average of explicit tariff vectors. The matrix weights are the national substitution matrices as a «proportion» of the world substitution matrix, reflecting the importance of the various nations in this overall measure of tariff distortion. An obvious implication of (3.13) is that if there are no explicit tariff distortions (due to no explicit tariffs or lack of substitution) the world shadow tariff vector is the zero vector and hence the world shadow price vector \( \bar{p} \) equals the world market price vector \( p \).\(^6\) However, if there are any explicit tariff distortions then \( \bar{\tau} \neq 0 \) and hence \( \bar{p} \neq p \).

---

5. The importance of shadow prices in the evaluation of tariff and quota reforms has also been recently emphasised by Anderson and Neary (1992), Neary (1993), and Turunen-Red and Woodland (1995).

6. The result that \( \bar{p} = p \) when there are no explicit tariffs does not depend, of course, upon the assumption that the numeraire good is not subject to a tariff. In general, using the homogeneity conditions and the definition of \( S_{pp} \) in (2.9) it can be readily shown that \( p^T S_{pp} = 0 \) and hence from (3.12) that \( \bar{p} = p \).
A second pure tariff reform that can be successful under certain conditions is of the form

\[ d\tau^k = (\delta \tilde{p} - \varepsilon^k \, p^k) d\alpha \quad k \in K^1 \]
\[ d\tau^k = (\delta \tilde{p}_1 - \varepsilon^k \, p_1^k) d\alpha \quad k \in K^2 \]
\[ d\tau^k = 0 \quad k \in K^2, \]

(3.14)

where \( \delta > 0 \) and \( d\alpha > 0 \). It turns out that the size of \( \varepsilon^k \) is arbitrary and, indeed, it plays no role in the results concerning the welfare implications of the reform. However, if \( \varepsilon^k = \delta \) is chosen, the tariff reform may be expressed as \( d\tau^k = (\tau^* - \tau^k) \, d\alpha \) for \( k \in K^1 \) and \( d\tau^k = (\tau^* - \tau^k) \, d\alpha \) for \( k \in K^2 \), where \( \tau^* \) is a carefully chosen common tariff vector towards which all tariff vectors are moved (see TURUNEN-RED and WOODLAND [1993; 158-159]).

In this special form, the reform embodies the idea of all nations moving towards a common tariff structure which, if attained, would involve no tariff distortions. One particular choice of special interest for the common tariff vector is the world shadow tariff vector, whence \( \tau^* = \tilde{\tau} \). The welfare implications of this family of tariff reforms are set out in the following theorem.

**Theorem 5:** Suppose that Assumption A holds, the numeraire good is not taxed, and the world substitution matrix \( S_{qq} \) has an inverse. Then a multilateral tariff and quota reform described by directional vectors \( a^k \) in (3.14) yields a strict Pareto improvement in welfare if

\[ t^k = p^T S_{pp}^k \tilde{p} > 0 \quad k \in K^1 \]
\[ t^k = p_1^T S_{pp}^k \tilde{p}_1 > 0 \quad k \in K^2. \]

(3.15)

4. SIMULATION OF POLICY REFORMS

The purpose of this section is to set out the model, procedures and results of some simulations of international trade policy reforms. It is a logical extension of the theoretical analysis of tariff and quota reforms in a multilateral context considered above, where fairly complex conditions were developed to determine whether particular reforms are Pareto improving in welfare. In particular, since international transfers of income are not permitted, the concrete reform proposals developed were based upon a simplifying assumption regarding consumer tastes, namely Assumption A. The purpose of the simulation experiments is to try to provide some numerical results to help clarify a complex situation and, more specifically, to see whether our theoretical results concerning the welfare implications of concrete reform proposals are sensitive or robust to Assumption A.
Model Specification for Simulation

The model of international trade with distortions due to tariffs and quotas, as modelled in Section 2, may be rewritten, using the functions $S^k$ for all nations, as

\[
\sum_{k \in K} S^k_i (p_t + \tau^k_i, p_r + \tau^r_i, u^k_i) = 0 \quad i \in N
\]  

(4.1)

\[
\sum_{i \in N} p_i S^k_i (p_t + \tau^k_i, p_r + \tau^r_i, u^k_i) = b^k \quad k \in K
\]  

(4.2)

\[
S^k_i (p_t + \tau^k_i, p_r + \tau^r_i, u^k_i) = x^k_i \quad i \in N^r, k \in K^2
\]  

(4.3)

where $S^k_i (p_t + \tau^k_i, p_r + \tau^r_i, u^k_i)$ are the net export functions. In previous sections, the model and various conditions were expressed in terms of the derivatives of the functions $\delta^k (p_t + \tau^k_i, x^k_i, u^k_i)$ for $k \in K^2$, where $x^k_i$ is the vector of net exports of quota-restricted goods in $N^r$ in country $k$. The derivatives of these functions can obviously be computed from our competitive solution to (4.1) – (4.3) using the relationship between the $\delta^k$ and $S^k$ functions given by (2.2).

Given the exogenous variables $b^k, k \in K$ and the policy instruments

\[
\tau^k_i \quad k \in K^1, i \in N
\]

\[
x^k_i \quad k \in K^2, i \in N^r
\]

\[
\tau^k_i \quad k \in K^2, i \in N^r
\]

system (4.1) – (4.3) can be solved for the endogenous variables

\[
p_i \quad i \in N
\]

\[
u^k \quad k \in K
\]

\[
\tau^k_i \quad k \in K^2, i \in N^r
\]

where $p_1 = 1$ due to the price normalisation.

In order to undertake a numerical simulation analysis of the trade policy reforms, it is necessary to choose particular functional forms for the functions used to describe the technologies and tastes, numerical values for their parameters, and values for all the exogenous variables. While it is desirable to make these choices consistent with empirical evidence to reflect real world technologies and tastes, the information requirements are substantial and so this approach is not taken. Rather, for the purposes of this study,
functional forms are chosen to be parsimonious, consistent with economic theory and to facilitate ease of computation.

Ignoring the country superscript $k$ we assume that

$$G(p) = v \left( \sum_{i \in N} b_i p_i^2 \right)^{0.5} \equiv vg(p) , \quad (4.4)$$

$$E(p, u) = \sum_{i \in N} e_i p_i + u \prod_{i \in N} p_i^{e_i} \equiv e(p) + u \alpha(p) , \quad (4.5)$$

where the parameters obey the following restrictions:

$$v > 0, \quad b_i \geq 0, \quad e_i \leq 0, \quad a_i \geq 0, \quad \sum_{i \in N} a_i = 1 . \quad (4.6)$$

The GNP function describes a technology that exhibits constant elasticities of transformation ([POWELL and GRUEN [1968]]), and all goods are substitutes in production. The expenditure function generates the linear expenditure system of [STONE (1954)] that has been extensively used in empirical studies of demand. As is well known, the implied preferences are homothetic to the point $e$. The chosen GNP and expenditure functions have the important feature that they are globally consistent with economic theory under simple parameter restrictions that we invoke.

Of particular importance in the present context is the fact that the functional form for expenditure function given by (4.5) allows a parametric representation of Assumption A. In particular, $S_{iu} = -a_i \alpha(p) / p_i$ and $a_i$ is equal to the marginal budget share (change in expenditure on good $i$ as a result of a unit increase in the consumer’s income). Thus, Assumption A holds if and only if $a_1 = 1$ and $a_j = 0$ for all $j \neq 1$.

The resulting equations (4.1) – (4.3) are non-linear in the endogenous variables, and they are solved using the NLSYS procedure in GAUSS. Following the solution for the competitive equilibrium, a range of properties of that solution are computed; for example, the first and second derivatives of the $S^k$ functions and various functions of these.

**Example**

To make the model operational, numerical values have to be chosen for the parameters of the revenue and expenditure functions and for the exogenous variables. The chosen values for the parameters are expressed in matrix form as follows:
where the columns refer to countries and the rows to products. There are 4 countries trading in 4 products. The elements of $B$ are the parameters of the revenue functions, while the elements of $E$ and $A$ are parameters of the expenditure functions. Since $E = 0$, preferences in all countries are homothetic and the expenditure functions are Cobb-Douglas functional forms.

First, assume that there are no tariff or quota distortions to trade. Hence we have a situation of free world trade and the competitive equilibrium solution for the world price vector is $p = (1\ 1\ 1\ 1)^T$. The utility vector is $u = (3.162\ 3.162\ 3.162\ 3.162)^T$ and the equilibrium net export matrix, indicating the pattern of trade, is

$$X = \begin{pmatrix} -0.9487 & -0.3162 & 0.3162 & 0.9487 \\ -0.3162 & -0.9487 & 0.9487 & 0.3162 \\ 0.3162 & 0.9487 & -0.9487 & -0.3162 \\ 0.9487 & 0.3162 & -0.3162 & -0.9487 \end{pmatrix}.$$ 

Looking at row 1 we see that product 1 is imported by countries 1 and 2 and exported by countries 3 and 4. In fact, due to the symmetry of the parameter choices, trade in each product is bilateral or specialised. Looking at column 1 we see that country 1 imports products 1 and 2 and exports the other two products.

Second, consider the imposition of tariffs by countries on all goods except the numeraire. If the tariff matrix is

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ .1 & .2 & -.2 & -.1 \\ -.1 & -.2 & .2 & .2 \\ -.1 & -.1 & .1 & .4 \end{pmatrix},$$

the new equilibrium has the domestic price and net export matrices, $P$ and $X$, and utility vector, $u$, given as:

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1.117 & 1.217 & .817 & .917 \\ .896 & .796 & 1.196 & 1.196 \\ .853 & .853 & 1.053 & 1.353 \end{pmatrix}, \quad X = \begin{pmatrix} -.89 & -.25 & .31 & .83 \\ -.07 & -.61 & .66 & .02 \\ .22 & .72 & -.73 & -.21 \\ .79 & .16 & -.27 & -.68 \end{pmatrix}.$$
The pattern of trade has not altered but the volume of trade has reduced for every country in every product and it is no longer the case that trade is specialised and bilateral. All nations are worse off in terms of utility compared to free trade.

Starting from this tariff-distorted competitive equilibrium, the question is whether we can find directions of policy change that will improve welfare for every country. More specifically, will the specific reform proposals that have been suggested in the previous section have this desirable outcome despite the fact that Assumption A, which was used to establish the theoretical results, does not hold in this example world?

Three specific policy reforms were proposed in section 3. These are defined by (3.9), (3.10) and (3.14) and will be henceforth referred to as Policy Reforms #1, #2 and #3. In the context of the current example, policy reform #1 involves tariffs only as there are no quotas present and so the directions of tariff change satisfy the equations $S_{kp}^a = S_p^k$ for all $k \in K$. Policy reform #2 involves an equiproportional reduction in all tariff rates. For policy reform #3 we choose $\delta = \epsilon$ and $x^* = \bar{x}$ so this policy reform proportionately reduces the difference between actual tariffs and world shadow tariffs.

The tariff change matrices corresponding to the three trade policy reform proposals are

$$dT_1=egin{pmatrix}
0 & 0 & 0 & 0 \\
1.2798 & -1.0702 & 0.0547 & -0.8469 \\
1.2963 & 0.4846 & -1.9490 & -1.4044 \\
1.6479 & 0.1139 & -0.8869 & -2.6526
\end{pmatrix}$$

$$dT_2=egin{pmatrix}
0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.0164 & -0.0146 & 0.0197 & 0.0142 \\
0.0035 & 0.0176 & -0.0103 & -0.0130 \\
0.0018 & 0.0094 & -0.0056 & -0.0267
\end{pmatrix}$$

$$dT_3=egin{pmatrix}
0.0000 & 0.0000 & 0.0000 & 0.0000 \\
-0.0265 & -0.0248 & 0.0094 & 0.0040 \\
-0.0039 & 0.0101 & -0.0176 & -0.0203 \\
-0.0013 & 0.0064 & -0.0086 & -0.0296
\end{pmatrix}.$$
increase in all net export vectors at the initial prices and utility levels, as explained in section 3.

The resulting changes in the utility vector, obtained by comparative statics calculations, are

\[ du = (0.3371 \ 0.3709 \ 0.2482 \ 0.1117)^T \]
\[ du = (0.0036 \ 0.0075 \ 0.0026 \ 0.0053)^T \]
\[ du = (0.0068 \ 0.0066 \ 0.0032 \ 0.0022)^T \]

showing that all nations benefit from each of the three tariff reforms. That all nations will benefit has been proved under the very restrictive assumption that all income effects are concentrated on good 1 and hence income effects on the consumption of all other goods are zero. That assumption is, of course, sufficient but not necessary for the Pareto-improvement outcome. The above example illustrates this quite clearly, since Assumption A is not satisfied and yet each policy reform yields a Pareto improvement.

The calculation of \( du \) presented above is obtained by evaluating the slopes of the relationships between utilities and tariffs at the initial equilibrium. If a small finite step-size is chosen for the tariff reform and the new tariffs are enacted, there will be a new equilibrium with new utility levels. Using a step size \( da = 0.05 \) the changes in the utility vector are given by

\[ \Delta u = (0.0162 \ 0.0265 \ 0.0146 \ 0.0116)^T \]
\[ \Delta u = (0.0165 \ 0.0271 \ 0.0148 \ 0.0120)^T \]
\[ \Delta u = (0.0170 \ 0.0276 \ 0.0150 \ 0.0122)^T \]

each of which has positive elements.

In summary, this example illustrates a pretend world of trade in four goods between four countries. Policy reforms #1 – #3 were evaluated numerically and the welfare implications calculated. Although each policy reform has only been proved to be welfare-improving for all countries under a very restrictive assumption on consumer preferences, which does not hold in the model considered here, the resulting utility changes computed from our example model are all positive. This suggests that the restrictive assumption on consumer preferences may not be important in practice. To more confidently establish its importance, attention is now turned to a more extensive set of numerical experiments.

**Simulation of Trade Policy Reforms**

The primary purpose of the simulation experiments is to determine the extent to which the Pareto improvement outcome is dependent upon the validity of Assumption A. To
this end, we alter the preference parameters in the example model described in the previous subsection. In particular, we alter the preference parameter $a^k$ over the range 0.1 to 0.9, with the remaining preference parameters being determined by $a^k = (1 - a^k) / (N - 1)$ to ensure that they sum to unity for each country.

In the present context of a linear expenditure system, the parameter $a^k$ is interpreted as the marginal budget share for good one in country $k$. If it is unity then Assumption A holds exactly (and globally); and the closer $a^k$ is to zero the greater is the violation of Assumption A. Accordingly, it is expected that the success of the three trade policy reforms considered in section 3 will be poor when $a^k$ is close to zero and be high when it is close to unity. Below, we provide results showing the failure rate of each policy reform using the criteria that a failure occurs when the reform does not lead to a Pareto improvement, i.e., at least one country experiences a decrease in welfare.

The success of each reform will depend upon the extent and nature of the initial set of trade policy distortions and the competitive equilibrium. While parameters of the technology are not varied in our numerical simulations, we allow for a range of initial policy settings and a number of model structures. For each model, there are 9 settings for the preference parameters ($a^k = 0.1, 0.2, ..., 0.9$) and for each of these parameter settings 100 initial policy settings are chosen randomly from a distribution. The competitive equilibrium is solved for each of these 900 situations, the three policy reforms are numerically evaluated, and the comparative static solution for the resulting response of the utilities is calculated. The number of cases where not all countries gain is recorded and plotted against the value of $a^k$.

There are a number of variations to the models used in this simulations:
(a) We consider a four country, four good model for the most part ($K = 4, N = 4$), but also consider a two country, two good model, and a two country, four good model.
(b) In the four country, four good model we consider (i) only tariffs, and (ii) quotas on two goods only by two countries and tariffs elsewhere ($K^2 = 2; N^r = 2$).
(c) The tariffs and quotas are drawn from a uniform distribution and so the spread of policies is quite large within certain restrictions. Under one regime we allow tariffs or quotas on all goods, but we also consider the case where they are only imposed on goods with greatest trades. In all cases we tax or restrict trade rather than subsidise it.
(d) Also, we vary the ad valorem equivalent of tariffs from a maximum of 30% to a maximum of 15% (for quotas these are the two maxima as a percentage of the country's trade) to evaluate the role played by the severity of distortions.

While the results from all of our experiments cannot be presented here, the main points arising from them can be seen in the sample of results given in Figures 1-6.

Figure 1 shows the effect of allowing for 1,000 samples to be drawn for each parameter setting compared to the 100 samples used throughout the remainder of the study. It is evident that the results, showing the number of failures per 1,000 samples, are quite similar and so we feel confident that 100 samples are sufficient for our purposes.

Figure 2 shows the failure rate for each of three policy reforms (Policy Reforms #1-#3) considered in section 3 in a model with four countries trading in four goods, when no
Figure 1
Comparison of Sample Size (Reform #1)

Figure 2
Policy Reform: Tariffs (K=4, N=4)

Figure 3
Policy Reform: Tariffs and Quotas (K=4, N=4)
Figure 4
Tariff Reform with Smaller Tariffs

Figure 5
Policy Reform: Tariffs (K=2, N=2)

Figure 6
Tariffs and Trade Related (Reform #3)
quotas are permitted. Several features emerge from this figure. First, our expectation that the success of policy reforms would be least when $a_1^{*}$ is close to zero and be very greatest when $a_1^{*}$ is close to unity is borne out very clearly in Figure 2 for policy reforms #1 and #3. In fact, policy reform #3 is successful in all 100 cases when $a_1^{*} = 0.9$ and the success rate is very high when $a_1^{*} = 0.8$. Policy reform #1 is less successful but it fails in only about 10-15% of cases when $a_1^{*}$ is in the range 0.5 to 0.9. Second, policy reform #2 does not follow this pattern and its number of failures actually increases as $a_1^{*}$ gets larger. It will be recalled that each reform requires conditions in addition to Assumption A to hold to ensure success, and that reform #2’s conditions are much tighter than the other reforms (the additional condition, that tariff revenue is positive in each nation always being satisfied for policy reform #1). Third, the general pattern of success of the three policy reforms as shown in Figure 2 tends to carry over to other situations.

Figure 3 shows the failure rates for policy reforms #1 and #3 for the case where two countries can impose quotas on two goods and tariffs on two goods, while the other two countries can impose tariffs on all goods. In this case, policy reform #1 involves both tariffs and quotas while policy reform #3 leaves quotas untouched but alters tariffs towards the world shadow tariff vector. The most important observation about Figure 3 is that the failure rates for both policy reforms, but especially reform #1, are higher when quota distortions are present than when tariffs are the only form of distortion (see Figure 2). The reason for this is not clear.

In our experiments we considered two different maximum rates of tariff and quota settings. When the maximum allowable tariffs and quotas are halved the failure rates of policy reforms #1 and #3 decline. Figure 4 shows the failure rates for the case of a model involving tariff distortions only; they are very low for values of $a_1^{*}$ above 0.5, and even when $a_1^{*}$ is close to zero the failure rates amount to only about 30%. This suggests that the severity of the initial distortions is important. What could be relevant here is whether the initial tariffs are above or below their Nash equilibrium values.

The experiments were also run with a simple model of trade in two products by two countries, with tariff distortions. Figure 5 shows that while policy reform #2 continues to perform badly, policy reform #3 has an average failure rate of about 10% over the whole range, and policy reform #1 conforms to our initial expectations. The failure rate for policy reform #1 declines as $a_1^{*}$ gets bigger and, moreover, the failure rates are very low. These results suggest that the robustness of the Pareto improvement outcome to Assumption A improves as the number of countries and goods is reduced. A similar improvement was observed when the number of countries was reduced but the number of goods was maintained at four (results not presented). This is perhaps to be expected since the number of policy instruments available to each country is the same but the number of utility levels required to increase is reduced as the number of countries is reduced.

Figure 6 shows the effect of relating tariff policy to the size of trade. In the model considered here, tariffs are imposed on the good with the largest quantity of imports and export taxes are imposed on the good with the largest exports. Other goods have no tariffs
or taxes. Thus, the size of tariff distortions is positively related to the volume of trade under this trade policy. Figure 6 shows the results for this policy setting in comparison with the case where all goods except the numeraire get taxed. For policy reform #3 the failure rates are much higher when tariffs and trade are positively related. However, for policy reform #1 the failure rates are almost identical. This difference suggests that further investigation is warranted.

In summary, our results are generally, though not always, consistent with our expectation that the failure rate for the three policy reforms would be greatest when Assumption A was severely violated \((a^e_i \text{ close to zero})\) and very low when Assumption A was close to being satisfied \((a^e_i \text{ close to unity})\). On the whole, this expectation has been borne out for policy reforms #1 and #3. Policy reform #2, on the other hand, has very high failure rates over most of the parameter range and they seldom fall as \(a^e_i\) gets bigger. Evidently, the requirement in Theorem 5 that its \(t^k\) be positive is particularly stringent.

5. CONCLUDING REMARKS

This paper has examined the conditions under which a multilateral piecemeal reform of tariffs and quota restrictions on the international trade of commodities can achieve a Pareto improvement in welfare. The model is general in that it allows for arbitrary numbers of countries and goods and considers an arbitrary initial equilibrium involving both tax and quota distortions to international trade. Attention is restricted to piecemeal (differential) trade policy reforms since it is normally the case that small changes to policy are made rather than sudden significant shifts. The Pareto welfare criterion is adhered to since it provides a stringent test for a trade policy reform to pass and one that seems appropriate when the multilateral trade policy reform has to be agreed to by all parties.

Within this context, conditions for trade policy reforms to be strict Pareto improving in welfare were established under the assumption that the trade policy reforms are not accompanied by a set of international income transfers. In this context, the quota/tariff policy reform instruments have the dual tasks of creating efficiency gains and distributing them to effect a strict Pareto improvement in welfare. These tasks are onerous and specific reform proposals are more difficult to construct than when international transfers of income form part of the reform package. Also, given that our focus is on multilateral reforms in a general equilibrium framework, it is necessary to take account of changes in the world prices of goods arising as a result of the reforms. The consequence is that the welfare evaluation of the reforms must make use of the shadow prices for goods, not market prices, in the net benefit conditions.

Apart from a general existence result, the paper has provided some specific reform formulae that will yield strict Pareto improvements in welfare. Our theorems concerning the welfare implications of these policy reform formulae based upon the very stringent Assumption A on preferences in the neighbourhood of the initial distorted equilibrium.
We have employed numerical simulation techniques to evaluate the extent to which our theoretical results are robust to a relaxation of this assumption.

The simulation results are, on the whole, consistent with our prior expectation that each trade policy would continue to yield a strict Pareto improvement in welfare when Assumption A was only mildly violated and that the likelihood of success of each policy reform would decline with greater deviations from Assumption A. For policy reforms #1 and #3, the failure rates were generally low when Assumption A was close to being satisfied, rising as the assumption was more severely violated. The results suggest that Assumption A may not be vitally important; the trade policy reforms are often Pareto improving in welfare when this assumption does not hold.

While simulation methods are common in other areas such as econometrics, they are seldom used in areas such as international trade to complement theoretical analysis. The results from our simulation model are very encouraging and suggest that numerical simulations might be profitably used in a variety of areas of international trade theory to provide another way of evaluating the importance of assumptions.

APPENDIX

The following Lemma based upon Motzkin's Theorem of the Alternative is taken from Dievert, Turunen-Red and Woodland (1989) and is used in the subsequent proofs of theorems.

**Lemma (Motzkin's Theorem of the Alternative):** Let $D_1$ and $D_2$ be given matrices and let $x_1$, $x_2$ and $y$ be conformable vectors. Then either

$$D_1 x_1 + D_2 x_2 = 0, \quad x_1 \succ 0,$$  \hspace{1cm} (A.1)

has a solution $(x_1, x_2)$, or

$$y^T D_1 < 0, \quad y^T D_2 = 0,$$  \hspace{1cm} (A.2)

has a solution $y$, but never both.

**Preliminary Results:** Equation system (2.7) - (2.8) may be written in matrix notation as

$$S_{qq} dq + S_{q\tau} d\tau + S_{qx} dx + S_{qu} du = 0$$  \hspace{1cm} (A.3)

$$T_q dq + T_\tau d\tau + T_x dx + T_u du = db = 0 ,$$  \hspace{1cm} (A.4)
where the normalisation of the price for the numeraire good is used so the change in the world price vector is \( dp^T = (0, dq^T) \) and the market equilibrium condition for the numeraire good is ignored in accordance with Walras' Law.

Since \( S_{qq} \) is assumed to have full rank its inverse exists and so (A.3) may be solved for \( dq \) as

\[
dq = -S_{qq}^{-1} [S_{q\tau} d\tau + S_{qx} dx + S_{qu} du]
\]

and substituted into (A.4) to obtain

\[
(T_u - T_q S_{qq}^{-1} S_{qu} du + (T_\tau - T_q S_{qq}^{-1} S_{q\tau}) d\tau + (T_x - T_q S_{qq}^{-1} S_{qx}) dx = 0,
\]

which may be written as

\[
R_u du + R_\tau d\tau + R_x dx = 0,
\]

where \( R_i = T_i - T_q S_{qq}^{-1} S_{qi} \) for \( i = u, \tau, x \). The matrices \( R_i \) are complicated but system (A.7) may nevertheless be written in component form as

\[
p^T S_{pp}^k du^k + \tau^k \left[ \sum_{j \in K^1} S_{ju}^k du^j + \sum_{j \in K^1} S_{j\tau}^k d\tau^j + \sum_{j \in K^2} S_{j\nu}^k d\nu^j + \sum_{j \in K^2} S_{j\xi}^k d\xi^j \right] + p^T S_{pp}^k dx^k = 0, \quad k \in K^1
\]

\[
p^T S_{uu}^k du^k + \tau^k \left[ \sum_{j \in K^1} S_{ju}^k du^j + \sum_{j \in K^1} S_{j\tau}^k d\tau^j + \sum_{j \in K^2} S_{j\nu}^k d\nu^j + \sum_{j \in K^2} S_{j\xi}^k d\xi^j \right] + p^T S_{uu}^k dx^k + (p^T S_{hu}^k + p^T S_{hu}^k) dx^k = 0, \quad k \in K^2,
\]

where the \( \tau^k \) are defined in (3.5).

**Proof of Theorem 1:** Substituting \( d\tau^k = a_k^\tau d\alpha \) for \( k \in K^1 \), \( d\xi^k = a_k^\xi d\alpha \) and \( dx^k = a_k^x d\alpha \) for \( k \in K^2 \) into equations (A.8) and then collecting like terms yields the equations of the form

\[
p^T S_{pu}^k du^k + \tau^k \left[ \sum_{j \in K^1} S_{ju}^k du^j + \sum_{j \in K^1} S_{j\tau}^k d\tau^j + \sum_{j \in K^2} S_{j\nu}^k d\nu^j + \sum_{j \in K^2} S_{j\xi}^k d\xi^j \right] + \theta^k d\alpha = 0, \quad K \in K^1
\]

\[
p^T S_{hu}^k du^k + \tau^k \left[ \sum_{j \in K^1} S_{ju}^k du^j + \sum_{j \in K^1} S_{j\tau}^k d\tau^j + \sum_{j \in K^2} S_{j\nu}^k d\nu^j + \sum_{j \in K^2} S_{j\xi}^k d\xi^j \right] + \theta^k d\alpha = 0, \quad K \in K^2.
\]
where $\theta^k$ is the sum of all the coefficients of $d\alpha$ in equation $k$. By the Lemma, system (A.9) has a solution with $du > 0$ and $d\alpha > 0$ if and only if there does not exist a solution $\lambda \in R^K$ to

$$\lambda^T R_u < 0, \quad \theta \equiv \sum_{k \in K} \lambda_k \theta^k < 0,$$

(A.10)

where $\theta$ is given by the expression

$$\theta = \sum_{k \in K^1} \bar{p}^{kT} S_{pp}^k d^k + \sum_{k \in K^2} \bar{p}^{kT} \hat{S}_{iu}^k d_i^k + \sum_{k \in K^2} (\bar{p}_{ir}^{kT} \hat{S}_{ir}^k + \bar{p}_r^{kT}) d_r^k.

(A.11)

Condition (A.10) may be written as (3.2) in the statement of Theorem 1.

**Proof of Theorem 2:** First, it is shown that the assumptions imply $[\bar{\beta}^1, ..., \bar{\beta}_K^1] > 0$ if and only if $\lambda > 0$. Under Assumption A, that the income effects of all goods except the numeraire are zero and that the numeraire good is normal, the $\bar{\beta}^k$ terms defined by (3.3) simplify to

$$-\bar{\beta}^k = \bar{p}_{1T}^k S_{pu}^k = \bar{p}_i^k S_{iu}^k = \lambda_k p_1 S_{iu}^k, \quad k \in K^1$$

$$-\bar{\beta}^k = \bar{p}_{1T}^k \hat{S}_{iu}^k = \bar{p}_i^k \hat{S}_{iu}^k = \lambda_k p_1 \hat{S}_{iu}^k, \quad k \in K^2,

(A.12)

where it is noted that $\bar{p}_i^k = \lambda_k p_1$ from (3.4) and (3.5). Since $p_1 > 0$ and each $S_{iu}^k < 0$ by the normality of good 1 in every country, it follows from (A.12) that $[\bar{\beta}^1, ..., \bar{\beta}_K^1] > 0$ if and only if $\lambda > 0$.

Second, the expression for $\theta$ given by (A.11) above may be rewritten as

$$\theta = \sum_{k \in K} \lambda_k t^k,

(A.13)

where

$$t^k = p^T S_{pp}^k d^k - (p^T S_{pq}^k + S_{qq}^k) S_{qq}^{-1} f_q^k, \quad k \in K^1$$

$$p^T \begin{bmatrix} \hat{S}_{iu}^k & \hat{S}_{ir}^k \\ 0 & I \end{bmatrix} \begin{bmatrix} a_i^k \\ a_r^k \end{bmatrix} - \left[ p^T \hat{S}_{iu}^k + S_{iu}^k, \chi_T^k \right] S_{qq}^{-1} f_q^k, \quad k \in K^2

(A.14)

and
\[ f_q = \sum_{j \in K^1} S_{qp} a^j + \sum_{j \in K^2} \begin{bmatrix} 0 \\ 1 \\ 0 \\ a_j^r \end{bmatrix} \begin{bmatrix} s_{qr} \\ a_j^r \end{bmatrix}. \]  

(A.15)

Third, it is now shown that (3.2) cannot have a solution and hence by Theorem 1 there does exist a Pareto improving trade policy reform of the type under consideration. If (3.2) is to have a solution then \([\tilde{\theta}_1, ..., \tilde{\theta}_K] > 0\). But this, under the assumptions, implies that \(\lambda > 0\), as shown above. In this case \(\theta > 0\) since \(\lambda > 0\) and each \(\tilde{\theta}_k\) is positive by assumption. Thus, if \(\tilde{\theta}_k > 0\) for all \(k \in K\) there cannot be a solution to (3.2) and hence a Pareto improving trade policy reform exists.

**Proof of Theorem 3:** In view of Theorem 2, which applies under Assumption A, it is sufficient to prove that the \(\tilde{\theta}_k\) terms defined by (3.7) and (3.8) are all positive under present assumptions. First, it is shown that the policy reform defined by (3.8) implies that \(\tilde{\theta}_k = 0\) in (3.8). Indeed, if (3.9) is substituted into (3.8) we get

\[ f_q = \sum_{j \in K^1} S_{qp} + \sum_{j \in K^2} \begin{bmatrix} s_{qr} \\ x_j^r \end{bmatrix} = 0 \]

since \(f_q\) becomes the world net export vector, which is zero because all markets clear. Second, it is observed from (3.7) that \(f_q = 0\) reduces each expression for \(\tilde{\theta}_k\) to the first term. Thus, for country \(k \in K^1\), \(\tilde{\theta}_k = p^T S_{pp} a^k = -\tau^T S_{pp} a^k\) (using price homogeneity of \(S^k\)) = \(-\tau^T S_{pp} a^k\) (since \(\tau_1 = 0\)) = \(-\tau^T S_q a^k\) (using [3.9]), which is the tariff revenue earned by country \(k\). Similarly, it can be shown that for \(k \in K^2\)

\[ \tilde{\theta}_k = -\tau^T \begin{bmatrix} s_{qr} \\ x_j^r \end{bmatrix} \]

which is the tariff (explicit and implicit) revenue earned by country \(k \in K^2\). Finally, if the tariff (explicit and implicit) revenue earned by each country is positive, each \(\tilde{\theta}_k\) is positive and hence, by Theorem 2, the policy reform yields a strict Pareto improvement in welfare.

**Proof of Theorem 4:** Again, to prove the theorem it suffices to show that each \(\tilde{\theta}_k\) defined by (3.7) and (3.8) takes the form given by (3.11) in the statement of Theorem 4. First, it may be shown (using the price homogeneity conditions for \(S^k\) and \(\tilde{S}^k\)) that

\[ S_{pp} a^k = \varepsilon^k S_{pp} \ p, \quad k \in K^1, \]

\[ \begin{bmatrix} \tilde{s}^k_{nt} \\ \tilde{s}^k_{nr} \end{bmatrix} \begin{bmatrix} a^t \\ a^r \end{bmatrix} = \varepsilon^k \begin{bmatrix} s^k_{nt} \\ 0 \\ p_t \end{bmatrix} = \varepsilon^k \tilde{s}^k_{nt} p_t, \quad k \in K^2. \]
Second, this result implies that \( f_q = \varepsilon S_{qp} p \) and hence \( S_{qq}^{-1} f_q = \varepsilon S_{qq}^{-1} S_{qp} p = \varepsilon (q - \bar{q})^T \).

Third, using these results and the assumption that there are zero trade balances, i.e., \( p^T S_k = 0 \) for \( k \in K^1 \) and \( p^T \tilde{S}^k = 0 \) for \( k \in K^2 \), the \( k^k \) terms in (3.7) may be written as in (3.11) apart from the multiplicative term \( \varepsilon > 0 \).

**Proof of Theorem 5:** Again, to prove the theorem it suffices to show that each \( k^k \) takes the form given by (3.11) in the statement of Theorem 5. First, it may be shown (using the price homogeneity conditions for \( S^k \) and \( \tilde{S}^k \)) that

\[
S_{pp}^k a^k = \delta S_{pp} \tilde{p}, \quad k \in K^1, \\
\begin{bmatrix} S_{tt}^k & \tilde{S}_{tr}^k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_t^k \\ \tilde{a}^k \end{bmatrix} = \delta \begin{bmatrix} \tilde{S}_{tt}^k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p}_t \\ \tilde{p}_r \end{bmatrix} = \delta \tilde{S}_{tt}^k \tilde{p}_t, \quad k \in K^2.
\]

Second, this result implies that \( f_q = \delta S_{qp} \tilde{p} = 0 \) since

\[
S_{qq} \tilde{p} = S_{qq} \left[ p - \left( 0, p^T S_q p^{-1} S_q^T \right) \right] = S_{qq} p - S_{qq} S_q^{-1} S_q p = 0.
\]

Third, these results may be used to show that the \( k^k \) terms in (3.7) may be expressed as (3.14) apart from a multiplicative term \( \delta > 0 \).

**REFERENCES**


SUMMARY

This paper is concerned with proposals for multilateral reforms of tariffs taxes and quotas that are Pareto-improving in welfare when the reforms are not accompanied by any income transfers between countries. Starting from a tariff and quota distorted competitive equilibrium for a model of a trading world consisting of an arbitrary number of nations engaged in international trade in an arbitrary number of commodities, we examine the possibilities of undertaking gradual multilateral reforms of tariffs and quotas to attain a strict Pareto improvement in welfare.

Since we exclude the use of international transfers of income as a redistributive device, the trade policy reforms are required to simultaneously create efficiency gains and to redistribute them to attain a strict welfare improvement for all countries. In addition to providing conditions that can be numerically checked in order to determine whether a particular direction of change in policy parameters is a Pareto improvement, we also give concrete reform recommendations that can result in welfare improvements.

Unfortunately, these policy reform recommendations are shown to yield Pareto improvements in welfare under a very strict sufficiency condition on consumer preferences. To evaluate the likely importance of this sufficiency condition for the Pareto-improvement outcome, a series of numerical simulations are undertaken. Given the numerical values for the parameters of the technologies and preferences in each country of our model, together with the policy (tariffs and quotas) parameters, the competitive equilibria are solved numerically. The specific policy reforms are then evaluated and their welfare implications obtained. In short, the restrictiveness of our sufficiency condition used in the theoretical analysis is evaluated using simulation methods. The results suggest that the strict normality assumption on preferences is a rather weak sufficiency condition.