Mean Reversion on Global Stock Markets

WOLFGANG DROBETZ* and PATRICK WEGMANN**

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Keywords: Asset Pricing, Mean Reversion, Variance Ratios, Regime Switching, Monte Carlo Simulation

1. INTRODUCTION

In his seminal paper SAMUELSON (1965) argues that in an informationally efficient market, not to be confused with an allocationally or Pareto-efficient market, price changes must not be forecastable if they are properly anticipated. Prices follow a random walk and fully incorporate the information and expectations of all market participants. Fama's (1970) dictum that "prices fully reflect all available information" has become the holy grail throughout the academic community and has triggered virtually hundreds of empirical investigations supporting market efficiency since then. Starting in the late 1980s, however, several empirical studies have reported deviations of stock returns from the random walk in that asset prices exhibit mean reversion, i.e., a tendency to return to a trend path. Mean reversion exists in asset return data if autocorrelations are negative. In contrast, positive autocorrelations imply momentum (see Jegadeesh and Titman, 1993). However, the existence of mean reversion is not without controversy. Poterba and Summers (1988) and Fama and French (1988) find significant negative autocorrelation in long-horizon returns using monthly and annual U.S. data, while Lo and MacKinlay (1988) detect positive autocorrelation for weekly holding-period returns. Kim, Nelson and Startz (1991) show that mean reversion is only observed in pre-war data. Richardson and Stock (1989) and Richardson (1993) argue that a correction for possible small-sample biases may affect the mean reversion results. Accordingly, the evidence on whether there is mean reversion in actual stock market data remains mixed due to the small sample size of available data. A possible remedy is to

* Wolfgang Drobetz, University of Basel, Department of Finance, and Otto Beisheim Graduate School of Management (WHU), Holbeinstrasse 12, 4051 Basel, Switzerland, Phone: +41-61 267 33 29, Email: wolfgang.drobetz@unibas.ch.
** Patrick Wegmann, University of Basel, Department of Finance, Holbeinstrasse 12, 4051 Basel, Switzerland, Phone: +41-61-267 33 16, Email: patrick.wegmann@unibas.ch.

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include data from other countries outside the United States, as in Kasa (1992). He includes, in addition to the United States, stock index data from Canada, Germany, Japan, and the United Kingdom. Balvers, Wu and Gilliland (2000) propose a powerful cross-sectional test for their sample of 18 stock markets and find significant mean reversion which can be exploited by contrarian strategies.

Another strand of the literature focuses on the "why" of mean reversion. LeRoy (1973) and Lucas (1978) have shown that the random walk hypothesis is neither a necessary nor a sufficient condition for rationally determined asset prices. Intuitively, unforecastable prices do not necessarily imply a well-functioning financial market with rational investors, and forecastable prices need not imply the opposite. Samuelson's and Fama's original version of efficient markets implicitly assumes constant expected returns. More recently, Harvey (1991), Chen (1991), and Ferson and Harvey (1991, 1993) have shown that stock returns are predictable to some extent using macroeconomic instrument variables. They argue that expected returns vary with the business cycle, and that carefully chosen forecasting variables must be indicators for recent and/or future real activities. Balvers, Cosimano and McDonald (1990) and Cecchetti, Lam and Mark (1990) demonstrate that mean reversion is fully consistent with general equilibrium asset pricing models. The ingredients required for their models are time-variation in the endowment or production process and preferences for consumption smoothing. Unfortunately, the empirical evidence is limited to U. S. data. Kasa (1992) and Balvers, Wu and Gilliland (2000) show the existence of mean reversion in international stock market data. In addition, it would be interesting to extend the equilibrium analysis to other countries as well to see whether the U. S. results can be duplicated. This exercise is likely to shed some more light on the determinants of the international phenomenon of mean reversion.

In this paper we extend the analysis by Cecchetti, Lam and Mark (1990) to an international context and demonstrate that the serial correlation in stock market returns from Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States is consistent with a variant of the Lucas (1978) model of an exchange economy. As in Cecchetti, Lam and Mark (1990), we model the endowment to follow a regime-switching model with two states. The two states have a direct business cycle interpretation. The standard representative agent model posits that consumption, output, and dividends are all the same. Since no series is more appropriate per se, we choose consumption to proxy for the endowment in our empirical work. As usual, we assume that the period utility function belongs to the constant relative risk aversion family. This has the well known disadvantage that it is impossible to disentangle relative risk aversion from the elasticity of intertemporal substitution in consumption. For the sake of our economic story we interpret the concavity of the utility function in terms of the consumption smoothing motive. In particular, asset prices are determined in equilibrium and agents attempt to smooth consumption. In this type of model asset returns can well be negatively serially correlated and reflect all fundamental information. In many instances, an empirically plausible statistical model of the price process may be more important than a
detailed economic paradigm of equilibrium. In particular, the pricing of complex financial claims depends critically on the specific stochastic process driving underlying asset returns. However, the construction of a single stochastic process that fits actual return data is left for further investigation.

We study the measures of mean reversion proposed in Cecchetti, Lam and Mark (1990). Specifically, these are variance ratio statistics and autoregression coefficients of returns for varying time horizons. We start by calculating these statistics from historical equity return data. Then we calibrate the asset pricing model by setting the parameters of the consumption process equal to their maximum likelihood estimates. To see whether mean reversion is indeed a result of rational investors' investment and consumption decisions, we generate Monte Carlo distributions of the simulated variance ratios and regression coefficients. This allows us to determine whether the estimates from historical data were drawn from these simulated distributions implied by equilibrium returns. Our results support the consumption-based model. For most countries the actual variance ratios and regression coefficients for holding periods up to ten quarters lie within a 66 percent confidence interval of the simulated distribution generated by our equilibrium model. What we cannot explain with our model, however, is the phenomenon of persistence, or momentum, in short horizon stock returns, as reported by Lo and MacKinlay (1988).

The remainder of this paper is as follows. Section 2 starts with a description of the Lucas (1978) type exchange economy. We then estimate a regime-switching model with two different states of the economy, extending the analysis of Cecchetti, Lam and Mark (1990) by allowing for a state-dependent volatility of consumption. To our knowledge this is the first study to examine mean reversion within an equilibrium model for other than U.S. stock market data. We derive closed solutions for equilibrium asset prices and returns at the end of this section. Section 3 reports the results for maximum likelihood estimation of the transition probabilities using quarterly consumption data for the G-7 countries from January 1973 to January 1998. In section 4 we compute actual variance ratios using historical data for all seven stock markets. The asset pricing model is then calibrated by setting the parameters of the consumption process equal to the maximum likelihood estimates. To see whether mean reversion is a result of rational investors' investment and consumption decisions, we generate Monte Carlo distributions of the simulated variance ratios. In section 5 we repeat the Monte Carlo experiment and calculate long-horizon regression coefficients. Section 6 summarizes the findings and concludes.

2. THE CONSUMPTION BASED MODEL

As in Lucas (1978) and Cecchetti, Lam and Mark (1990), we consider a pure exchange economy with a single perishable consumption good. We assume the existence of a representative consumer with a period utility function of the constant relative risk
aversion (CRRA) type. The first-order condition of the consumer’s intertemporal utility maximization problem leads to the following fundamental asset pricing formula:

\[ p_t = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (p_{t+1} + y_{t+1}) \right], \]

(1)

where \( c_t \) denotes current consumption, \( p_t \) is the current ex-dividend price of the risky asset, \( y_t \) is the asset’s payoff at time \( t \), \( \gamma \) is the coefficient of relative risk aversion, and \( \beta \) is the time preference parameter. Finally, \( E_t \) denotes the expectation conditional on information available at time \( t \). Since this expression is standard in the asset pricing literature, we do not further discuss the derivation of this expression. In the same spirit, we obtain the following expression for the price of a riskless asset:

\[ p^f_t = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \right]. \]

(2)

Since in equilibrium consumption must equal endowment in every period, the risky asset is simply a claim on the aggregate consumption good. Therefore, we can replace the asset’s payoff at time \( t + 1 \), denoted as \( y_{t+1} \), with future consumption, \( c_{t+1} \). Equation (1) can then be rewritten as:

\[ p_t = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} (p_{t+1} + c_{t+1}) \right]. \]

(3)

Dividing both sides of equation (3) by \( c_t \) and rearranging terms, we obtain the asset pricing restriction in terms of the price/consumption ratio, denoted as \( \rho \):

\[ \rho_t = \frac{p_t}{c_t} = \beta E_t \left[ \left( \frac{c_{t+1}}{c_t} \right)^{1-\gamma} \left( \frac{p_{t+1}}{c_{t+1}} + 1 \right) \right]. \]

(4)

Accordingly, the realized gross one-period return of the risky asset is:

\[ r_{t+1} = \frac{p_{t+1} + c_{t+1}}{p_t} = \frac{p_{t+1}}{c_t} \frac{c_{t+1}}{p_t} + \frac{1}{c_t} c_{t+1} = \rho_{t+1} + \frac{1}{c_t} c_{t+1}, \]

(5)

and the risk-free rate from equation (2) is:

\[ r^f_t = \frac{1}{p^f_t}. \]

(6)

This model has been rejected on many grounds. Most important, it is not able to explain the magnitude of the historical equity premium and the low risk-free rate. Our specification is not intended to capture all stylized facts of the empirical equity and consumption data. The aim of this paper is to show that the model is consistent with time-varying expected returns and mean reversion in asset prices. Therefore, we continue with an empirical specification of the endowment process and compare the resulting endogenous asset prices with the empirical ones.

3. ESTIMATION OF THE ENDOWMENT PROCESS

Modern formulations of asset pricing models not only interpret a market equilibrium as a particular intertemporal condition for optimal portfolio formation, but also posit that the stochastic properties of the discount factor directly determine the underlying price process. Therefore, the first part of this section gives a short technical description of how we formulate a regime switching model for international consumption data. In the second part we report the results of our maximum likelihood estimations and interpret them within the framework of modern asset pricing theories.

3.1. Hamilton’s Markov switching model and equilibrium prices

Hamilton (1989) introduced a new technique to model non-stationary time series based on Markov processes. The basic idea is that a variable is allowed to follow a different time series process over different subsamples. Hamilton considers the process to depend on an unobserved random variable called ‘state’ or ‘regime’. The probability law governing the changes in the regime is described by a Markov chain with constant transition probabilities. This type of model is able to capture asymmetry and leptokurtosis that are often found in empirical data. The technique has found applications in different areas and has been extended in many regards. Following Cecchetti, Lam and Mark (1990), we restrict ourselves to a simple model with two states. Specifically, the endowment process is parameterized as follows:

3. In their pathbreaking analysis Hansen and Jagannathan (1991) derived volatility bounds for stochastic discount factors to be consistent with asset return data.
4. Bonomo and Garcia (1996) allow for three states, but their empirical results are only hard to interpret.
\[\Delta \ln(c_t) = \mu_{s_t,t} + \epsilon_{s_t,t}\]
\[P[s_{t+1} = 1 \mid s_t = 1] = p\]
\[P[s_{t+1} = 0 \mid s_t = 1] = 1 - p\]
\[P[s_{t+1} = 0 \mid s_t = 0] = q\]
\[P[s_{t+1} = 1 \mid s_t = 0] = 1 - q\]
\[\epsilon_{s_t,t} = \text{i.i.d. } \sim N\left(0, \sigma^2_{s_t,t}\right)\]

where \(s_t\) denotes the state of the economy at time \(t\), and \(p\) and \(q\) are the probabilities of remaining in state 1 and 0, respectively. These probabilities are constant through time. The state of the economy is defined as a sequence of Markov random variables, \(\{s_t\}\), that take on values of 1 or 0 with transition probabilities \(1 - p\) and \(1 - q\), respectively. Given that \(\Delta \ln(c) = \ln(c_{t+1}/c_t)\), the consumption process follows a random walk in logarithms with state-dependent drift, \(\mu_{s_t,t}\), and state-dependent volatility, \(\sigma_{s_t,t}\). Our specification is slightly more general than the one by Cecchetti, Lam and Mark (1990) in that we allow for state-dependent variances. Accordingly, this specification is able to capture time-dependence in variances, i.e., ARCH-effects.

The model in equation (7) is economically appealing, because the two states can be interpreted as directly linked to the business cycle. Specifically, we interpret state 1 as an economic expansion and state 0 as a recession. The notion that the statistical properties of consumption growth are different across the states of the economy, i.e., over the business cycle, is crucial to obtain time variation in expected stock returns. Given the specification of the consumption process in (7), we can solve for the equilibrium price/consumption ratios. From equation (1) it follows that:

\[(1 - \gamma)\Delta c_{t+1} \sim N\left((1 - \gamma)\mu_{s_t,t}, (1 - \gamma)^2 \sigma^2_{s_t,t}\right)\]

By the properties of the lognormal distribution the following expression must also hold:

\[E\left[e^{(1-\gamma)\Delta c_{t+1}}\right] = e^{(1-\gamma)\mu_{s_t,t} + \frac{1}{2}(1-\gamma)^2\sigma^2_{s_t,t}}\]

To solve for the equilibrium price/consumption ratio, we plug equation (9) into the pricing restriction in (4). For the price/consumption ratio in the upstate, denoted as \(p_1\) we can then write (omitting time subscripts):

5. In a different context, Whitelaw (1997) experiments with time-varying transition probabilities. However, the estimated parameters in his analysis turn out to be statistically insignificant.

6. Alternatively, Campbell and Cochrane (1999) calibrate a 'habit' model with increasing risk aversion as consumption declines toward the habit level. Their model explains the predictability in U.S. stock returns, but is unable to duplicate a high risk premium with low risk aversion.
\[ \rho_1 = \beta \left( \frac{p \times e^{(1-\gamma)\mu_1 + \frac{1}{2}(1-\gamma)^2 \sigma_1^2} (\rho_1 + 1)}{(1-p) \times e^{(1-\gamma)\mu_0 + \frac{1}{2}(1-\gamma)^2 \sigma_0^2} (\rho_0 + 1)} \right). \]  

(10)

The analogue expression also holds for the price/consumption ratio in the downstate, \( \rho_0 \). Collecting both price/consumption ratios in a vector, denoted as \( \rho \), plugging into (4), and solving the resulting expression for the price/consumption ratio, we finally get:

\[ \rho = (I - \beta \pi^*)^{-1} \beta \pi^* \mathbf{1} \]

(11)

with

\[ \rho = \begin{pmatrix} \rho_1 \\ \rho_0 \end{pmatrix} \]

(12)

and

\[ \pi^* = \begin{pmatrix} p \times e^{(1-\gamma)\mu_1 + \frac{1}{2}(1-\gamma)^2 \sigma_1^2} & (1-p) \times e^{(1-\gamma)\mu_0 + \frac{1}{2}(1-\gamma)^2 \sigma_0^2} \\ (1-q) \times e^{(1-\gamma)\mu_1 + \frac{1}{2}(1-\gamma)^2 \sigma_1^2} & q \times e^{(1-\gamma)\mu_0 + \frac{1}{2}(1-\gamma)^2 \sigma_0^2} \end{pmatrix}, \]

(13)

and where \( I \) and \( \mathbf{1} \) are the identity matrix and a vector of ones, respectively. Plugging this expression into equation (5), closed-form solutions for gross (one period) returns on the risky asset are easy to compute. Armed with this pricing restriction we are now able to compare the model's implications for realized returns with actual stock market data.

3.2. Maximum likelihood estimates of the Markov switching model

Although the underlying state of nature, \( s_t \), is not observable, given the normality assumptions on the \( \epsilon \)'s in equation (7), HAMILTON (1989) showed that the parameters of the endowment process, \( (p, q, \mu_1, \mu_0, \sigma_1^2, \sigma_0^2) \), can be estimated using maximum likelihood. Quarterly consumption data for non-durable goods and services in the G-7 countries for the period from January 1973 to January 1998 are taken from the OECD database.\(^7\) Following HAMILTON (1990), we estimate the parameters of the consumption process in equation (7) by maximum likelihood.\(^8\)

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7. This is important as we assume a standard time-separable CRRA utility function. Only for Germany the numbers contain total consumption. This is problematic because consumers derive utility from investments in durable goods over several periods. Therefore, the presumed time-separability of utility is lost.

8. The algorithm requires starting values for the conditional probabilities of being in the two states. We choose as the starting values the ergodic (i.e., unconditional) probabilities for each of the two different states.
The estimation results are reported in table 1. Similar to Cecchetti, Lam and Mark (1990) for U. S. data, most parameters are accurately estimated for many of the G-7 countries. This can be inferred from the estimates' standard errors reported in parenthesis underneath the actual values. An interesting observation is the persistence in states. Remember, when the economy is in a boom, \( p \) describes the probability that it remains in the good state during the next quarter. For all countries, except Germany, \( p \) is estimated to be well above 0.8. Once the economy has hit a slump, the probabilities to remain in a low state in the next quarter, denoted as \( q \), are somewhat lower, but still considerably persistent.\(^9\) To give an example, for the United States the probability that an expansion will be followed by another quarter of expansion is \( p = 0.951 \), so that this regime can be expected to persist on average for roughly \( 1/(1 - p) = 20 \) quarters. The probability that a contraction will be followed by another quarter of contraction is \( q = 0.878 \), which implies that the bad state typically persists for approximately \( 1/(1 - q) = 8 \) quarters. This is consistent with the observation in the business cycle literature that phases of economic expansions are usually longer in duration than phases of economic downturns. It is also consistent with the practice of the National Bureau of Economic Research (NBER) to define business cycle fluctuations in macroeconomic series as cycles of 6 to 32 quarters in length. We interpret our results as evidence for the proposition that the mean reverting components in consumption relate to the business cycle.

Most interesting, there are pronounced differences in the state dependent moments characterizing growth in consumption throughout the G-7 countries. The numbers in table 1 are reported in percent per quarter. Our parsimonious regime switching model identifies good states as those with quarterly expected consumption growth rates, \( \mu_1 \), ranging from 0.452% (for Canada) to 0.768% (for Italy). Overall, the circumstances were extremely fortunate throughout the G-7 countries during our sample period. With the exception of Canada, it is therefore not surprising to observe that bad states have not been particularly disastrous. In general, recessions implied a mere flattening of consumption growth. Only for the Canadian case table 1 reports an actual decrease of consumption by \( \mu_0 = -0.054\% \) per quarter. We extend the analysis by Cecchetti, Lam and Mark (1990) by allowing for state dependent volatilities. Inspecting the coefficient estimates for \( \sigma_1^2 \) and \( \sigma_0^2 \) in table 1, this seems to be an important addition to the model. Consumption risk clearly varies across states in most G-7 countries; it is higher in recession states than in boom states. However, the absolute magnitude of the estimated variances demonstrates that G-7 consumption series tend to be fairly smooth. A noteworthy anomaly is Japan, where \( \sigma_0^2 \) is estimated huge, but statistically insignificant. This could be explained by the dynamic development of the Japanese economy. Given estimates of the parameters in our regime-switching model, it is possible to make inferences about which regime was more likely to have been responsible for producing the date \( t \) observation of consumption, \( \xi_t \). The 'filter probabilities' for the value of \( \xi_t \) depend on all obser-

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\(^9\) The lower values for Germany might be explained by reunification in 1990.
vations through date $t$ and are based on the knowledge of the population parameters.\textsuperscript{10} Our analysis shows that for Japan the upstate probabilities are close to one during the entire sample period, except for the oil shock in 1974. It should then come as no surprise that the downstate parameters lack statistical precision. A second anomaly in our results is the United Kingdom, where the variance in consumption is lower in the downstate than in the upstate. Unfortunately, we cannot come up with an appealing economic story for this anomaly.\textsuperscript{11}

Table 1: Maximum likelihood estimates of the Markov model

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>$q$</th>
<th>$\mu_1$</th>
<th>$\mu_0$</th>
<th>$\sigma^2_1$</th>
<th>$\sigma^2_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.939</td>
<td>0.725</td>
<td>0.452</td>
<td>-0.054</td>
<td>0.252</td>
<td>0.945</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.174)</td>
<td>(0.069)</td>
<td>(0.303)</td>
<td>(0.050)</td>
<td>(0.412)</td>
</tr>
<tr>
<td>France</td>
<td>0.936</td>
<td>0.981</td>
<td>0.557</td>
<td>0.525</td>
<td>0.066</td>
<td>0.595</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.020)</td>
<td>(0.058)</td>
<td>(0.084)</td>
<td>(0.021)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.681</td>
<td>0.398</td>
<td>0.691</td>
<td>0.280</td>
<td>0.424</td>
<td>2.647</td>
</tr>
<tr>
<td></td>
<td>(0.174)</td>
<td>(0.203)</td>
<td>(0.107)</td>
<td>(0.327)</td>
<td>(0.149)</td>
<td>(0.939)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.888</td>
<td>0.875</td>
<td>0.768</td>
<td>0.230</td>
<td>0.070</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.064)</td>
<td>(0.048)</td>
<td>(0.060)</td>
<td>(0.015)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.987</td>
<td>0.675</td>
<td>0.702</td>
<td>0.201</td>
<td>0.592</td>
<td>10.013</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.312)</td>
<td>(0.076)</td>
<td>(1.564)</td>
<td>(0.089)</td>
<td>(8.787)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.969</td>
<td>0.917</td>
<td>0.620</td>
<td>0.081</td>
<td>1.026</td>
<td>0.252</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.062)</td>
<td>(0.135)</td>
<td>(0.110)</td>
<td>(0.096)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>United States</td>
<td>0.951</td>
<td>0.878</td>
<td>0.613</td>
<td>0.067</td>
<td>0.111</td>
<td>0.209</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.071)</td>
<td>(0.049)</td>
<td>(0.121)</td>
<td>(0.020)</td>
<td>(0.059)</td>
</tr>
</tbody>
</table>

Overall, consumption growth in the G-7 sample seems to be properly characterized by our two-state regime switching model. Modern formulations of asset pricing models imply that the stochastic properties of the discount factor directly determine the underlying price process. In particular, assuming a constant expectation of the pricing kernel and, hence, a constant risk-free rate boils down to the old Samuelson (1965) and Fama (1970) version of the random-walk model. This insight goes back to LeRoy (1973). Our regime-switching model provides evidence that consumption growth is cyclical. The two states are statistically significant and the volatility of consumption growth tends to be higher in recessions than in booms.\textsuperscript{12} With two clearly identified states in the endowment process, return simulations using our version of the Lucas model in section 2 can be expected to refute the random-walk model. The states are highly persistent because of positive autocorrelation in consumption streams. If expected returns were the same in both

\textsuperscript{11} We checked for the sensitivity of our parameter estimates with regards to the starting values. It turns out that the maximum likelihood estimates are extremely stable.
\textsuperscript{12} This latter observation is especially noteworthy because standard results in asset pricing theory allow to interpret the volatility of the pricing kernel as the market price of risk. For a thorough treatment see Zimmermann (1998).
states, the autocorrelation characteristics of the consumption data would transform to asset returns and we would not find mean reversion in the model simulations. Therefore, in order to generate mean reversion in stock returns we need sufficiently different expected returns in the two states. Our sensitivity analysis below will show what this implies for the parameter values in the calibration.

Early work by LEROY (1973) and LUCAS (1978) shows that rational expectation equilibrium prices need not follow a random walk, and not even a martingale sequence. Without an explicit economic model of the price generating-mechanism, a rejection of the random walk hypothesis has few implications for the efficiency of market price formations. As LO and MACKINLAY (1988) emphasize, mean reversion may be interpreted as a rejection of some economic model of efficient price formation, but there may be other (and maybe more) plausible models that are consistent with empirical findings. In the next section we will therefore examine whether mean reversion is observable in historical stock return data, and if so, whether the LUCAS model is able to reproduce this pattern.

4. VARIANCE RATIOS

The first part of this section explains and applies the concept of variance ratios to G-7 stock market data. We find clear evidence of mean reversion in historical time series for most countries in this group. In the second part, we use the returns obtained from the equilibrium model to generate Monte Carlo distributions of the variance ratio statistics. This allows us to determine whether the empirically observable patterns in stock returns are consistent with our version of the LUCAS model. While we cannot fully explain the positive serial correlation of stock returns in the short-run ('overshooting'), our results clearly show that mean reversion is perfectly consistent with the standard equilibrium model of modern asset pricing in the spirit of LUCAS (1978).

4.1. Serial correlation in historical returns

Variance ratios have been used extensively to examine mean reversion in stock return series, e.g., POTERBA and SUMMERS (1988) and LO and MACKINLAY (1988). The idea is to exploit the fact that the variance of the increments of a random walk is linear in the sampling interval. A simple example might be helpful to illustrate this concept. Assume that stock prices are generated by a random walk (possibly with drift). In this case, the variance of yearly sampled continuously compounded returns must be four times as large as the variance of a quarterly sample. Therefore, comparing the variance estimates obtained from yearly and quarterly data is an appropriate way to examine the plausibility of the random walk theory.

More specifically, let $R_t$ be the continuously compounded one period real rate of re-
return, and $R^*_k$ the k period return. Given that stock prices follow a random walk, we can write: $R^*_k = \sum_{j=0}^{k-1} R_{t-j}$. Accordingly, POTERBA and SUMMERS (1988) define the variance ratio, $VR(k)$, for returns at the k-th horizon as:

$$VR(k) = \frac{\text{Var}(R^*_k)}{k \times \text{Var}(R_t)}.$$  \hspace{1cm} (14)

LO and MACKINLAY (1988) show that this expression converges asymptotically to a linear combination of the first $k - 1$ autocorrelation coefficients of $\{R_t\}$, with linearly declining weights:

$$VR(k) = 1 + \sum_{j=1}^{k-1} \frac{2(k-j)}{k} \rho_j,$$  \hspace{1cm} (15)

where $\rho_j$ denotes the $j$-th autocorrelation of quarterly returns. When returns are serially uncorrelated, the variance ratio is equal to one in large samples for all $k$. Therefore, the random-walk model is usually taken as the null hypothesis, and significant deviations of variance ratios from the theoretical value are interpreted as market inefficiencies. In sharp contrast, our simple model in section 2 posits that mean reversion is a natural result to be expected under rational asset pricing. Given that consumption is cyclical and investors attempt to smooth consumption over time, stock returns must be negatively serially correlated. In other words, we should expect the variance ratios to fall below one for a high enough $k$.

Table 2: Variance ratios for historical returns

<table>
<thead>
<tr>
<th></th>
<th>$k = 2$</th>
<th>$k = 3$</th>
<th>$k = 4$</th>
<th>$k = 5$</th>
<th>$k = 6$</th>
<th>$k = 7$</th>
<th>$k = 8$</th>
<th>$k = 9$</th>
<th>$k = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>1.139</td>
<td>1.126</td>
<td>1.085</td>
<td>1.035</td>
<td>0.987</td>
<td>0.932</td>
<td>0.886</td>
<td>0.851</td>
<td>0.748</td>
</tr>
<tr>
<td>France</td>
<td>1.071</td>
<td>1.066</td>
<td>1.096</td>
<td>1.110</td>
<td>1.107</td>
<td>1.082</td>
<td>1.013</td>
<td>0.944</td>
<td>0.842</td>
</tr>
<tr>
<td>Germany</td>
<td>1.020</td>
<td>0.938</td>
<td>0.938</td>
<td>0.944</td>
<td>0.908</td>
<td>0.879</td>
<td>0.832</td>
<td>0.771</td>
<td>0.687</td>
</tr>
<tr>
<td>Italy</td>
<td>1.043</td>
<td>1.195</td>
<td>1.277</td>
<td>1.376</td>
<td>1.423</td>
<td>1.433</td>
<td>1.417</td>
<td>1.447</td>
<td>1.278</td>
</tr>
<tr>
<td>Japan</td>
<td>1.134</td>
<td>1.248</td>
<td>1.328</td>
<td>1.346</td>
<td>1.342</td>
<td>1.351</td>
<td>1.343</td>
<td>1.326</td>
<td>1.162</td>
</tr>
<tr>
<td>UK</td>
<td>0.064</td>
<td>1.018</td>
<td>1.003</td>
<td>0.921</td>
<td>0.868</td>
<td>0.843</td>
<td>0.849</td>
<td>0.884</td>
<td>0.839</td>
</tr>
<tr>
<td>USA</td>
<td>1.135</td>
<td>1.022</td>
<td>0.946</td>
<td>0.911</td>
<td>0.883</td>
<td>0.865</td>
<td>0.857</td>
<td>0.839</td>
<td>0.776</td>
</tr>
</tbody>
</table>

Table 2 shows the variance ratios obtained from historical G-7 real stock returns up to a sampling period of 10 quarters. We use Morgan Stanley Capital International (MSCI) total return indices on a quarterly basis from January 1973 to January 1998. Taking the

13. POTERBA and SUMMERS (1988) use Monte Carlo simulations to obtain standard errors for their variance ratios under the null hypothesis of a random walk. In contrast, LO and MACKINLAY (1988) develop simple, asymptotically standard normal tests.
standpoint of a Swiss investors, we compute continuously compounded returns in Swiss francs and deflate returns using a Swiss consumer price index. The ratios indicate positive autocorrelations for $k$ up to six quarters, but eventually fall below one for all countries, except Italy and Japan. The numbers in table 2 confirm previous research for the United States using international data. Fama and French (1988) report negative serial autocorrelations for long-holding period returns, while Lo and MacKinlay (1988) find positive autocorrelations for weekly holding period returns. The fact that the ratios for Italy and Japan are still above one in the final column of table 2 might be due to the restriction of the length of our sampling interval. Further increasing $k$ would bring the ratios down below one for these countries as well.

4.2. Serial correlation in equilibrium returns

In this section we use the return generating equation in section 2 to compute Monte Carlo distributions of variance ratios. The simulated distributions allow to draw inferences about the equilibrium model. We generate distributions for the case in which the utility function is concave. In particular, we assume a coefficient of relative risk aversion of $\gamma = 3$ and a time discount factor of $\beta = 0.98$. Following Cecchetti, Lam and Mark (1990), we calibrate our model to the estimated consumption process, as reported in section 3.2. The parameters of the forcing consumption process, $(p, q, \mu_1, \mu_0, \sigma_1^2, \sigma_0^2)$, are set equal to the values in table 1 for every country in our sample. The exact procedure is as follows: First, given the estimates of $p$ and $q$, we generate a sequence of 113 $s_t$'s according to equation (7). This particular length of the sequence is determined by the number of quarterly historical return observations we use to compute the variance ratios in table 2. Second, using the state dependent moments, we generate a sequence of 113 consumption increments as independent draws from a normal distribution with mean $\mu_1(\mu_0)$ and variance $\sigma_1^2(\sigma_0^2)$, respectively. Third, given risk aversion, $\gamma$, and the time preference, $\beta$, we calculate the price/consumption ratios for the 113 periods using equation (11) and the 113 asset returns from equation (5). For each sample of returns, the variance ratios are calculated as shown in equation (16) for horizons of 1 to 10 quarters. We repeat this experiment 10,000 times. The tabulation of our calculations allows to derive the Monte Carlo distribution of the statistic from which we draw inference. Finally, we calculate the median as well as 90 % and 66 % confidence intervals of the resulting Monte Carlo distribution. The whole procedure is repeated for all G-7 stock markets.

14. The starting values for the first period are sampled from the ergodic probabilities of the Markov chain.
Figure 1: Variance ratios

Canada

France
MEAN REVERSION ON GLOBAL STOCK MARKETS

Japan

United Kingdom
For the ease of exposition we report our results graphically in figure 1. Each graph displays the point estimates of historical variance ratios, the median, and both the 66% and 90% confidence intervals of our Monte Carlo simulations. The horizontal axis shows the number of quarters $k$ used to calculate returns, and the vertical axis depicts the variance ratios. Overall, the results strongly support our argument. Given risk aversion of $\gamma = 3$, the medians of the sample distributions of variance ratios tend to be well below one for every time horizon. It is further evident that the variance ratios calculated from MSCI data eventually fall within the 66% confidence interval of the Monte Carlo distribution. The fit of the model is almost perfect for the United States, Germany, the United Kingdom, and Canada. The medians of the simulated distributions closely track the historical variance ratios for these countries. The empirical variance ratios eventually fall within the 66% confidence interval for France and Japan, and the 90% confidence interval for Italy. We conclude that mean reversion is consistent with rational equilibrium models of asset pricing.

An important implication of our results is that the value for the intertemporal rate of substitution should be in the range suggested above to explain the empirically observed mean reversion. However, as the corresponding value of relative risk aversion is not able to duplicate historical excess returns, our results impose another puzzle to equilibrium asset pricing: the mean reversion puzzle. A level of risk aversion consistent with the equity premium would result – apart from excessive volatility in the risk-free rate of return – in a too high speed of mean reversion.

In addition to mean reversion, a second effect becomes apparent when looking at the graphs for France and Italy. Although the empirical variance ratios fall within the simulated confidence regions for large enough time horizons, we observe that they lie well
above the 90% confidence interval for shorter holding periods. This phenomenon is known as 'momentum' in the literature (e.g., Lo and MacKinlay (1990)). Hence, our simple equilibrium model in the tradition of Lucas (1978) is not able to capture stock market overreaction for these two G-7 countries. However, Lo and MacKinlay observe that there is a pronounced cross-sectional lead-lag structure in U.S. stock returns, where large-capitalization stocks almost always lead small caps. Given this observation, together with negative serial correlation of individual securities over shorter holding periods, they argue that positive autocorrelation in market indices over shorter horizons might be attributable to cross-effects. In particular, they find that less than 50% of the expected profits from a contrarian investment rule may be attributed to overreaction.

4.3. Sensitivity analysis

In the analysis above we arbitrarily set $\gamma$ equal to 3. To check the sensitivity of our results, we now examine whether the same mean reversion result is also obtained with other values of risk aversion. The Monte Carlo simulations are repeated with different values for $\gamma$. We find that for values of $\gamma$ below 2.7 asset returns are positively autocorrelated, i.e., they do not exhibit any mean reversion. This implies that $\gamma$ has to be above 2.7 in order to transform the positive serial correlation of a highly persistent consumption stream into a negative serial correlation of asset returns. In other words, the elasticity of intertemporal substitution has to be sufficiently low to reproduce the characteristics of G-7 stock return data.

5. REGRESSION COEFFICIENTS ON RETURNS OF VARYING HORIZONS

As in Cecchetti, Lam and Mark (1990), we consider a second statistic for mean reversion, long-horizon regression coefficients as originally introduced by Fama and French (1988). The first-order serial correlation coefficient on $\tau$-year returns is estimated by the following regression:

$$R_{t,t+\tau} = a_\tau + b_\tau \times R_{t-\tau,t} + u_{t,t+\tau},$$

where $\tau$ ranges from 1 to 10 years. $R_{t,t+\tau}$ is the continuously compounded real rate of return from $t$ to $t + \tau$. Fama and French (1988) show that the regression coefficient $b_\tau$ can be expressed in terms of the autocorrelations of returns at different horizons. Intuitively, positive autocorrelations correspond to positive values of $b_\tau$, and negative autocorrelations correspond to negative values of $b_\tau$. Given the distinct variance ratio patterns reported in the previous section, we expect negative and declining values of $b_\tau$ for the G-7 stock markets. We repeat our Monte Carlo simulations from above to examine whether this feature of historical data can also be duplicated by the model.
Table 3 contains the regression coefficients obtained from historical returns. A quick inspection shows that the statistics exhibit similar patterns as the variance ratios in table 2. For all countries, except Japan, the coefficients end up below zero for longer horizons. Again, there is ‘momentum’ at short-horizon sampling intervals due to positive autocorrelation, as already found in the variance ratios. This finding is in accordance with the results in Fama and French (1988) and Lo and MacKinlay (1988).

<table>
<thead>
<tr>
<th>Country</th>
<th>( \tau = 1 )</th>
<th>( \tau = 2 )</th>
<th>( \tau = 3 )</th>
<th>( \tau = 4 )</th>
<th>( \tau = 5 )</th>
<th>( \tau = 6 )</th>
<th>( \tau = 7 )</th>
<th>( \tau = 8 )</th>
<th>( \tau = 9 )</th>
<th>( \tau = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.128</td>
<td>0.134</td>
<td>0.007</td>
<td>-0.201</td>
<td>-0.289</td>
<td>-0.340</td>
<td>-0.436</td>
<td>-0.495</td>
<td>-0.488</td>
<td>-0.470</td>
</tr>
<tr>
<td>France</td>
<td>0.018</td>
<td>-0.007</td>
<td>0.020</td>
<td>-0.115</td>
<td>-0.279</td>
<td>-0.330</td>
<td>-0.350</td>
<td>-0.309</td>
<td>-0.253</td>
<td>-0.226</td>
</tr>
<tr>
<td>Germany</td>
<td>0.084</td>
<td>0.017</td>
<td>0.065</td>
<td>-0.055</td>
<td>-0.192</td>
<td>-0.219</td>
<td>-0.271</td>
<td>-0.261</td>
<td>-0.218</td>
<td>-0.177</td>
</tr>
<tr>
<td>Italy</td>
<td>0.043</td>
<td>0.264</td>
<td>0.164</td>
<td>0.009</td>
<td>-0.140</td>
<td>-0.192</td>
<td>-0.239</td>
<td>-0.260</td>
<td>-0.292</td>
<td>-0.292</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.107</td>
<td>0.050</td>
<td>0.010</td>
<td>0.060</td>
<td>0.178</td>
<td>0.320</td>
<td>0.371</td>
<td>0.348</td>
<td>0.292</td>
<td>0.210</td>
</tr>
<tr>
<td>UK</td>
<td>-0.117</td>
<td>-0.113</td>
<td>-0.298</td>
<td>-0.338</td>
<td>-0.129</td>
<td>-0.003</td>
<td>-0.040</td>
<td>-0.009</td>
<td>-0.011</td>
<td>-0.129</td>
</tr>
<tr>
<td>USA</td>
<td>0.046</td>
<td>-0.128</td>
<td>-0.172</td>
<td>-0.319</td>
<td>-0.362</td>
<td>-0.317</td>
<td>-0.339</td>
<td>-0.300</td>
<td>-0.203</td>
<td>-0.135</td>
</tr>
</tbody>
</table>

In a next step, we calculate the distribution of the regression coefficients using our simulated returns from the Monte Carlo experiment in section 4.2. The results are displayed in figure 2. We observe that for all countries the coefficients from historical returns fall within the 66% confidence region (except for Canada and Japan). This again supports our view that mean reversion is fully compatible with rational asset pricing.

Figure 2: Return regression coefficients

Canada
MEAN REVERSION ON GLOBAL STOCK MARKETS

France

![Graph showing the relationship between time horizon in quarters (k) and autoregression coefficient for France. The graph includes 90% and 66% confidence intervals, median, and estimated from data markers.]

Germany

![Graph showing the relationship between time horizon in quarters (k) and autoregression coefficient for Germany. The graph includes 90% and 66% confidence intervals, median, and estimated from data markers.]
Italy

![Graph showing the autoregression coefficient over time for Italy. The graph includes lines for 90% and 66% confidence intervals, median, and estimated from data.]

Japan

![Graph showing the autoregression coefficient over time for Japan. The graph includes lines for 90% and 66% confidence intervals, median, and estimated from data.]

United Kingdom

- 90% Confidence interval
- 66% Confidence interval
- Median
- Estimated from data

United States

- 90% Confidence interval
- 66% Confidence interval
- Median
- Estimated from data
6. CONCLUSION

This paper has presented a two-state regime-switching model for the consumption process, estimated with consumption data from the countries in the G-7 group. We have shown that for most countries this simple model is able to duplicate the interesting empirical fact that variance ratios of longer time horizons tend to be less than 1, and that long-horizon autoregression coefficients tend to be negative. In other words, empirical stock prices are to some extent mean-reverting. We have demonstrated that in this respect the explanatory power of the standard consumption-based model is not restricted to U.S. data, as demonstrated by Cecchetti, Lam and Mark (1990), but rather seems to be a more general feature of world economies.

We have not been able to explain the short-run momentum of stock prices within our simple framework, i.e., variance ratios greater than 1 for shorter time horizons, combined with variance ratios smaller than 1 for longer horizons. It must remain an open question in what form it is possible to capture this stylized fact within a model of the rational expectations/utility maximization paradigm.

From an investment professional's point of view, a final question is certainly of interest: Does mean reversion imply that it is optimal for investors to exploit predictability to earn excess profits using contrarian strategies, as shown in Balvers, Wu and Gilliland (2000)? Modern asset pricing stories allow to interpret a capital market equilibrium in terms of portfolio optimality conditions. In particular, in equilibrium the marginal utility gain from an additional asset bought at time $t$ to be sold for consumption purposes at $t+1$ is exactly offset by the utility loss from consuming less in the current period. Hence, as Balvers, Cosimano and McDonald (1990) argue, an investor would have to give up consumption right in times when he or she likes to consume much. This, however, is not consistent with utility-maximizing behavior. Any possible advantage from higher expected returns will be offset by the disadvantage of a less smooth consumption pattern.
REFERENCES


Lo, A. and A.C. MacKinlay (1988), “Stock Market Prices do not Follow Simple ran-


**SUMMARY**

This paper focuses on mean reversion on international stock markets and explores whether this empirical observation is compatible with a rational, general equilibrium asset pricing model. We consider a simple time series model with switching regimes for the consumption process in the G-7 countries and compare the simulated returns with historical stock market data. Our results show that for most countries the empirical mean reversion produces no challenge for an equilibrium model. Short-run momentum, however, cannot be explained within the same simple framework.

**ZUSAMMENFASSUNG**

Gegenstand der Untersuchung ist die Frage, ob die auf den internationalen Aktienmärkten beobachtbare Mean-Reversion mit einem rationalen Bewertungsmodell kompatibel ist. Wir analysieren ein durch unterschiedliche Umweltzustände charakterisiertes Zeitreihenmodell für die Wachstumsraten des aggregierten Konsums in den G-7 Ländern und vergleichen simulierte Renditen mit den empirischen Renditerealisationen.
Unsere Ergebnisse zeigen, dass für die Mehrzahl der Länder Mean-Reversion durch das Standardmodell der konsumbasierten Bewertungstheorie erklärt werden kann. Gegen vermag das Modell Momentum in den kurzfristigen Renditen nicht zu erklären.

RÉSUMÉ

Cet article examine la question si la convergence des rendements vers la valeur moyenne (mean-reversion) observable sur les marchés internationaux des actions est compatible avec un modèle d’évaluation rationnel. Pour les pays du G-7, nous analysons un modèle de séries temporelles pour les taux de croissance de la consommation agrégée avec des états différents de l’économie et comparons les rendements simulés avec les réalisations empiriques. Nos résultats montrent que pour la plupart des pays, cette convergence (mean-reversion) peut être expliquée par un modèle standard de consommation. Cependant, le modèle ne peut pas expliquer le momentum dans les rendements sur un horizon à court terme.