The Informational Role of Volume Vis-à-Vis No-Trade Results: A Note

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1. INTRODUCTION

The informational role of trading volume in financial markets is not well understood. On the one hand, a large amount of empirical research has shown several regularities between volume and prices in financial markets, such as a positive relation between volume and volatility of price changes as well as between volume and the absolute value of price changes (see e.g. GALLANT, ROSSI and TAUCHEN, (1992, 1993); HSU (1996); KARPOFF (1987)), or a negative relation between volume and the serial correlation in price changes (e.g. CAMPBELL, GROSSMAN and WANG (1993); LEBARON (1992)). On the other hand, it is very difficult to derive these empirical regularities in theoretical models (some examples are WANG (1993, 1994); HSU (1996); OROSEL (1997)). In the context of rational expectations models with private information one reason for this difficulty are the so-called «no-trade theorems» (MILGROM and STOKEY (1982); see also VARIAN (1989)), which imply that differences in private information alone cannot give rise to trade among rational agents. That is, either there are *non-informational* reasons for trade or there is no trade. Basically this result is due to the fact that if differential information is the only reason for trade and a rational agent wants to trade a certain asset, this agent must have superior private information. In such a situation the less informed trading partner would loose from this trade, and knowing this will not trade. This implies that if differential private information is the only potential reason for trade, there is no trade in equilibrium, i.e., trading volume is always zero. This is one reason, among others, why rational expectations models that deal with private information in financial markets, generally assume either random endowments or the existence of so-called «noise traders». These noise traders are not rational; their trades are given by exogenous random variables.

The no-trade theorems assume that all agents are fully rational. It is not clear what type of boundedly rational – but plausible – behavior would avoid no-trade results. In a recent article Blume, Easley and O'Hara (1994) introduce a purely informational model where agents are not fully rational and the no-trade results seem to fail. The aim

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of their model is to investigate the informational role of trading volume when there is private information. In their setup, agents receive private signals, which differ in precision, about an asset's value. Moreover, the agents are not fully rational. Rather, it is assumed that when selecting their trades agents do not use the information contained in the contemporaneous price although they observe this price and, in fact, need to observe it because their demand (but not their expectations) depends on this price. The unique feature of the model is that volume would convey important information to uninformed agents. Thus, the model seems to circumvent the no-trade results and to provide an explanation of pervasive activity of technical analysis on price and volume.

In their introduction BLUME, EASLEY and O'HARA (1994) – hereafter quoted as BEO – assert the following: *«We show that although all traders will learn the asset's value, and prices will thus converge to the full information or strong form efficient price, volume does not converge to zero. In fact, volume has a limit distribution that is nondegenerate. This demonstrates that markets do not shut down as beliefs converge and has the important implication that the «no-trade» equilibrium results so prevalent in the literature may not describe the limit behavior of equilibrium models with learning.»* (BEO, p. 155)

Thus, it seems that the no-trade theorems are not valid any more, if full rationality is substituted by a certain type of bounded rationality. A model that combines the assumptions of private information and boundedly rational agents appears to be the right vehicle to investigate the informational role of volume.

In this note we show that this is not true. Contrary to the quotation given above, no-trade theorems, which we derive below, do apply to the BEO model although agents are only boundedly rational. Therefore, in the BEO model volume is always zero and consequently cannot play any informational role. Given that the only equilibrium is one without trade, the BEO model cannot be used to describe volume or traders learning from volume.

The note is organized as follows. Section 2 summarizes the relevant sections of the BEO model. Section 3 consists of two subsections. In Subsection 3.1 we accept, for the sake of the argument, the demand functions of the BEO model for those agents who participate in the market. However, we consider the agents' decisions whether or not to participate in the market and show that in an equilibrium no agent will participate. Thus, in equilibrium trading volume is zero. In Subsection 3.2 we don't force the participating agents to behave according to the demand functions of the BEO model. Rather, we analyze their demand directly assuming only that they are boundedly rational in the sense that they do not infer any information from the contemporaneous price. We show that, again, in equilibrium there is no trade, i.e., trading volume is zero. The Appendix contains the mathematical proofs.

2. THE BEO MODEL

The BEO model for the first period[1] can be summarized as follows. Consider an economy with two assets, risk-free and risky, in fixed supply. The value of the risky asset is a normally distributed random variable ψ with mean ψ_0 and variance $1/\rho_0$, i.e. $\psi \sim N(\psi_0, 1/\rho_0)$.[2] All agents begin with identical beliefs. Further, each agent receives a signal on the value of the asset. There are two types of agents differing in the precision of the signal they receive: Type 1 (informed) agent i, $i = 1, \ldots N_1$, receives a signal[3]

$$y^{1i} = \psi + \omega + e^i$$

whereas type 2 (uninformed) agent $i = 1, \ldots N_2$, receives a signal

$$y^{2i} = \psi + \omega + \varepsilon^i$$

where ω is a common error distributed as $\omega \sim N(0, 1/\rho_\omega)$, e^i and ε^i represent an idiosyncratic error of group 1 and group 2 traders, respectively. The distribution of e^i and ε^i are $e^i \sim N(0, 1/\rho^1)$ and $\varepsilon^i \sim N(0, 1/\rho^2)$. BEO assume that $\rho^1 \in (\rho^2, \rho_\omega)$, i.e., the signals group 1 agents receive are more precise than those of group 2 agents.[4] Further, with exception of ρ^1, all the parameters are publicly known; ρ^1 is known by group 1 agents, but not by group 2 agents. All the random variables are assumed to be independent. All agents understand the model and maximize their expected utility. The utility function is identical for all agents and has the form:

$$U(W_i) = \exp(-W_i) \tag{1}$$

where W_i is agent i's terminal wealth. Each agent is endowed with N_0 units of the riskless asset and zero units of the risky asset.[5]

Define $\rho^{s1} := \rho_\omega \rho^1 (\rho_\omega + \rho^1)^{-1}$ and $\rho^{s2} := \rho_\omega \rho^2 (\rho_\omega + \rho^2)^{-1}$. BEO assume that when selecting their trades agents do not use the information contained in the price. As a

1. For simplicity we deal only with the single-period case. But it is obvious that our results are also relevant for the multiperiod case since in the BEO model the multiperiod case is just a sequence of single-period cases to each of which our analysis applies.
2. We use BEO's notation, but for simplicity we omit the subscripts which refer to period 1 or t respectively, i.e. we write p instead of p_1, ρ^1 instead of ρ^1_t etc.
3. To distinguish signals received by agents in different groups, the number of the group is added in the superscript.
4. See BEO p. 171, last sentence. We have omitted the time index t.
5. On page 162, BEO state that *«Each trader begins with zero endowment of the risky asset and some exogenous endowment, N_0, of the riskless asset»*. However, the last part of the second sentence of Section II (BEO, p. 161), *«there is no exogenous supply of any asset»*, is not consistent with $N_0 > 0$ and should read: there is no exogenous supply of the risky asset.

consequence, the agents are only boundedly rational because they do not use all the available information. According to BEO this implies that the demand for the risky asset of agent i in group 1 is

$$\rho_0(\psi_0 - p) + \rho^{s1}\,(y^{1i} - p) \tag{2}$$

and

$$\rho_0(\psi_0 - p) + \rho^{s2}\,(y^{2i} - p) \tag{3}$$

for agent i in group 2.[6] BEO's equilibrium price is then

$$p = \frac{\rho_0\psi_0 + \mu\rho^{s1}\,\bar{y}^1 + (1-\mu)\,\rho^{s2}\bar{y}^2}{\rho_0 + \mu\rho^{s1} + (1-\mu)\,\rho^{s2}} \tag{4}$$

where $\mu = N_1/(N_1+N_2)$ and $\bar{y}^1$ and $\bar{y}^2$ are the mean signals of group 1 and 2 agents, respectively. In the large economy, the Strong Law of Large Numbers applies, so that for each group the average signal converges to $\theta := \psi + \omega$. Hence, the equilibrium price is given by[7]

$$p = \frac{\rho_0\psi_0 + \left[\mu\rho^{s1} + (1-\mu)\,\rho^{s2}\right]\theta}{\rho_0 + \mu\rho^{s1} + (1-\mu)\,\rho^{s2}} \tag{5}$$

BEO consider a sequential economy in which agents infer information from past prices. Because ρ^1 (and thus ρ^{s1}) is unknown to group 2 agents, they cannot learn the value of θ by observing the price alone. Thus, there is a reason for group 2 agents to look at volume. In Proposition 3 (p. 172) BEO show that, given their assumptions, price and volume together reveal θ and ρ^1 .

However, in their model BEO do not take account (i) of the model's (implicit) assumption that all agents understand the model (– otherwise the agents could not draw any inferences from price or volume data); and (ii) of the fact that an agent cannot be forced to trade and, consequently, will only trade if this increases her expected utility. Because of this neglect the price derived as the equilibrium price in the BEO model, i.e. the price given by equation (5), is not market clearing.

6. Below we will argue that these demand functions are not correct for rational agents even if they are not allowed to draw any inferences from the contemporaneous price p .
7. Equations (2), (3), (4) and (5) are identical to the equations (9), (10), (11) and (12) in BEO, respectively.

In the following, we will prove two results. For the first result we take account of the restriction that an agent will participate in the market only if she expects a non-negative gain from trade. Adopting for those agents who do participate the demand functions derived by BEO, we show that one can never get the results of the BEO model, since volume is never positive.

For the second result we argue that because the agents understand the model their demand functions are different from those derived by BEO, *although no agent infers any information from the contemporaneous price.* We consider a certain, rather general class of equilibria, which includes the equilibrium proposed by BEO, and show that in this class there is no equilibrium with positive trade.[8] These two results imply that the BEO model – though sophisticated in many respects – cannot explain how volume behaves or how traders learn from volume data or perform volume based technical analysis.

Although the no-trade theorem of Milgrom and Stokey (1982) is not relevant for the BEO model, since agents are only *boundedly* rational, our results prove that no-trade theorems hold nevertheless for the BEO model. This is easy to understand intuitively. Since in the BEO model all agents have identical endowments and preferences, there are no mutual gains from trade. The initial allocation of risk is already efficient and the aggregate profit from trade is always zero. Therefore, if some agents have positive expected profits from trade, there must exist other agents who expect a negative profit from market participation. Consequently, the latter agents will not participate and the situation cannot be an equilibrium. Hence, expected profits from trade must be zero for all agents. But then, since each agent is risk-averse and has a risk-free initial endowment, no agent will participate in the market and volume is zero. In the following we spell out this argument more rigorously.

3. THE NON-EXISTENCE OF EQUILIBRIUM WITH POSITIVE VOLUME

In the subsequent analysis we adhere strictly to all the assumptions of the BEO model, in particular to the assumption that the agents do not use the information contained in the contemporaneous price. However, of these assumptions only two are crucial for our (no-trade) results, (i) that all agents understand the model, and (ii) that no agent can be forced to trade. In subsection 3.1 we accept, for the sake of the argument, that demand for the risky asset is given by (2) and (3) respectively, as postulated by BEO. Then, the case of agents who are boundedly rational in the sense that they are rational except that they do not use any information contained in the (contemporaneous) price will be considered in subsection 3.2. In both cases we will derive no-trade results.

8. Our conjecture is that there exists no equilibrium at all with positive trade. However, for this note it would be excessive to investigate very complicated cases for which one cannot get an explicit solution.

3.1. EQUILIBRIUM WITH BEO DEMAND FUNCTIONS

We assume that for each agent who trades, i.e. participates in the market, demand is given by (2) and (3) respectively. Since all agents are risk-averse and have a risk-free initial endowment, a necessary (but not sufficient) condition for an agent to trade is that the expected profit from market participation is positive.

It makes a difference whether an agent has to decide about her market participation *before* or *after* she has observed the signal. In the BEO model it seems more appropriate to assume that an agent decides about market participation *after* having observed the signal. However, for completeness we also have to consider the case where agents must decide about market participation *before* the signal is observed. Otherwise we would not know whether or not the results of the BEO model about the informational role of volume are correct when agents must decide about market participation before the signal is observed.

We assume that for agents who participate in the market demand for the risky asset is given by (2) and (3), respectively. Thus, for each agent in each group the demand for the risky asset is either zero (if the agent does not participate) or it is given by equation (2) and (3), respectively (if the agent participates). Implicitly BEO assume that *all* agents participate in the market. Based on this implicit assumption they derive from (2) and (3) the market clearing price given in equation (5). Define

$$\lambda_0 := \frac{\rho_0}{\rho_0 + \mu\rho^{s1} + (1-\mu)\,\rho^{s2}}$$

and

$$\lambda_1 := \frac{\mu\rho^{s1} + (1-\mu)\,\rho^{s2}}{\rho_0 + \mu\rho^{s1} + (1-\mu)\,\rho^{s2}}.$$

Then we can rewrite equation (5) as

$$p = \lambda_0\psi_0 + \lambda_1\theta. \tag{6}$$

Consider first the case in which each agent has to decide about market participation *before* the signal is observed. The following proposition shows that volume is always zero in equilibrium.

Proposition 1 *Assume that each agent decides about market participation conditional only on her a priori information. Then there exists no equilibrium with positive volume.*
Proof. *Appendix.*

We now turn to the case where each agent can condition her decision about market participation on the observed signal. As in the BEO model we do not allow the agents to use the information contained in the realization of the (contemporaneous) price, i.e., we use exactly the same information structure as it is used in the BEO model. However, in contrast to the BEO model we take account of the fact that the agents have to know *the model.* Otherwise the agents could not make inferences even from past prices or volume and consequently volume could have no informational role in the BEO model. The whole analysis of the BEO model breaks down, if the agents don't know the model. Their knowledge of the model implies that the agents know the *joint probability distribution* of the equilibrium price, the signals and the asset's true value (whereas by assumption they cannot extract the additional information contained in the *realization* of the equilibrium price). From this the no-trade result follows.

For the following proposition, we consider only equilibria which preserve two main properties of the equilibrium in the BEO model: (i) the equilibrium price is linear with respect to the average signal of each group, and (ii) for each group the average signal is equal to the realization of θ. If not all agents participate, the average signal $\bar{y}^j$ of group j, $j = 1, 2$, has to be calculated considering only the *participating* agents. Formally we restrict ourselves to the class of equilibria which fulfill

$$p = \chi_0\psi_0 + \chi_1\bar{y}^1 + \chi_2\bar{y}^2 \tag{7}$$

for some constant parameters χ_0, χ_1 and χ_2, and

$$\bar{y}^1 = \bar{y}^2 = \theta. \tag{8}$$

In order to simplify the formal analysis (but without deviating from the BEO model in substance) we model BEO's «large economy» (BEO, p. 163) as a continuum of agents on the unit interval $[0, 1]$, where $[0, \mu]$ is the set of agents of type 1 and $[\mu, 1]$ is the set of agents of type 2.

Proposition 2 below shows that, given the demand functions (2) and (3) derived by BEO, there exists no equilibrium with positive volume which fulfills (7) and (8).

Proposition 2 *Assume (i) that for each agent demand is given by (2) and (3) respectively, and (ii) that each agent decides about market participation conditional on the observed signal (but, in accordance with the BEO model, does not condition on the market price). Then there exists no equilibrium with positive volume such that (7) and (8) hold.*

Proof. Appendix.

Proposition 2 is due to the fact that the percentage of the agents who participate in the market depends on the realization of θ. The reason for this is as follows. Denote the realization of y^j as Y^j. Since an agent can condition her action only on the observed signal

Y^j ($j = 1, 2$) the decision whether or not to participate is a function (only) of Y^j. However, the distribution of the signal Y^j over the agents varies with the realization of θ. If, for example the realization of θ is high, more agents will receive high signals. The proof of Proposition 2 shows that this implies that in general the average signal Y^j of group j is not equal to θ. Thus, (8) cannot be satisfied and the proposition follows.

One may think that (8) is too strong an assumption because it demands that for each group taken *separately* the average signal equals the true θ, whereas it should only be demanded that for both groups taken *together*, i.e. for *all* the participating agents, the average signal equals the true θ. In this case the question is whether an equilibrium with positive volume exists, if we replace (8) by the weaker requirement $\mu^* \bar{y}^1 + (1-\mu^*)\bar{y}^2 = \theta$ where μ^* and $(1-\mu^*)$ are the weights of the two groups of (participating) agents in the *participating* population. The answer to this question is that this is not the case. A side remark (n.14) in the proof of Proposition 2 shows that this weaker requirement cannot be satisfied either, and thus there does not exist an equilibrium that (a) satisfies (7) and $\mu^* \bar{y}^1 + (1-\mu^*)\bar{y}^2 = \theta$ and (b) has positive volume.

3.2. EQUILIBRIUM WHEN AGENTS ARE BOUNDEDLY RATIONAL

We now consider the case in which all agents are boundedly rational in the sense that they are rational except that, as in the BEO model, they do not use any information contained in the (contemporaneous) price. As in Section 3.1 we assume that there is a continuum of agents.

Since the equilibrium price p of the risky asset must clear the market for each realization θ and ρ^1, it will be a function $p(\theta, \rho^1)$ of θ and ρ^1. In this subsection we analyze the class of equilibria which are characterized by the property that $p(\theta, \rho^1)$ has the following structure:

$$p(\theta, \rho^1) = [1 - b(\rho^1)]\psi_0 + b(\rho^1)\theta, \tag{9}$$

where $b(\rho^1)$ is some function of ρ^1 with range $[0, \rho_\omega/(\rho_0 + \rho_\omega)]$, i.e.

$$0 \leq b(\rho^1) \leq \frac{\rho_w}{\rho_0 + \rho_w} \text{ for all } \rho^1. \tag{10}$$

Clearly, the equilibrium price suggested by BEO in equation (5) satisfies these restrictions. The justification for these restrictions is as follows. If $p(\theta, \rho^1)$ is not linear in θ, the model is intractable. Given linearity with respect to θ, i.e. $p(\theta, \rho^1) = a(\rho^1) + b(\rho^1)\,\theta$ for some function $a(\rho^1)$, the (reasonable) condition that $p(\psi_0, \rho^1) = \psi_0$ for all ρ^1, implies $a(\rho^1) = [1 - b(\rho^1)]\psi_0$, that is (9). If $b(\rho^1) = \rho_\omega/(\rho_0 + \rho_\omega)$, then $p(\theta, \rho^1) = E(\psi|\theta)$. Since by assumption an agent i in group j conditions her demand only on the signal

$y^{ji} = \theta + \eta^{ji}$, $j = 1, 2$, where η^{ji} is her idiosyncratic error (i.e. $\eta^{1i} = e^i$ and $\eta^{2i} = \varepsilon^i$), θ will influence the equilibrium price by a non-negative factor not exceeding $\rho_\omega/(\rho_0 + \rho_\omega)$. This justifies restricting ourselves to the set of equilibria which satisfy (9) and (10). The following proposition shows that in this set there is no equilibrium with trade.

Proposition 3 *In any equilibrium for which (9) and (10) hold volume is zero.*
Proof. Appendix.

The reason for this result is that the boundedly rational agents understand the model and therefore take account of (9) when deciding about their demand. Although by assumption the agents do not infer any information about θ or ρ^1 from the observed price, they will still make use of the stochastic relation between ψ and $p(\theta, \rho^1)$. Because of this, the demand functions of the boundedly rational agents who understand the model differ from the demand functions (2) and (3) that BEO assert, even though no agent uses any information contained in the price realization of the random variable $p(\theta, \rho^1)$.

4. CONCLUSIONS

The conclusion of our analysis is that volume is always zero in the BEO model. No-trade theorems apply to the BEO model although the agents are only boundedly rational. This shows that the no-trade theorems can be generalized to certain cases of bounded rationality.

For the BEO model our results imply that it is not a good vehicle to investigate the informational role of volume and to explain volume based technical analysis. Given that the only equilibrium in the BEO model is one without trade, the BEO model's results cannot be used to describe volume or traders learning from volume. In order to analyze the informational content of volume data one has to construct a model which includes non-informational reasons for trade in addition to differences in information. This is a rather unfavorable situation because such models very soon become intractable.

APPENDIX

A PROOF OF PROPOSITION 1

The proof is indirect. Assume that there exists an equilibrium with $M_1 \geq 0$ agents of group 1 and $M_2 \geq 0$ agents of group 2 participating in the market such that $M_1 + M_2 > 0$. For the present proof define $\mu := M_1 / (M_1 + M_2) \in [0,1]$;[9] then the market clearing price is given by equation (4) and (5) respectively. Let η^{ji},j = 1, 2, denote the idiosyncratic error of the signal y^{ji}, i.e. $\eta^{1i} = e^i$ and $\eta^{2i} = \varepsilon^i$. Because of (2), (3), (6) and $\lambda_0 = 1 - \lambda_1$, the expected profit $E(\pi_j)$ from trade of an agent of group j is given by

$$\begin{aligned}
E(\pi_j) &= E\left\{(\psi - p)\left[\rho_0 (\psi_0 - p) + \rho^{sj}(y^{ji} - p)\right]\right\} \\
&= E\left\{(\psi - \lambda_0\psi_0 - \lambda_1\theta)\left[\rho_0 (\psi_0 - \lambda_0\psi_0 - \lambda_1\theta) + \rho^{sj}(\theta + \eta^{ji} - \lambda_0\psi_0 - \lambda_1\theta)\right]\right\} \\
&= E\left\{(\theta - \lambda_1\theta - \lambda_0\psi_0 - \omega)\left[\rho_0\lambda_1(\psi_0 - \theta) + \rho^{sj}\lambda_0(\theta - \psi_0) + \rho^{sj}\eta^{ji}\right]\right\} \\
&= E\left\{\left[(\lambda_0(\theta - \psi_0) - \omega\right]\left[(\rho^{sj}\lambda_0 - \rho_0\lambda_1)(\theta - \psi_0) + \rho^{sj}\eta^{ji}\right]\right\}
\end{aligned}$$

j = 1, 2, where ω is the common error (denoted by ω_t in BEO) of all signals. Since

$$E(\omega|\theta) = \frac{\rho_0}{(\rho_0 + \rho_\omega)}(\theta - \psi_0)$$

and

$$E(\eta^{ji}|\theta) = 0, \text{j} = 1, 2,$$

we get

$$E(\pi_2) = E\left[E(\pi_2|\theta)\right] = \left(\lambda_0 - \frac{\rho_0}{\rho_0 + \rho_\omega}\right)(\rho^{s2}\lambda_0 - \rho_0\lambda_1)\, E\left[(\theta - \psi_0)^2\right].$$

Clearly,

$$E\left[(\theta - \psi_0)^2\right] = \frac{1}{\rho_0} + \frac{1}{\rho_\omega} > 0.$$

9. This definition of μ coincides with the one in BEO (only) if all agents participate. However, it simplifies the notation and saves us from rewriting equation (4) and (5).

Further,

$$\rho^{s2}\lambda_0 - \rho_0\lambda_1 = \rho_0 \frac{\rho^{s2} - [\mu\rho^{s1} + (1-\mu)\rho^{s2}]}{\rho_0 + \mu\rho^{s1} + (1-\mu)\rho^{s2}} \leq 0.$$

since $\rho^{s1} > \rho^{s2}$. Finally,

$$\rho^{s1} := \frac{\rho_\omega \rho^1}{\rho_\omega + \rho^1} < \rho_\omega$$

and

$$\rho^{s2} := \frac{\rho_\omega \rho^2}{\rho_\omega + \rho^2} < \rho_\omega$$

which implies

$$\mu\rho^{s1} + (1-\mu)\,\rho^{s2} < \rho_\omega$$

and thus

$$\lambda_0 - \frac{\rho_0}{\rho_0 + \rho_\omega} = \rho_0\left(\frac{1}{\rho_0 + \mu\rho^{s1} + (1-\mu)\,\rho^{s2}} - \frac{1}{\rho_0 + \rho_\omega}\right) > 0.$$

Consequently, $E(\pi_2) \leq 0$ and therefore $M_2 = 0$. But this gives

$$E(\pi_1) = \left(\lambda_0 - \frac{\rho_0}{\rho_0 + \rho_\omega}\right)(\rho^{s1}\lambda_0 - \rho_0\lambda_1)\, E[(\theta - \psi_0)^2] = 0$$

since $M_2 = 0$, i.e. $\mu = 1$, implies $\rho^{s1}\lambda_0 - \rho_0\lambda_1 = \rho_0(\rho_0 + \rho^{s1})^{-1}(\rho^{s1} - \rho^{s1}) = 0$. Consequently, $M_1 = 0$ as well as $M_2 = 0$ and the proposition follows.

B PROOF OF PROPOSITION 2

First we proof two lemmas; then the proof of Proposition 2 follows. In order to state Lemma 1 we note that an agent receives utility $-e^{-N_0}$ if she does not participate in the market. Therefore an agent will participate if

$$E\left(-e^{(\pi_j + N_0)} \mid y^j = \hat{Y}^j + \psi_0\right) \geq -e^{-N_0},$$

where π_j denotes the profit from trade and $\hat{Y}^j$ denotes the realization of demeaned signals, i.e. $\hat{Y}^j := \hat{Y}^j - \psi_0$.[10] Let the set Z_j be the set of signals $\hat{Y}^j$ which induce participation:

$$Z_j = \left\{ \hat{Y}^j \middle| E\left(-e^{(\pi_j + N_0)} \mid y^j = \hat{Y}^j + \psi_0\right) \geq -e^{-N_0} \right\}, j = 1, 2.$$

For Proposition 2 it is important that the sets Z_j, j = 1, 2, are symmetric and do not contain an interval around zero. In the following lemma we demonstrate that the sets have these properties. In this lemma we use provisionally the additional assumptions that $\chi_0 + \chi_1 + \chi_2 = 0$ and $\chi_0 \in (\rho_0 / (\rho_0 + \rho_\omega), 1)$. However, in the proof of Proposition 2 at the end of this subsection we show that these assumptions of the lemma are always fulfilled. Thus, these provisional assumptions are just an intermediate step in the proof.

Lemma 1 *Let* $\chi_0 + \chi_1 + \chi_2 = 0$ *and* $\chi_0 \in (\rho_0 / (\rho_0 + \rho_\omega), 1)$. *Then the set* Z_j *is symmetric around zero and, furthermore, there is an interval* $(-a_j, a_j)$, $a_j > 0$, *such that* $(-a_j, a_j) \not\subset Z_j$, $j = 1, 2$.[11]

Proof: We first derive the result that the sets $Z_j, j = 1, 2$, are symmetric around zero. Let $\hat{\psi} := \psi - \psi_0$ and $\hat{y}^j := y^j - \psi_0$ denote the demeaned variables. Further, let $\Psi(\hat{Y}^j)$, $H^j(\hat{Y}^j)$ and $\Upsilon^j(\hat{Y}^j)$ denote the random variables that are distributed according to the conditional distribution function of $\hat{\psi}$ given $\hat{y}^j = \hat{Y}^j$; of η^j given $\hat{y}^j = \hat{Y}^j$ (where $\eta^j = e$ for $j = 1$ and $\eta^j = \varepsilon$ for $j = 2$); and of $(\eta^j)^2$ given $\hat{y}^j = \hat{Y}^j$, respectively. Using (7), (8), $\theta = \hat{y}^j - \eta^j + \psi_0$ and $\chi_1 + \chi_2 = 1 - \chi_0$, the (unconditional) profit of an agent in group j is

$$\begin{aligned}
\pi_j(y^j) &= (\psi - p)[\rho_0(\psi_0 - p) + \rho^{sj}(y^j - p)] \\
&= [\psi - \psi_0 + (1-\chi_0)\psi_0 - (\chi_1+\chi_2)(\hat{y}^j - \eta^j) - (\chi_1+\chi_2)\psi_0] \times \\
&\quad \{\rho_0[(1-\chi_0)\psi_0 - (\chi_1+\chi_2)(\hat{y}^j - \eta^j) - (\chi_1+\chi_2)\psi_0] \\
&\quad \rho^{sj}[\hat{y}^j + (1-\chi_0)\psi_0 - (\chi_1+\chi_2)(\hat{y}^j - \eta^j) - (\chi_1+\chi_2)\psi_0]\} \\
&= [\hat{\psi} - (1-\chi_0)\hat{y}^j + (1-\chi_0)\eta^j]\{[\chi_0\rho^{sj} - (1-\chi_0)\rho_0]\hat{y}^j + (1-\chi_0)(\rho_0+\rho^{sj})\eta^j\} \\
&= -[\chi_0\rho^{sj} - (1-\chi_0)\rho_0](1-\chi_0)(\hat{y}^j)^2 + [\chi_0\rho^{sj} - (1-\chi_0)\rho_0]\hat{\psi}\,\hat{y}^j + \\
&\quad \{[\chi_0\rho^{sj} - (1-\chi_0)\rho_0] - (1-\chi_0)(\rho_0+\rho^{sj})\}(1-\chi_0)\hat{y}^j\eta^j + \\
&\quad (1-\chi_0)(\rho_0+\rho^{sj})\hat{\psi}\,\eta^j + (1-\chi_0)^2(\rho_0+\rho^{sj})(\eta^j)^2,
\end{aligned}$$

10. The terminal wealth of an agent who participates in the market is the profit from trading the risky asset plus the initial endowment N_0, i.e. it is $\pi_j + N_0$.
11. The lemma makes no assertion whether $-a_j$, or a_j or both are elements of Z_j. Also, it does not necessarily imply $(-\infty, -a_j) \subseteq Z_j$ or $(a_j, \infty) \subseteq Z_j$.

$j = 1, 2$. The profit conditional on the signal $\hat{y}^j = \hat{Y}^j$, denoted by $\pi_j(\hat{Y}^j)$ is

$$\begin{aligned}\pi_j(\hat{Y}^j) \;=\; & -[\chi_0\rho^{sj} - (1-\chi_0)\,\rho_0]\,(1-\chi_0)\,(\hat{Y}^j)^2 + \\ & [\chi_0\,\rho^{sj} - (1-\chi_0)\,\rho_0]\,\hat{Y}^j\,\Psi(\hat{Y}^j) + \\ & \{[\chi_0\rho^{sj} - (1-\chi_0)\,\rho_0] - (1-\chi_0)\,(\rho_0+\rho^{sj})\}(1-\chi_0)\,\hat{Y}^j H^j(\hat{Y}^j) + \\ & (1-\chi_0)\,(\rho_0+\rho^{sj})\Psi(\hat{Y}^j)\,H^j(\hat{Y}^j) + (1-\chi_0)^2\,(\rho_0+\rho^{sj})\,\Upsilon^j(\hat{Y}^j).\end{aligned} \tag{A–1}$$

Since $\Psi(\hat{Y}^j)$is normally distributed with

$$\Psi(\hat{Y}^j) \sim N\left(\frac{\rho^j\rho_w}{\rho_0\,\rho^j + \rho^j\rho_w + \rho_0\rho_w}\hat{Y}^j,\ \frac{\rho^j + \rho_w}{\rho_0\,\rho^j + \rho^j\rho_w + \rho_0\rho_w}\right),$$

it is obvious that $\Psi(-\hat{Y}^j)$ and $-\Psi(\hat{Y}^j)$ have the same distribution. Analogously, $H^j(-\hat{Y}^j)$ and $-H^j(\hat{Y}^j)$ have the same distribution. This implies that $\Psi(\hat{Y}^j)\,H^j(\hat{Y}^j)$ and $\Psi(-\hat{Y}^j)H^j(-\hat{Y}^j)$ have the same distribution. Further, conditional on $\hat{y}^j = \hat{Y}^j$, $\Upsilon^j(\hat{Y}^j)$ has a χ^2- distribution such that

$$\left(\frac{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega}{\rho_0 + \rho_\omega}\right)\Upsilon^j(\hat{Y}^j) \sim \chi^2\left(1, m_j^2\right) \text{ with}$$

$$m_j^2 = \frac{(\rho_0\,\rho_\omega)^2}{(\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega)(\rho_0+\rho_\omega)}\,(\hat{Y}^j)^2\ , j = 1, 2.$$

Clearly, $\Upsilon^j(\hat{Y}^j)$ and $\Upsilon^j(-\hat{Y}^j)$ have the same distribution. Therefore, $\pi_j(\hat{Y}^j)$ and $\pi_j(-\hat{Y}^j)$ are identically distributed random variables. Hence,

$$E\left(-e^{-(\pi_j+N_0)}\,\middle|\,\hat{y}^j = \hat{Y}^j\right) = E\left(-e^{-(\pi_j+N_0)}\,\middle|\,\hat{y}^j = -\hat{Y}^j\right).$$

As a consequence, Z_j is symmetric around zero.

For the remaining part of proof we need the following results for some conditional moments:[12]

12. The following notation uses that $\Psi(\hat{Y}^j)$, $H^j(\hat{Y}^j)$ and $\Upsilon^j(\hat{Y}^j)$ respectively are defined as random variables that are distributed according to the *conditional* distribution function, given $\hat{y}^j = \hat{Y}^j$, of $\hat{\psi}$, η^j and $(\eta^j)^2$.

$$E\left[\Psi(\hat{Y}^j)\right] = E(\hat{\psi}\,|\hat{Y}^j) = \frac{\rho^j\rho_\omega}{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega}\,\hat{Y}^j,$$

$$E\left[H^j(\hat{Y}^j)\right] = E(\eta^j|\hat{Y}^j) = \frac{\rho_0\rho_\omega}{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega}\,\hat{Y}^j,$$

$$E\left[\Psi(\hat{Y}^j)\,H^j(\hat{Y}^j)\right] = E(\hat{\psi}\;\eta^j|\hat{Y}^j) = -\frac{\rho_\omega}{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega} + E(\hat{\psi}\,|\hat{Y}^j)\,E(\eta^j|\hat{Y}^j)$$

and

$$E\left[\Upsilon^j(\hat{Y}^j)\right] = E\left[(\eta^j)^2|\hat{Y}^j\right] = \left(\frac{\rho_0\rho_\omega}{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega}\hat{Y}^j\right)^2 + \frac{\rho_0 + \rho_\omega}{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega}.$$

We use these results and (A–1) to derive the expected profit conditional on the signal $\hat{y}^j = \hat{Y}^j$. After some lengthy but simple calculations we get

$$E\left[\pi_j\,(\hat{Y}^j)\right] = -\alpha_j + \beta_j\,(\hat{Y}^j)^2, \tag{A–2}$$

where

$$\alpha_j = \frac{(1-\chi_0)\,(\rho_0 + \rho^{sj})\left[\chi_0(\rho_0 + \rho_\omega) - \rho_0\right]}{\rho_0\,\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega}, \tag{A–3}$$

$$\beta_j = \frac{\left\{\rho^j\left[\chi_0(\rho_0 + \rho_\omega) - \rho_0\right]\right\}^2}{(\rho^j + \rho_\omega)(\rho_0\rho^j + \rho^j\rho_\omega + \rho_0\rho_\omega)}\,. \tag{A–4}$$

From $\chi_0 \in (\rho_0/(\rho_0 + \rho_\omega), 1)$ and $\chi_1 + \chi_2 = 1 - \chi_0 > 0$, we get $\alpha_j/\beta_j > 0$. Further, a necessary condition for an agent to participate in the market is that the expected profit conditional on the signal is positive. For an agent of group j this condition is

$$E\left\{\pi_j\,(\hat{Y}^j)\right\} > 0. \tag{A–5}$$

Hence, agents who receive the signal

$$(\hat{Y}^j)^2 < \frac{\alpha_j}{\beta_j} \tag{A–6}$$

will *not* participate in the market. This implies that $(-a_j, a_j) \not\subset Z_j$, $j = 1, 2$, where $a_j = \sqrt{\alpha_j / \beta_j} > 0$.

For Lemma 2 below we define ξ_j, $j = 1, 2$, as the percentage of participating agents of group j relative to all group j agents.

Lemma 2 *If Z_j has positive Lebesgue measure, if $(-a_j, a_j) \not\subset Z_j$, and if $a_j > 1 / \sqrt{\rho_j}$, $j = 1, 2$, then for some $\zeta > 0$ it holds that*

$$(i)\ \frac{d\xi_j}{d\theta} < 0 \text{ for } \theta \in (\psi_0 - \zeta, \psi_0) \text{ and } \frac{d\xi_j}{d\theta} > 0 \text{ for } \theta \in (\psi_0, \psi_0 + \zeta);$$

$$(ii)\ \bar{y}^j < \theta \text{ for } \theta \in (\psi_0 - \zeta, \psi_0) \text{ and } \bar{y}^j > \theta \text{ for } \theta \in (\psi_0, \psi_0 + \zeta).$$

Proof Consider the intervals $I^+ = [x, x + \delta]$ and $I^- = [-x - \delta, -x]$ for $x \geq 0$ and some given $\delta > 0$ where δ may be infinite. Let $\hat{y} = \kappa + \eta$ be a normally distributed random variable, where $\kappa \in \Re$ is a deterministic variable and $\eta \sim N(0, 1 / \rho_\eta)$. The probability mass $G(x, \delta, \kappa)$ over I^+ is given by

$$G(x, \delta, \kappa) = \sqrt{\frac{\rho_\eta}{2\pi}} \int_x^{x+\delta} e^{-\frac{1}{2}\rho_\eta (t-\kappa)^2} dt = \frac{1}{\sqrt{2\pi}} \int_{(x-\kappa)\sqrt{\rho_\eta}}^{(x+\delta-\kappa)\sqrt{\rho_\eta}} e^{-\frac{1}{2}t^2} dt.$$

With $c: = \sqrt{\rho_\eta / 2\pi}$ we get

$$\frac{\partial G(x, \delta, \kappa)}{\partial x} = c\left[e^{-\frac{1}{2}\rho_\eta (x+\delta-\kappa)^2} - e^{-\frac{1}{2}\rho_\eta (x-\kappa)^2}\right]$$

and

$$\frac{\partial G^2(x, \delta, \kappa)}{\partial x^2} = c\sqrt{\rho_\eta}\left[\sqrt{\rho_\eta}(x-\kappa)e^{-\frac{1}{2}\rho_\eta (x-\kappa)^2} - \sqrt{\rho_\eta}(x+\delta-\kappa)e^{-\frac{1}{2}\rho_\eta (x+\delta-\kappa)^2}\right].$$

Since the function $f(z): = ze^{-\frac{1}{2}z^2}$ decreases for $z \geq 1$, $\partial^2 G(x, \delta, \kappa) / \partial x^2 > 0$ for $x - \kappa \geq 1/\sqrt{\rho_\eta}$. Further, note that $\partial G(x, \delta, \kappa) / \partial \kappa = -\partial G(x, \delta, \kappa) / \partial x$ and $\partial G(-x-\delta, \delta, \kappa) / \partial x = -\partial G(x + 2\kappa, \delta, \kappa) / \partial x$. The probability mass $P(x, \delta, \kappa)$ over $I^+ \cup I^-$ is given by

$$P(x, \delta, \kappa) = G(x, \delta, \kappa) + G(-x - \delta, \delta, \kappa).$$

Differentiating $P(x, \delta, \kappa)$ with respect to κ for $\kappa > 0$ results in

$$\frac{\partial P(x, \delta, \kappa)}{\partial \kappa} = -\left[\frac{\partial G(x, \delta, \kappa)}{\partial x} - \frac{\partial G(x+2\kappa, \delta, \kappa)}{\partial x}\right] > 0 \text{ for } x - \kappa \geq \frac{1}{\sqrt{\rho_\eta}}.$$

Because of Lemma 1, Z_j consists of a set of intervals $\{I_k^{j+}, I_k^{j-}\}$, $k = 1,\ldots, K$, (K need not be finite), where $I_k^{j+} = [x_k^j, x_k^j + \delta_k^j]$, $I_k^{j-} = [-x_k^j - \delta_k^j, -x_k^j]$ and $x_{k+1}^j > x_k^j$. By assumptions of the Lemma 2, $x_1^j > a_j > 0$ where $a_j > 1/\sqrt{\rho^j}$. For $\rho_\eta = \rho^j$ and a realization $\hat{\theta} = \kappa$ we get

$$\begin{aligned}
\xi_j(\kappa) &= \sqrt{\frac{\rho^j}{2\pi}} \int_{Z_j} e^{-\frac{1}{2}\rho^j(t-\kappa)^2} dt \\
&= \sum_{k=1}^{K} \sqrt{\frac{\rho^j}{2\pi}} \left[\int_{I_k^{j+}} e^{-\frac{1}{2}\rho^j(t-\kappa)^2} dt + \int_{I_k^{j-}} e^{-\frac{1}{2}\rho^j(t-\kappa)^2} dt\right] \\
&= \sum_{k=1}^{K} P(x_k^j, \delta_k^j, \kappa).
\end{aligned}$$

If $\kappa > 0$ and $x_k^j - \kappa \geq 1/\sqrt{\rho^j}$. $\partial P(x, \delta, \kappa)/\partial\kappa > 0$ for all k. As a result, $d\xi_j(\kappa)/d\kappa > 0$ for $\kappa \in (0, \zeta)$, provided $\zeta > 0$ is sufficiently small. Further, due to symmetry $d\xi_j(\kappa)/d\kappa < 0$ for $\kappa \in (-\zeta, 0)$, for sufficiently small $\zeta > 0$. This proves Lemma 2(i).

In order to prove Lemma 2(ii) we differentiate $\xi_j(\kappa)$ with respect to κ, which gives

$$\frac{d\xi_j(\kappa)}{d\kappa} = \rho^j \sqrt{\frac{\rho^j}{2\pi}} \int_{Z_j} (t-\kappa) e^{-\frac{1}{2}\rho^j(t-\kappa)^2} dt.$$

Since, given $\hat{\theta} = \kappa$,

$$\bar{y}^j(\kappa) - \psi_0 = \frac{1}{\xi_j(\kappa)} \sqrt{\frac{\rho^j}{2\pi}} \int_{Z_j} t e^{-\frac{1}{2}\rho^j(t-\kappa)^2} dt \qquad \text{(A-7)}$$

we get

$$\frac{1}{\rho^j}\frac{d\xi_j(\kappa)}{d\kappa} = (\bar{y}^j(\kappa) - \psi_0 - \kappa)\, \xi_j(\kappa)$$

and therefore

$$\bar{y}^j(\kappa) - \psi_0 = \frac{1}{\rho^j \xi_j(\kappa)} \frac{d\xi_j(\kappa)}{d\kappa} + \kappa > \kappa \text{ for } \frac{d\xi_j(\kappa)}{d\kappa} > 0$$

Due to part (i) of Lemma 2 shown above, this implies part (ii) for $\kappa \in (0, \zeta)$ and $\zeta > 0$ sufficiently small. For $\kappa \in (-\zeta, 0)$ the result follows from the symmetry of Z_j.

PROOF OF PROPOSITION 2

If volume is positive, $\xi_1 + \xi_2 > 0$. Denote the ratio of *participating* agents in group 1 to the whole *participating* population by μ^*. Then

$$\mu^* = \frac{\mu\xi_1}{\mu\xi_1 + (1-\mu)\xi_2}.$$

Because of (2) and (3) the market clearing price is given by

$$p = \frac{\rho_0\,\psi_0 + \mu^*\rho^{s1}\bar{y}^1 + (1-\mu^*)\rho^{s2}\bar{y}^2}{\rho_0 + \mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}}.$$

Therefore, $\chi_0 = \rho_0 / (\rho_0 + \mu^*\rho^{s1} + (1-\mu^*)\rho^{s2})$, $\chi_1 = \mu^*\rho^{s1} / (\rho_0 + \mu^*\rho^{s1} + (1-\mu^*)\rho^{s2})$, $\chi_2 = (1-\mu^*)\rho^{s2} / (\rho_0 + \mu^*\rho^{s1} + (1-\mu^*)\rho^{s2})$ and thus $\chi_0 + \chi_1 + \chi_2 = 1$. Further, $\chi_0 \in (\rho_0/(\rho_0 + \rho_\omega), 1)$ since $\rho^{sj} < \rho_\omega, j = 1, 2,$ implies

$$\chi_0(\rho_0 + \rho_\omega) - \rho_0 = \rho_0(\rho_0 + \rho_\omega)\left[\frac{1}{\rho_0 + \mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}} - \frac{1}{(\rho_0 + \rho_\omega)}\right]$$

$$= \rho_0(\rho_0 + \rho_\omega)\,\frac{\rho_0 + \rho_\omega - \rho_0 - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}}{\left[\rho_0 + \mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}\right](\rho_0 + \rho_\omega)}$$

$$= \chi_0\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right] > 0 \qquad \text{(A-8)}$$

A necessary condition for an agent in group 2 to participate in the market is that $E\left[\pi_2(\hat{Y}^2)\right] > 0$, i.e. $(\hat{Y}^2)^2 > \alpha_2 / \beta_2$, because of (A–2). From (A–3) and (A–4),

$$\alpha_2 = \frac{(1-\chi_0)(\rho_0 + \rho^{s2})\left[\chi_0(\rho_0 + \rho_\omega) - \rho_0\right]}{(\rho_0\rho^2 + \rho^2\rho_\omega + \rho_0\rho_\omega)}, \text{ and}$$

$$\beta_2 = \frac{\left\{\rho^2\left[\chi_0(\rho_0 + \rho_\omega) - \rho_0\right]\right\}^2}{(\rho^2 + \rho_\omega)(\rho_0\rho^2 + \rho^2\rho_\omega + \rho_0\rho_\omega)}.$$

Because of $\rho^{s2} = \rho^2\rho_\omega / (\rho^2 + \rho_\omega)$, the expression for χ_0 derived above and (A–8), we get

$$\frac{\alpha_2}{\beta_2} = \frac{(1-\chi_0)(\rho_0 + \rho^{s2})(\rho^2 + \rho_\omega)}{(\rho^2)^2\left[\chi_0(\rho_0 + \rho_\omega) - \rho_0\right]}$$

$$= \frac{(1-\chi_0)(\rho_0\rho^2 + \rho^2\rho_\omega + \rho_0\rho_\omega)}{\chi_0(\rho^2)^2\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right]}$$

$$= \frac{\left[\mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}\right](\rho_0\rho^2 + \rho^2\rho_\omega + \rho_0\rho_\omega)}{\rho_0(\rho^2)^2\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right]}$$

$$= \frac{\left[\mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}\right]\rho_\omega}{\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right]\rho^2}\left(\frac{1}{\rho_0} + \frac{1}{\rho_\omega} + \frac{1}{\rho_2}\right)$$

$$> \frac{\left[\mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}\right]\rho_\omega}{\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right]\rho^2}\left(\frac{1}{\rho_2}\right).$$

Since $\mu^*\rho^{s1} + (1-\mu^*)\rho^{s2} > \rho^{s2} = \rho^2\rho_\omega / (\rho^2 + \rho_\omega)$,

$$(\rho^2 + \rho_\omega)\left[\mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}\right] > \rho^2\rho_\omega,$$

which implies

$$\rho_\omega\left[\mu^*\rho^{s1} + (1-\mu^*)\rho^{s2}\right] > \rho^2\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right]$$

and

$$\frac{\left[\mu^* \rho^{s1} + (1-\mu^*)\rho^{s2}\right]\rho_\omega}{\left[\rho_\omega - \mu^*\rho^{s1} - (1-\mu^*)\rho^{s2}\right]\rho^2} > 1.$$

Thus

$$\sqrt{\frac{\alpha_2}{\beta_2}} > \frac{1}{\sqrt{\rho_2}}. \tag{A-9}$$

Consider first the case in which Z_2 has positive Lebesgue measure. For $\theta = \psi_0$ we get $\bar{y}^2 = \theta$ because of (A–7) and symmetry of Z_2. Therefore, Lemma 2(ii) and (A–9) imply $\bar{y}^2 \neq \theta$ for $\theta \in (\psi_0 - \zeta, \psi_0)$ and $\theta \in (\psi_0, \psi_0 + \zeta)$ where ζ is some positive number. Thus, (8) cannot hold.

If Z_2 has Lebesgue measure zero, $\xi_2 = 0$. This implies $\mu^* = 1$ and $\chi_0 = \rho_0/(\rho_0 + \rho^{s1})$, $\chi_1 = \rho^{s1}/(\rho_0 + \rho^{s1})$, $\chi_2 = 0$. Then, because of (A–3), (A–4) and $\rho_\omega > \rho^{s1}$

$$\begin{aligned}
\frac{\alpha_1}{\beta_1} &= \frac{(1-\chi_0)(\rho_0 + \rho^{s1})(\rho^1 + \rho_\omega)}{(\rho^1)^2\left[\chi_0(\rho_0 + \rho_\omega) - \rho_0\right]} \\
&= \frac{(1-\chi_0)(\rho_0\rho^1 + \rho^1\rho_\omega + \rho_0\rho_\omega)}{\chi_0(\rho^1)^2(\rho_\omega - \rho^{s1})} \\
&= \frac{\rho^{s1}(\rho_0\rho^1 + \rho^1\rho_\omega + \rho_0\rho_\omega)}{\rho_0(\rho^1)^2(\rho_\omega + \rho^{s1})} \\
&= \frac{\rho^{s1}\rho_\omega}{(\rho_\omega - \rho^{s1})\rho^1}\left(\frac{1}{\rho_0} + \frac{1}{\rho_\omega} + \frac{1}{\rho^1}\right) \\
&= \left(\frac{1}{\rho_0} + \frac{1}{\rho_\omega} + \frac{1}{\rho^1}\right) > \frac{1}{\rho^1}.
\end{aligned}$$

It follows that

$$\sqrt{\frac{\alpha_1}{\beta_1}} > \frac{1}{\sqrt{\rho^1}}.$$

As above, Lemma 2(ii) implies that $\bar{y}^1 \neq \theta$ in a neighborhood of ψ_0 and therefore (8) cannot hold. This completes the proof of Proposition 2.[13,14]

C PROOF OF PROPOSITION 3

Let $\hat{\psi} := \psi - \psi_0$, $\hat{\theta} = \theta - \psi_0$ and $\hat{y}^j := y^j - \psi_0$, $j = 1, 2$, denote the demeaned random variables. Then,

$$\psi - p(\theta, \rho^1) = \hat{\psi} - b(\rho^1)\hat{\theta}$$

because of (9). We denote realizations of $\hat{y}^j$ by $\hat{Y}^j$. An agent of group j, $j = 1, 2$, who has observed signal $\hat{y}^j = \hat{Y}^j + \psi_0$. maximizes

$$E\left\{-e^{-[\psi - p(\theta, \rho^1)]x} \middle| \hat{y}^j = \hat{Y}^1\right\} = E\left\{-e^{-[\hat{\psi} - b(\rho^1)\hat{\theta}]x} \middle| \hat{y}^j = \hat{Y}^j\right\}$$

with respect to x. For any $\hat{Y}^j \in \Re$ the maximizer (which will be shown to be unique) gives the demand $d_j(\hat{Y}^j)$ of an agent of group j who has received signal $y^j = \hat{Y}^j + \psi_0$. Agents of group 1 know ρ^1, thus $\hat{\psi} - b(\rho^1)\hat{\theta}$ conditional on $\hat{y}^j = \hat{Y}^1$ is normally distributed and their demand $d_1(\hat{Y}^1)$ is given by[15]

$$d_1(\hat{Y}^1) = \alpha(\rho^1)\hat{Y}^1$$

13. It would be possible to show that the result that $\frac{d\xi_j}{d\theta} < 0$ for $\theta \in (\psi_0 - \zeta, \psi_0)$ and $\frac{d\xi_j}{d\theta} > 0$ for $\theta \in (\psi_0 + \zeta, \psi_0)$, implies that (7) cannot be fulfilled. However, this would be a bit tedious and is not necessary for the proof of Proposition 2.
14. If we substitute (8) by the weaker condition

$$\mu^* \bar{y}^1 + (1 - \mu^*)\, \bar{y}^2 = \theta$$

we still have no equilibrium with positive volume. The reason for this is as follows. If Z_2 has positive Lebesgue measure, ξ_2 is not independent of θ because of Lemma 2(i) and consequently, χ_1 will depend on θ unless $\mu^* = 0$, which implies $\mu^* \bar{y}^1 + (1 - \mu^*)\bar{y}^2 = \bar{y}^2 \neq \theta$ for θ close to ψ_0, as shown above. If Z_2 has measure zero, $\mu^* = 1$, $\frac{\alpha_1}{\beta_1} > \frac{1}{\rho_1}$ and $\mu^* \bar{y}^1 + (1 - \mu^*)\bar{y}^2 = \bar{y}^1 \neq \theta$ for θ close to ψ_0, as shown above.

15. Normality of returns implies

$$d_1(\hat{Y}^1) = E\left[\hat{\psi} - b(\rho^1)\hat{\theta} \mid \hat{y}^1 = \hat{Y}^1\right]\left\{\mathrm{Var}\left[\hat{\psi} - b(\rho^1)\hat{\theta} \mid \hat{y}^j = \hat{Y}^1\right]\right\}^{-1}.$$

Since

$$E\left[\hat{\psi} - b(\rho^1)\hat{\theta} \mid \hat{y}^1 = \hat{Y}^1\right] = \frac{\rho^1}{R_1}\left\{\left[1 - b(\rho^1)\right]\rho_\omega - b(\rho^1)\rho_0\right\}\hat{Y}^1,$$

we get $d_1(\hat{Y}^1) = \alpha(\rho^1)\hat{Y}^1$.

with

$$\alpha(\rho^1) := \frac{\rho^1\left[\rho_\omega - b(\rho^1)(\rho_0 + \rho_\omega)\right]}{R_1 Var\left[\hat{\psi} - b(\rho^1)\,\hat{\theta} \mid \hat{y}^j = \hat{Y}^1\right]} \geq 0$$

where $R_1 := \rho_0\rho_\omega + \rho_0\rho^1 + \rho_\omega\rho^1$. Aggregating over group 1 gives

$$D_1(\hat{\theta}, \rho^1) = \mu\alpha(\rho^1)\,\hat{\theta} \qquad \text{(A–10)}$$

as total demand of group 1.
Agents of group 2 do not know ρ^1; they know, however, the probability distribution of ρ^1. Thus for this group the returns conditional on information are not normal and demand is not straightforward to derive. The first and second order conditions of the maximization problem are

$$E\left\{\left[\hat{\psi} - b(\rho^1)\hat{\theta}\right]e^{-[\hat{\psi} - b(\rho^1)\hat{\theta}].x}\,\middle|\,\hat{y}^2 = \hat{Y}^2\right\} = 0\,, \qquad \text{(A–11)}$$

and

$$-E\left\{\left[\hat{\psi} - b(\rho^1)\hat{\theta}\right]^2 e^{-[\hat{\psi} - b(\rho^1)\hat{\theta}].x}\,\middle|\,\hat{y}^2 = \hat{Y}^2\right\} < 0\,, \qquad \text{(A–12)}$$

respectively. Obviously, the second order condition (A-12) always holds and for each $\hat{Y}^2$ (A–11) determines uniquely a solution $d_2(\hat{Y}^2)$, which is the demand of an agent of group 2 who has observed the signal $\hat{Y}^2$. From (A–11),

$$d_2(\hat{Y}^2) \geq 0 \text{ if } E\left\{\left[\hat{\psi} - b(\rho^1)\hat{\theta}\right]\middle|\,\hat{y}^2 = \hat{Y}^2\right\} \geq 0$$

and

$$d_2(\hat{Y}^2) \leq 0 \text{ if } E\left\{\left[\hat{\psi} - b(\rho^1)\hat{\theta}\right]\middle|\,\hat{y}^2 = \hat{Y}^2\right\} \leq 0$$

because e^{-z} is decreasing (increasing) for $z > 0$ $(z < 0)$. We show that this implies that total demand of group 2 agents has the same sign as $\hat{\theta}$.

Since $\hat{y}^2 = \hat{\theta} + \varepsilon$ (with ε being the idiosyncratic error of group 2 agents), total demand of group 2 is

$$D_2(\hat{\theta}) = (1-\mu) \int_{-\infty}^{\infty} d_2(\theta + \varepsilon) f_\varepsilon (\varepsilon)\, d\varepsilon$$

where $f_\varepsilon (\varepsilon)$ is the density function of ε. The next step is to prove that

$$\hat{\theta}\, D_2(\hat{\theta}) \geq 0 \tag{A–13}$$

holds, i.e. $D_2(\hat{\theta}) \geq 0$ if and only if $\hat{\theta} \geq 0$.[16]

Let $\Psi(\hat{Y}^2)$ and $\Theta(\hat{Y}^2)$ denote the random variables «$\hat{\psi}$ conditional on $\hat{y}^2 = \hat{Y}^2$» and «$\hat{\theta}$ conditional on $\hat{y}^2 = \hat{Y}^2$» , respectively. Bayesian updating gives

$$\Psi(\hat{Y}^2) = \frac{\rho^2 \rho_\omega}{R_2}\, \hat{Y}^2 + \left(1 - \frac{\rho^2 \rho_\omega}{R_2}\right) \hat{\psi} - \frac{\rho^2 \rho_\omega}{R_2} (\omega + \varepsilon)$$

where $R_2 := \rho_0\, \rho_\omega + \rho_0\, \rho^2 + \rho_\omega\, \rho^2$; and

$$\Theta(\hat{Y}^2) = \frac{\rho^2 (\rho_0 + \rho_\omega)}{R_2}\, \hat{Y}^2 + \left[1 - \frac{\rho^2 (\rho_0 + \rho_\omega)}{R_2}\right] \hat{\theta} - \frac{\rho^2 (\rho_0 + \rho_\omega)}{R_2} \varepsilon.$$

Define

$$\beta := \frac{\rho^2}{R_2} \left[\rho_\omega - b\,(\rho^1)(\rho_0 + \rho_\omega)\right] \geq 0$$

and

$$\Gamma(\hat{Y}^2) := \Psi(\hat{Y}^2) - b\,(\rho^1)\Theta(\hat{Y}^2).$$

Using $\hat{\theta} = \psi + \omega$, $1 - \rho^2\, \rho_\omega / R_2 = \rho_0 (\rho_\omega + \rho^2) / R_2$ and $1 - \rho^2 (\rho_0 + \rho_\omega) / R_2 = \rho_0\, \rho_\omega / R_2$, we get

16. This follows generally from the symmetry of all the probability distributions in our model. However, in the following we make use of the assumption that all random variables are normal rather than to apply a more abstract general argument.

$$\Gamma(\hat{Y}^2) = \beta\hat{Y}^2 + \frac{\rho_0}{R_2}\left[\rho_\omega + \rho^2 - b\,(\rho^1)\rho_\omega\right]\hat{\psi} -$$

$$\frac{\rho_\omega}{R_2}\left[\rho^2 - b\,(\rho^1)\rho_0\right]\omega - \frac{\rho^2}{R_2}\left[\rho_\omega - b\,(\rho^1)(\rho_0+\rho_\omega)\right]\varepsilon.$$

Since the distributions of $\hat{\psi}$, ω and ε are the same as the distributions of $-\hat{\psi}$, $-\omega$ and $-\varepsilon$ respectively, $\Gamma(\hat{Y}^2)$ has the same distribution as $-\Gamma(-\hat{Y}^2)$. This implies that if for some real number x^*

$$E\left[\Gamma(\hat{Y}^2)e^{-\Gamma(\hat{Y}^2)x^*}\right] = 0,$$

i.e., if x^* satisfies (A–11) for $\hat{Y}^2$, then

$$E\left[\Gamma(-\hat{Y}^2)e^{-\Gamma(-\hat{Y}^2)(-x^*)}\right] = -E\left[\Gamma(\hat{Y}^2)e^{-\Gamma(\hat{Y}^2)x^*}\right] = 0,$$

i.e., $-x^*$ satisfies (A–11) for $-\hat{Y}^2$. Therefore, $d_2\,(-\hat{Y}^2) = -d_2\,(\hat{Y}^2)$ for all $\hat{Y}^2 \in \Re$. Consequently, since $f_\varepsilon(-\varepsilon) = f_\varepsilon(\varepsilon)$ for all $\varepsilon \in \Re$,

$$D_2(-\hat{\theta}) = (1-\mu)\int_{-\infty}^{\infty} d_2(-\hat{\theta}+\varepsilon)\,f_\varepsilon\,(\varepsilon)\,d\varepsilon$$

$$= (1-\mu)\int_{-\infty}^{\infty} d_2(-\hat{\theta}-\varepsilon)\,f_\varepsilon\,(\varepsilon)\,d\varepsilon$$

$$= -(1-\mu)\int_{-\infty}^{\infty} d_2(\hat{\theta}+\varepsilon)\,f_\varepsilon\,(\varepsilon)\,d\varepsilon$$

$$= -D_2(\hat{\theta}).$$

This implies

$$D_2(0) = 0. \tag{A–15}$$

Consider now the derivative $D'(\hat{\theta})$. Since $\Gamma'(\hat{Y}^2) = \beta \geq 0$ because of (A–14), implicit differentiation of (A–11), i.e. of

$$E\left[\Gamma(\hat{Y}^2)e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] = 0,$$

with respect to $\hat{Y}^2$ gives

$$
\begin{aligned}
0 &= E\left\{\beta e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)} - \Gamma(\hat{Y}^2)\left[\beta d_2(\hat{Y}^2) + \Gamma(\hat{Y}^2)\, d'_2(\hat{Y}^2)\right] e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right\} \\
&= \beta E\left[e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] - \beta d_2(\hat{Y}^2) E\left[\Gamma(\hat{Y}^2)\, e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] - \\
&\quad d'_2(\hat{Y}^2)\, E\left\{\left[\Gamma(\hat{Y}^2)\right]^2 e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right\} \\
&= \beta E\left[e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] - d'_2(\hat{Y}^2) E\left\{\left[\Gamma(\hat{Y}^2)\right]^2 e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right\},
\end{aligned}
$$

$$
\begin{aligned}
0 &= E\left\{\beta e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)} - \Gamma(\hat{Y}^2)\left[\beta d_2(\hat{Y}^2) + \Gamma(\hat{Y}^2) d'_2(\hat{Y}^2)\right] e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right\} \\
&= \beta E\left[e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] - \beta d_2(\hat{Y}^2) E\left[\Gamma(\hat{Y}^2)\, e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] - \\
&\quad d'_2(\hat{Y}^2)\, E\left\{\left[\Gamma(\hat{Y}^2)\right]^2 e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right\} \\
&= \beta E\left[e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right] - d'_2(\hat{Y}^2) E\left\{\left[\Gamma(\hat{Y}^2)\right]^2 e^{-\Gamma(\hat{Y}^2)d_2(\hat{Y}^2)}\right\},
\end{aligned}
$$

hence $d'_2(\hat{Y}^2) \geq 0$. Therefore,

$$D'_2(\hat{\theta}) = (1-\mu) \int_{-\infty}^{\infty} d'_2(\hat{\theta}+\varepsilon) f_\varepsilon(\varepsilon)\, d\varepsilon \geq 0.$$

and, together with (A–15), $\hat{\theta} D_2(\hat{\theta}) \geq 0$. This proves (A–13).

Finally, because of market clearing,

$$D_1(\hat{\theta}, \rho^1) + D_2(\hat{\theta}) = 0$$

for all $\hat{\theta}$ and ρ^1. But since $\alpha(\rho^1) \geq 0$, (A–10) and (A–13) imply that, given any $\hat{\theta}$ and ρ^1, $D_1(\hat{\theta}, \rho^1)$ and $D_2(\hat{\theta})$ are both either non-negative (if $\hat{\theta} \geq 0$) or non-positive (if $\hat{\theta} \leq 0$). Consequently, $D_1(\hat{\theta}, \rho^1) = 0$ for all $\hat{\theta}$ and ρ^1. Because of this, $\alpha(\rho^1) = 0$ for all ρ^1 and therefore $b(\rho^1) = \rho_\omega / (\rho_0 + \rho_\omega)$ and $p(\theta, \rho^1) = E(\psi|\theta)$. But this implies

$$E\left[\psi - p(\theta, \rho^1) \mid \hat{y}^j = \hat{Y}^j\right] = E\left\{E\left[\psi - p(\theta, \rho^1) \mid \theta\right] \middle| \hat{y}^j = \hat{Y}^j\right] = 0$$

for all $\hat{Y} \in \Re$, $j = 1, 2$, i.e., the expected return conditional on information is zero for all agents. Above we have shown that $E\left[\psi - p(\theta, \rho^1) \mid \hat{y}^j = \hat{Y}^j\right] = 0$ implies $d_j(\hat{Y}^j) = 0$, i.e., no agent wants to trade. This completes the proof of Proposition 3.

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SUMMARY

The prominent Milgrom-Stokey no-trade theorem shows that differences in private information alone cannot give rise to trade among fully rational agents. In this note we demonstrate that similar no-trade results hold as well for a model introduced recently by Blume, Easly and O'Hara although in this model the agents are only boundedly rational and therefore the Milgrom-Stokey theorem does not apply. Our conclusion is that for an analysis of the informational content of volume one needs a model that contains non-informational reasons for trade in addition to differences in information.

ZUSAMMENFASSUNG

Das bekannte Nicht-Handels-Theorem von Milgrom und Stokey zeigt, dass Unterschiede der privaten Information allein nicht zu Handel unter vollständig rationalen Agenten führt. In dieser Arbeit zeigen wir, dass ähnliche Nicht-Handels-Ergebnisse bei einem Modell gelten, dass kürzlich von Blume, Easly und O'Hara eingeftihrt wurde; dies, obwohl in deren Modell Agenten nur beschränkt rational sind und das Milgrom-Stokey-Theorem nicht anwendbar ist. Wir folgern daraus, dass zur Analyse des Informationsgehalts des Handelsvolumens ein Modell benötigt wird, dass zusätzlich zu den Unterschieden in der Informationsverteilung nichtinformationsbezogene Ursachen für das Zustandekommen von Handel untersucht.

RESUME

Le theoreme bien connu de non-commerce de Milgrom et Stokey demontre que des differences dans l'information privee a elles seules n'induisent pas le commerce entre des agents entierement rationnels. Dans cette contribution, nous montrons que des resultats semblables de non-cornmerce sont valables dans un modele recemment introduit par Blume, Easly et O'hara; ceci malgre le fait que les agents ne sont que partiellement rationnels dans ce modele et que le theoreme de Milgrom et Stokey n'y est pas applicable. Nous en deduisons qu'une analyse du contenu d'information du volume de commerce necessite un modele qui, outre les differences dans la distribution de l'information, analyse aussi des causes pour l'apparition du commerce qui ne sont pas liees a l'information.