Principles of Seigniorage

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1. INTRODUCTION

The economic literature on seigniorage has traditionally mentioned different concepts of measuring seigniorage: One of the basic concepts is what NEUMANN (1992) named monetary seigniorage, which is measured by the growth rate of the nominal money stock $\mu$ multiplied by the value of real balances $m$. BAILEY (1956) used the inflation rate $\pi$ multiplied by real balances $m$ as a seigniorage measure, neglecting revenues from growth. Another basic concept, e.g. discussed in MARTY (1978), is the opportunity cost concept of seigniorage, which is given by the nominal interest rate $i$ multiplied by real balances $m$. DRAZEN (1985) suggested a general measure of seigniorage including the revenue from expansion of the nominal money supply plus the profits earned by monopolistic issuers of money due to people's desire to hold a given level of real balances. NEUMANN (1992) advocated the use of an «extended» measure of monetary seigniorage.

The question of the relationship between different concepts of seigniorage has never been settled clearly and the discussions of seigniorage are marred by a certain confusion about the appropriate way to measure seigniorage. The aim of this paper is therefore to discuss some principal aspects of seigniorage and to clarify the relationship between alternative measures of seigniorage which have been suggested in the literature.

A meaningful measure and definition of seigniorage must capture all the revenue the government obtains due to its power over the creation of central bank money. Central to our analysis is the distinction between actual seigniorage and fund transfers from the central bank to the government. As far as seigniorage is concerned, we find it useful to distinguish between different sources of seigniorage and between different liquidities of seigniorage. For understanding seigniorage, it is also important to distinguish between

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1. See also KLEIN and NEUMANN (1990).
2. The concept of monetary seigniorage is also discussed in FRIEDMAN (1971) and in BARRO (1982).
3. The opportunity cost concept is also discussed in PHLEPS (1973) and in BARRO (1982).
equilibrium and disequilibrium situations and between honest and dishonest government behavior.\textsuperscript{4}

Within this framework, we consider distinctly different aspects of seigniorage and we are able to integrate different concepts of measuring seigniorage usually mentioned in the literature. We conclude that the opportunity cost concept of seigniorage \textit{im} does in fact, with the exception of the dishonest government case, represent the appropriate measure of seigniorage for most cases, whereas the monetary seigniorage \textit{mn} concept is incomplete and either overstates or underrates seigniorage revenue depending on the situation. However, neither concept alone is very informative, because neither fully reflects all the different aspects of seigniorage. To obtain a precise picture of the seigniorage situation, we have to analyze the different aspects of seigniorage individually and distinguish between actual seigniorage and the fund transfer from the central bank to the government. As a matter of fact, a large part of seigniorage stems from inflation and growth. In order to analyze the theory of seigniorage in an economically meaningful way and to capture different aspects of seigniorage, all variables must be defined relative to income. The amounts of seigniorage originating from different sources would appear quite different if variables were expressed in nominal terms.

The remainder of the paper is organized as follows: In Section 2, we will concentrate on an equilibrium situation where the wealth constraint of the central bank remains unchanged in real, per capita terms. Section 3 allows changes in the wealth constraint of the central bank. Section 4 discusses the question of comparative statics and maximization of seigniorage. Section 5 concludes the paper.

In order to concentrate on the basic theory of seigniorage, we make the following simplifying assumptions:
- The elasticity of real money demand with respect to real income has a constant value of one.
- The real interest rate \( r \) is constant and the Fisher-relation \( i = r + \pi \) is valid, where \( i \) is the nominal interest rate and \( \pi \) is the inflation rate.
- Expected inflation is equal to actual inflation.
- The maturity of government bonds held by the public and private securities held by the central bank is infinitesimally small, which is equivalent to an assumption that these securities are indexed bonds. Thus, a change in the inflation rate does not change their real value.
- The entire monetary base is currency or, equivalently, no interest is paid on reserves at the central bank.
- The production of money is costless. Subsidies of the central bank to the private sector (e.g. banks) are excluded.
- The real interest rate \( r \) is larger than the real growth rate \( n \). The economy is thus dynamically efficient.

\textsuperscript{4} The exact meaning of equilibrium and disequilibrium situations and honest and dishonest government behavior will be explained later in the paper.
2. EQUILIBRIUM ANALYSIS

The wealth constraint of the central bank is

\[ a_t + bc_t = m_t, \]  

(2.1)

where \( a_t \) is the amount of private securities held by the central bank, \( bc_t \) is the government debt held by the central bank, \( m_t \) is the monetary base, and the subscript \( t \) is a time index.\(^5\) The central bank creates real balances in exchange for private securities \( a_t \) or by giving credit to the government \( bc_t \).\(^6\) The government does not have to pay interest on \( bc_t \), whereas private securities pay interest at the market rate. In order to keep things as simple as possible, we neglect other assets of the central bank such as gold or foreign currency. It is important to note that because \( m_t \) is completely determined by demand, the central bank or the government can only determine the division between \( a_t \) and \( bc_t \), but not directly the total amount of \( m_t \). In fact, \( m_t \) can be interpreted as an interest-free credit of the private sector to the central bank. The central bank gives \( bc_t \) as an interest-free credit to the government and buys private securities to the amount of \( a_t (= m_t - bc_t) \).\(^7\) The wealth constraint of the government is

\[ bp_t + bc_t = \Omega_t, \]  

(2.2)

where \( bp_t \) is the government debt held by the private sector and \( \Omega_t \) is the total indebtedness of the government. The government is thus able to incur debt, vis-à-vis either the central bank or the private sector. The consolidated wealth constraint of the public sector is

\[ m_t + bp_t - a_t = \Omega_t, \]  

(2.3)

Total indebtedness is therefore the sum of real balances and the net interest bearing debt of the government vis-à-vis the private sector \( (bp_t - a_t) \). We will speak of an equilibrium situation as long as \( bp_t \), \( a_t \), and \( m_t \) are unchanged, which means that the wealth constraint of the central bank remains unchanged. The concentration on this equilibrium case allows us to analyze first of all some basic features of seigniorage. The flow constraint of the central bank is

5. Variables in small letters are variables relative to income. If there is no technical progress, small letter variables can be seen as per capita variables. Capital letters are nominal variables. Variables with a dot are differentiated with respect to time.

6. \( a \) can also be seen as an amount of indexed government bonds originally held by the private sector and later transferred to the central bank via open market operations.

7. The conclusions about total seigniorage do not depend on the fact that the government does not have to pay interest on this credit. If the government had to pay interest on this credit, it would pay it to itself via the central bank.
\[(\pi_t + n)m_t + (r - n)a_t - \tau_t = 0, \tag{2.4}\]

where \(\tau\) is the final transfer to the government, \(n\) is the growth rate of real income and \((\pi_t + n)m_t = \mu mM\). In the equilibrium case, we can consider \(\pi\) and \(\mu\) as constant. The income of the central bank stems from the possibility to increase nominal money by \(\mu mM\) without changing \(m\), which yields \(\mu mM\), and from the earnings on its assets \(a\), which yields \(i\alpha\). The central bank needs \((\pi_t + n)a\) in order to keep its asset position unchanged. In an economic sense, \((\pi_t + n)a\) is not revenue for the central bank. The cash fund transfer from the central bank to the government is

\[\mu m_t + (r - n)a_t, \tag{2.5}\]

which is in this case equal to the final transfer. As long as \(m_t\) and \(bc_t\) are to remain unchanged, the central bank is not able to transfer anything other than the final transfer \(\tau_t\). We will see in the disequilibrium case that the central bank transfers to the government can differ from \(\tau\), but this involves credit changes and not actual seigniorage.

The usual budget identity of the government is written as follows

\[g_t - t + (i - \pi_t - n)b_p = b_p + (\pi_t + n)m_t + (r - n)a_t, \tag{2.6}\]

where \(g_t\) is the government consumption and \(t\) are taxes. As long as there is no possibility of confusion, we will drop the time index. A crucial point in our analysis is to rewrite the budget or flow constraint of the government as

\[g - t + ibp + ibc = b_p + (\pi + n)b_p + (\pi + n)bc + (i - \pi - n)bc + (\pi + n)m + (r - n)a, \tag{2.7}\]

where the left-hand side of the equation

\[g - t + ibp + ibc \tag{2.8}\]

is the financing requirement of the government net of ordinary taxes. Thus, the left-hand side is independent of any central bank financing. It reflects a situation where the government would have to pay the nominal interest on its total debt. An inspection of the right-hand side shows that this financing requirement is met by three different funding origins. The first funding origin is the increase in government debt held by the private sector.

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8. In nominal terms, the transfers from the central bank have a different interpretation. \(iA\) are the final transfers to the government, whereas \((\pi + n)BC\) is the increase in the nominal interest-free credit.
In a steady state, $bp$ is obviously zero. The second funding origin can be called *bond-seigniorage* and amounts to

$$ (\pi + n)bp + (\pi + n)bc = (\pi + n)\Omega. \quad (2.10) $$

Bond-seigniorage means that the government can issue a certain amount of bonds without changing the economically relevant per capita real debt situation. The source of bond-seigniorage is thus inflation and growth. If total real debt per capita is kept constant, the government earns a bond-seigniorage to the amount of $(\pi + n)\Omega$. So the effective cost of the total debt is not $i\Omega$, but only $(i - \pi - n)\Omega$. Bond-seigniorage is independent of the division of the total debt. The government receives the same amount of bond-seigniorage for every composition of debt.

*Money-seigniorage* is the third funding origin. According to our definition in the introduction, money-seigniorage is the sum of all government revenue due to its base money monopoly. More specifically, in the present equilibrium context, we define money seigniorage as the government income which can be maintained as a constant flow as a result of the government monopoly in money creation. Money-seigniorage corresponds to the sum of all terms on the right-hand side of equation (2.7) which do not belong to the first two funding origins. Money-seigniorage thus amounts to

$$ (i - \pi - n)bc + (\pi + n)m + (r - n)a. \quad (2.11) $$

In the remainder of the paper, we are only interested in the analysis of money seigniorage, i.e., whenever the term seigniorage is not specified, we always mean money-seigniorage. We are now able to split up money-seigniorage into different aspects of seigniorage and to check whether equation (2.11) does in fact include only revenue due to the base money monopoly. The first distinction we make is the distinction between cash and non-cash seigniorage. *Cash seigniorage* is that part of seigniorage which is transferred in cash from the central bank to the government. In the present equilibrium context, cash seigniorage is identical to the cash flow fund transfer from the central bank to the government

$$ (\pi + n)m + (r - n)a. \quad (2.12) $$

*Non-cash seigniorage* is the complement to cash seigniorage and amounts to
\[(i - \pi - n)bc = (r - n)bc. \quad (2.13)\]

Non-cash seigniorage reflects the net opportunity cost revenue from the interest-free credit of the central bank to the government. The partition between cash and non-cash seigniorage depends on the asset structure of the central bank. If the entire money base is created by purchases of private securities, then cash seigniorage is equal to total seigniorage and non-cash seigniorage is zero. If the monetary base is created entirely by giving interest-free credit to the government, then cash seigniorage amounts to 
\[(\pi + n)m \quad \text{and non-cash seigniorage is } (r - n)m.\]

In addition to this liquidity distinction of seigniorage, we can distinguish between two different sources of seigniorage. DRAZEN (1985) was the first to mention that seigniorage sprang from two different sources. The distinction of different sources helps to clarify the nature of seigniorage. In his terminology, the first source is the taxation source (source I): revenues from expansion of nominal money when the private sector already holds a given level of real balances. His second source is the monopoly source (source II): profits earned from the monopolistic supply of the monetary base due to the desire of the private sector to hold given real balances. This terminology of seigniorage sources may be somewhat misleading, because ultimately both sources stem from the base money monopoly of the government. For this reason, we prefer to simply speak of sources I and II in the remainder of this paper.

**Seigniorage from source I** is thus the amount of revenue resulting from the emission of new nominal money

\[(\pi + n)m = \mu m. \quad (2.14)\]

As DRAZEN (1985) notes, this is the amount the private sector must pay to continue consuming a given amount of real balances \(m\). The question behind the definition of seigniorage from source II is how the outstanding quantity of real balances itself affects the revenue situation of the government. First of all, we note that the central bank gets interest from the private sector. The central bank, however, needs \((\pi + n)a\) in order to keep the value of a constant in the face of inflation and growth. So \((r - n)a\) is net revenue for the central bank. In addition to that revenue, there is opportunity cost revenue from the interest-free credit the government gets from the central bank to the amount of \(ibc\). \((\pi + n)bc\) is needed to keep the value of this credit constant. In fact, this equals the bond-seigniorage the government gets anyway, independent of the interest-free credit from the central bank. Therefore, the net opportunity cost revenue for the government is 
\[(i - \pi - n)bc. \quad \text{Seigniorage from source II amounts to}\]

\[(i - \pi - n)bc + (r - n)a = (r - n)m. \quad (2.15)\]
Our definition of the amount of seigniorage from source II differs from DRAZEN’s (1985). DRAZEN only considers \((r - n)a\) as seigniorage from source II. He just includes the actual revenue resulting from the acquisition of private assets by the monetary authority, rather than potential profits. However, the basis of source II seigniorage is the monopoly of the public sector for base money. Seigniorage from source II is the net opportunity cost revenue from the monetary base, that is, from the interest-free credit the central bank gets from the private sector. The exact manner in which the monetary base is created, either by purchasing private securities or by extending an interest-free credit to the government, is irrelevant.\(^{10}\) Total seigniorage is the sum of either cash seigniorage and non-cash seigniorage or the sum of seigniorage from sources I and II:

\[
\sum_{\text{source I}} \mu m + (r - n)a + \sum_{\text{source II}} (r - n)bc = im. \tag{2.16}
\]

Total seigniorage is the income flow resulting from the interest-free credit of the private to the public sector. Another way to analyze the amount of total seigniorage is to take a tax perspective (see also PHELPS [1973]). We compare the amount of taxes between a situation with \(t_1\) and one without \(t_2\) a government base money monopoly which is necessary to keep the total debt \(\Omega\) unchanged, holding the other variables constant. The budget constraint with a base money monopoly is

\[
g - t_1 + ibp + ibc = (\pi + n)(bp + bc) + im, \tag{2.17}
\]

whereas, without a base money monopoly, it is

\[
g - t_2 + ibp + ibc = (\pi + n)(bp + bc). \tag{2.18}
\]

The difference in taxes is

\[
t_2 - t_1 = im. \tag{2.19}
\]

This leads exactly to the result above (Eq. [2.16]). We conclude that in an equilibrium situation, the seigniorage measure \(im\) includes the total amount of seigniorage, whereas the seigniorage measure \(\mu m\) only reflects part of seigniorage revenue. However, either

\(^{10}\) DRAZEN’s general measure of seigniorage includes the revenue from expansion of the nominal supply of base money \(\mu m\) plus actual revenue resulting from the acquisition of private assets by the monetary authority \((r - n)a\). This measure is questionable and misleading, because it excludes potential profit. Therefore, the traditional opportunity cost measure of seigniorage, \(im\), is viewed by him as a special measure relevant only for the case where all changes in money supply are brought about via open market operations.
measure individually is not very informative, because either reflects different aspects of seigniorage which are not specified by one measure alone. It is also important to note that it is illegitimate to take the sum of both measures in order to capture the size of total seigniorage as it is sometimes done.\textsuperscript{11} By doing this, one creates a confusion between seigniorage proper and credit and asset changes.

One such confusion arises in NEUMANN (1992). In our notation, his definition of an extended monetary seigniorage translates to $\mu m + ia$. This amounts to the addition of $\mu m$ and $im$ if the entire monetary base is created by open market operations. In any case, the measure of the extended monetary seigniorage, on the one hand, overstates the cash seigniorage revenue from interest bearing assets ($ia$ instead of $(r-n)a$) but, on the other hand, neglects non-cash seigniorage $((r-n)bc)$. The source of the main differences in the interpretation of seigniorage between this paper and KLEIN and NEUMANN (1990) as well as NEUMANN (1992) is the «nominal» formulation used by KLEIN and NEUMANN.\textsuperscript{12} By a nominal formulation we mean a formulation where the variables are defined in nominal terms and are made real with the division by the price level only at the end, rather than a formulation with variables defined relative to income to begin with. Using a nominal formulation blurs different aspects of seigniorage and leads to a confusion between seigniorage proper and fund transfers from the central bank to the government. As we will see in the next section, the distinction between equilibrium and disequilibrium situations is important and necessary for clarifying this point.

3. DISEQUILIBRIUM ANALYSIS

We now allow changes in the wealth constraint of the central bank and assume that the central bank can determine its composition of assets. The distinction between equilibrium and disequilibrium situation is crucial for the analysis of seigniorage. However, this distinction is neglected in the literature. In a disequilibrium situation, fund transfers from the central bank to the government incorporate a component which is not part of seigniorage revenue. This component reflects the change in the debt situation between the government and the central bank.

3.1 A Pure Asset Change of the Central Bank

We assume a situation, where real money balances ($m = 0$) and the inflation rate ($\pi = \bar{\pi}$) remain unchanged, but the central bank changes its asset composition ($\hat{a} \neq 0$). The central bank now has two flow constraints. One flow constraint is

\textsuperscript{11} For this aspect see also JOHNSON (1969).
\textsuperscript{12} In KLEIN and NEUMANN (1990) the emphasis is on the distribution of seigniorage.
\[(\pi + n)m + (r - n)a - \dot{a} + \dot{bc} - \tau = 0, \quad (3.1)\]

and the other flow constraint

\[\dot{a} = -\dot{bc} \quad (3.2)\]

stems from the fact that the central bank changes only the composition of the asset side of its balance sheet and leaves real money balances unchanged \((\dot{m} = 0)\). This implies that the final transfer to the government is

\[\tau = \mu m + (r - n)a, \quad (3.3)\]

and that the cash fund transfer to the government is

\[\tau + \dot{bc} = \mu m + (r - n)a + \dot{bc}. \quad (3.4)\]

The final transfer to the government remains untouched by the asset change. However, the cash fund transfer is influenced by the amount of the change in the interest-free credit. The government can increase this credit or else must repay the credit depending on the sign of \(bc\). The budget constraint of the government changes to

\[g - t + ibp + ibc = bp + (\pi + n)bp + (\pi + n)bc + (i - \pi - n)bc + (\pi + n)m + (r - n)a + \dot{bc}. \quad (3.5)\]

The change in the asset position of the central bank leads to a change in the credit to the government. So the cash fund transfer to the government is influenced by this credit change. Based on the above discussion, we can state that the total amount of seigniorage is constant over time and remains \(\mu m\), as long as \(m\) is unchanged, but the fund transfer from the central bank to the government and the amount of cash and non-cash seigniorage change with the asset composition of the central bank. Seigniorage from sources I or II, however, remains unchanged, because it does not depend on asset composition. It would be very misleading to take the cash fund transfer as an indicator for seigniorage.

13. As long as \(\dot{bc}/\dot{bc} > -(\pi + n)\), we have \(\dot{BC}/\dot{BC} > 0\), and in this case, the nominal interest-free credit does not diminish.
3.2 A Change in the Demand for Money

3.2.1 Honest Government

We will now go a step further and analyze a situation where real money balances change. In this case, the liability side of the central bank's balance sheet is also affected. Money demand is a function \( m = m(\pi, \ldots) \). It depends on the long-run inflation rate and other factors. We assume that the government sets the inflation rate and lets \( m \) adjust in order to avoid jumps in the price level. This is the idea of AUERNHEIMER's (1974) definition of an honest government. A change in the demand for money can be caused either by a change in the inflation rate set by the government or by an exogenous influence, e.g., a change in the productivity of the payment sector.

A first flow constraint of the central bank is

\[
(\pi + n)m + \dot{m} + (r - n)a - \dot{\alpha} - bc - \tau = 0, \tag{3.6}
\]

where \( \tau \) is the final transfer to the government and \( (\pi + n)m + \dot{m} = \mu m \). The second flow constraint is

\[
\dot{m} = \dot{\alpha} + bc. \tag{3.7}
\]

This implies that the final transfers are

\[
\tau = (\pi + n)m + (r - n)a. \tag{3.8}
\]

The government budget identity is

\[
g - t + ibp + ibc = \dot{bp} + (\pi + n)bp + (\pi + n)bc + (i - \pi - n)bc + (\pi + n)m + (r - n)a + \dot{bc}. \tag{3.9}
\]

On the left-hand side, we have the financing requirement. On the right-hand side, the terms for the change in the amount of government debt held by the public and for the bond-seigniorage remain unchanged. So we are again able to distinguish between the fundamental aspects of seigniorage. The cash fund transfer from the central bank to the government amounts to

14. Obviously, the amount of bond-seigniorage changes with the amount of total debt.
\[
\tau + \dot{bc} = (\pi + n)m + (r - n)a + \dot{bc}.
\] (3.10)

Cash seigniorage is that part of the cash fund transfer which does not result from a change in the interest-free credit

\[(\pi + n)m + (r - n)a,\] (3.11)

and non-cash seigniorage is

\[(i - \pi - n)bc.\] (3.12)

Seigniorage from source I yields

\[(\pi + n)m,\] (3.13)

which is not equal to \(\mu m\) as long as \(\dot{m} \neq 0\). Seigniorage coming from source II amounts to

\[(i - \pi - n)bc + (r - n)a = (r - n)m.\] (3.14)

Total seigniorage (the sum of cash and non-cash seigniorage or the sum of seigniorage from sources I and II) yields \(im\). However, with a change in the real money balance \(m\) every aspect of seigniorage, the total amount of seigniorage, and the fund transfer from the central bank change over time, as long as \(\dot{m} \neq 0\). The exact change of every aspect also depends on the change of \(a\) and \(bc\) over time. It is important to note that \((\pi + n) \neq \mu\), because the government has to adjust the amount of real balances in order to avoid a jump in the price level. Seigniorage from source I is \((\pi + n)m\) and not \(\mu m\), because the change in the amount of real balances is a change in the interest-free credit of the private sector to the central bank, but not a seigniorage loss or gain. Of course, the size of this credit volume determines the amount of seigniorage. The change in this credit by itself, however, has nothing to do with seigniorage.

The difference in seigniorage revenues after a change in the inflation rate and or an exogenous change in the demand for real balances is

\[\pi_{0+}m_{0+} - \pi_{0-}m_{0-} + n(m_{0+} - m_{0-}) + (r - n)(a_{0+} - a_{0-}) + (r - n)(bc_{0+} - bc_{0-}).\]

(3.15)
where $x_{0+}$ is the corresponding variable after and $x_{0-}$ is the variable before the change. The change in total seigniorage thus depends on the change in money demand and in the inflation rate. The change in the different aspects of seigniorage further depends on the exact manner in which the change in money demand is managed.

### 3.2.2 Dishonest Government

As in the honest government case, we assume that the real money demand depends on the long-run inflation rate which is equal to $\pi^* = \mu - n$, as well as on other factors. A change in the demand for money can be caused by either a change in the long-run inflation rate or an exogenous influence. The dishonest government sets the growth rate of nominal money and, consequently, the inflation rate in the long-run. However, the dishonest government does not adjust $m$ in order to avoid jumps in the price level. Ergo, a decrease in the money demand leads to excess inflation in the short run in order to destroy the undesired part of real money balances. In the short run, inflation is higher than in the honest government case.

For the analysis of the situation of a dishonest government, we concentrate on a situation with $\dot{m} < 0$, $\pi \geq \mu$, and $\dot{m} = \dot{b}c$. One flow constraint of the central bank is

$$\mu m + (r - n)a + \dot{m} - \dot{b}c - \tau = 0,$$

(3.16)

where $\tau$ is again the final transfer to the government. The other flow constraint, of course, is

$$\dot{m} = \dot{b}c.$$ 

(3.17)

This is to say that the decrease in real money balances is reflected in a decrease in the interest-free credit to the same amount. This implies that the cash fund transfer from the central bank to the government amounts to

$$\tau = \mu m + (r - n)a.$$

(3.18)

Beside this cash fund transfer from the central bank, there is a non-cash fund transfer from the central bank to the government amounting to

15. In a situation with $-\dot{m} > \dot{b}c$ we end with $a > m$, so that an extra gain $a - m$ can be exploited by selling $a - m$ and transferring this gain to the government. The interpretation of the following discussion does not change in such a case. In a situation with $-\dot{m} > 0$, there is no incentive for the government to behave dishonestly.
This is the loss in the real value of the interest-free credit from the central bank to the
government or in the real value of the interest-free credit from the private sector to the
public sector. The budget constraint of the government is now\textsuperscript{16}

\begin{equation}
g - t + ibp + ibc = bp + \pi bp + (\pi + n)bc + (i - \pi - n)bc + \mu m + (r - n)a - bc + bc. \tag{3.20}
\end{equation}

Cash seigniorage is

\begin{equation}
\mu m + (r - n)a \tag{3.21}
\end{equation}

and non-cash seigniorage is

\begin{equation}
(i - \pi + n)bc - bc. \tag{3.22}
\end{equation}

Seigniorage from source I yields

\begin{equation}
\mu m, \tag{3.23}
\end{equation}

which is not \((\pi + n)m\), since \(\pi + n \neq \mu\). Seigniorage from source II amounts to

\begin{equation}
(i - \pi - n)bc + (r - n)a - bc = (r - n)m - \hat{m}. \tag{3.24}
\end{equation}

Seigniorage from source II yields an extra gain to the amount of \(-\hat{m}\), because the real
value of the credit to the public sector diminishes, ceteris paribus, by \(\hat{m}\). In order to see
the difference to the honest government case, we note that (given a change in the inflation
rate and the long-run growth rate of nominal money)

\begin{equation}
(\pi_H + n)m_H + \hat{m}_H = \mu_H m_H, \tag{3.25}
\end{equation}

where \(H\) stands for honest government and

\textsuperscript{16} At the point of change, \(\pi\) and \(i\) are higher than under an honest government regime. However, the cost
of government debt does not increase, because bond-seigniorage increases proportionally. From this
perspective, the government is not concerned about a higher inflation rate.
\( (\pi_H + n) = \mu_D, \) \hspace{1cm} (3.26)

where \( D \) stands for dishonest government. Further, we note that

\[ m_H = m_D, \quad \dot{m}_H = \dot{m}_D, \quad \text{and} \quad \dot{m}_{H,D} = \dot{b}c_{H,D}. \] \hspace{1cm} (3.27)

The sum of cash and non-cash seigniorage in the case of the dishonest government now amounts to

\[ (\pi_H + n)m + (r-n)m - \dot{m}. \] \hspace{1cm} (3.28)

The dishonest government makes an extra gain of seigniorage in comparison to the honest government in the order of \(-\dot{m}\), because the inflation rate overshoots to decrease real balances. This overshooting is necessary to destroy part of the real balances. Total seigniorage is substantially larger because the government gets an additional non-cash transfer from the central bank during the transition phase, which equals the change in the value of the interest-free credit. The decrease in the interest-free credit is thus an extra gain for the government, because there is no corresponding increase in another kind of liability.

4. COMPARATIVE STATICS AND MAXIMIZATION OF SEIGNIORAGE

We look first at the comparative static difference in total seigniorage for two different inflation rates or growth rates of nominal money. The asset composition of the central bank is not crucial for this analysis. The net present value of seigniorage for the government and for \( \mu_1 \) and \( m_1 \) is

\[ V_1 = \int_{t=0}^{\infty} \mu_1 m_1 e^{-(r-n)t} dt + \int_{t=0}^{\infty} (r-n)m_1 e^{-(r-n)t} dt, \] \hspace{1cm} (4.1)

and for \( \mu_0 < \mu_1 \) and \( m_0 > m_1 \)

\[ V_0 = \int_{t=0}^{\infty} \mu_0 m_0 e^{-(r-n)t} dt + \int_{t=0}^{\infty} (r-n)m_0 e^{-(r-n)t} dt. \] \hspace{1cm} (4.2)

In the honest government case, the difference amounts to
This is equivalent to a difference of

$$\mu_1 m_1 - \mu_0 m_0 + (r-n)(m_1 - m_0)$$

per period, which is identical to equation (3.15). The first term corresponds to the additional revenues gained by increasing the nominal amount of money. The second term reflects the costs arising from the decrease in the interest-free credit to the public sector. This is the same result as in AUERNHEIMER (1974). However, there is a difference in the deduction and the interpretation of the result. AUERNHEIMER (1974) takes only seigniorage from source I into account and adds the change in the demand for real balances as additional costs or revenues. We look here at the effective total stream of seigniorage. The open market operation necessary for adjusting real balances does not embody a cost or revenue factor.

However, the future stream of interest resulting from a change in real balances is the actual cost or revenue factor.

If we look at the dishonest government case, we have to add the one time extra gain from the jump in the price level $m_1 - m_0$. Thus, the comparative static difference for seigniorage amounts to

$$\frac{\mu_1 m_1}{r-n} - \frac{\mu_0 m_0}{r-n} + (m_1 - m_0) - (m_1 - m_0) = \frac{\mu_1 m_1}{r-n} - \frac{\mu_0 m_0}{r-n}.$$  

This is equivalent to a difference of

$$\mu_1 m_1 - \mu_0 m_0$$

per period.

We now consider the maximization of seigniorage. This question has been discussed intensively by AUERNHEIMER (1974) and by FRIEDMAN (1971). In the honest government case, the government maximizes

$$\max_{\pi} \int_{t=0}^{\infty} (\mu m + (r-n)m)e^{-(r-n)t}dt.$$  

The first order condition is
\[
\frac{d\left( \int_{t=0}^{\infty} (\mu m + (r-n)m)e^{-(r-n)t} dt \right)}{d\pi} = 0.
\] (4.8)

If we assume a demand for real money as \( m = e^{-\alpha(r+\pi)} \), we get the result that seigniorage revenues are maximized at an inflation rate of

\[
\pi^* = \frac{1}{\alpha} - r.
\] (4.9)

As long as the inflation rate is smaller than \( \pi^* \), seigniorage can be increased. The maximizing inflation rate is, in this case, independent of the growth rate (AUERNHEIMER [1974]). The dishonest government, on the other hand, maximizes

\[
\max_{\pi} \int_{t=0}^{\infty} (\mu m + (r-n)m)e^{-(r-n)t} dt + (m_0 - m).
\] (4.10)

The first order condition is

\[
\frac{d\left( \int_{t=0}^{\infty} (\mu m)e^{-(r-n)t} dt \right)}{d\pi} = 0.
\] (4.11)

In this case, the maximizing inflation rate is

\[
\pi^* = \frac{1}{\alpha} - n,
\] (4.12)

which is now dependent on the growth rate (FRIEDMAN [1971]). The difference in these results stems from the fact that the dishonest government is, following an increase in the inflation rate, always compensated for the additional interest costs by an extra gain to the amount of the same net present value. The dishonest government can thus maximize the stream of seigniorage from source II, which depends on the growth rate but not on the real interest rate. The honest government, on the other hand, maximizes the total amount of seigniorage. This amount is independent of the growth rate, but depends on the real interest rate.
5. CONCLUSIONS

Discussions of seigniorage are marred by a certain confusion about the appropriate way to measure seigniorage. It was the aim of this paper to discuss some fundamental aspects of seigniorage and to integrate different measures of seigniorage suggested in the literature. We also analyzed the measuring of seigniorage in disequilibrium situation, which is usually completely neglected in the literature. Furthermore, the paper looked at the comparative statics of seigniorage revenues for different inflation rates and at the maximization of seigniorage. For our analysis, we found it necessary to distinguish between fund transfers and actual seigniorage, between different sources of seigniorage, and between different liquidities of seigniorage, as well as between equilibrium and disequilibrium situations, and between honest and dishonest government behavior.

We conclude that the opportunity cost concept of seigniorage im includes the total amount of seigniorage in most cases. An exception is the dishonest government case, where the government gets additional seigniorage revenues in the amount of \(-\dot{m}\). However, the monetary seigniorage concept \(\mu m\) is incomplete and either overstates or underrates seigniorage revenue depending on the situation. Either concept alone is not very meaningful, because neither fully reflects all the different aspects of seigniorage. To obtain a precise picture of the seigniorage situation, it is necessary to distinguish between actual seigniorage and the fund transfer from the central bank to the government.

In order to simplify the analysis and to concentrate on the basic aspects of seigniorage, we abstracted from a number of issues which could be important for a more detailed and applied analysis of seigniorage. For example, we totally ignored inflation surprises. Allowing for inflation surprises could permit the government to also profit from changes in the real value of its outstanding interest-earning debt. Empirically, this can be very important. Another simplification was to disregard bank deposits and reserve requirements on deposits. Reserve requirements can be important determinants of the amount of central bank money which is held by the private sector. While these are interesting issues for an extended analysis they do not, however, affect the validity of the basic results on seigniorage reported in the present paper.

REFERENCES


**SUMMARY**

Traditionally, the literature on seigniorage has mentioned different concepts of measuring seigniorage, most prominently the monetary concept and the opportunity cost concept. The relationship between different concepts of seigniorage has never been settled clearly. This paper discusses principal aspects of seigniorage and clarifies the relationship between alternative measures of seigniorage. The conclusion is that the opportunity cost concept includes the total amount of seigniorage, whereas the monetary concept is incomplete. However, either concept alone is not very informative, because neither fully reflects all the different aspects of seigniorage. To obtain a precise picture of the seigniorage situation, it is crucial to make a distinction between actual seigniorage and the fund transfer from the central bank to the government.

**ZUSAMMENFASSUNG**

Opportunitätskosten-Konzept die gesamte Seigniorage erfasst, während das Konzept der monetären Seigniorage unvollständig ist. Beide Konzepte sind aber für sich alleine wenig informativ, weil sie die verschiedenen Aspekte der Seigniorage nicht genau zum Ausdruck bringen. Um ein klares Bild der gesamten Seignioragesituation zu erhalten, ist es wichtig, zwischen tatsächlicher Seigniorage und dem Mittelfluss von der Notenbank zur Regierung zu unterscheiden.

RESUME

Traditionnellement, la littérature sur le seignorage mentionne plusieurs concepts différents pour mesurer le seignorage dont le concept monétaire et le concept de coûts d'opportunité sont les plus éminents. La relation entre les différents concepts de seignorage n'a jamais été analysée à fond. Cet article discute les aspects principaux du seignorage et clarifie la relation entre les moyens alternatifs de mesurer le seignorage. La conclusion montre que le concept de coûts d'opportunité inclut la somme totale du seignorage tandis que le concept monétaire n'est pas complet. Les deux concepts, considérés séparément, ne sont pas très informatifs parce qu'ils ne reflètent pas entièrement tous les aspects différents du seignorage. Pour obtenir une image précise de la situation du seignorage il est important de faire une distinction entre le seignorage en-soi et le transfert de fonds de la banque centrale au gouvernement.