1. INTRODUCTION

According to the real interest parity condition, real interest rates in different countries should move together in the long run, leading to a world interest rate or at least to a fixed real interest rate differential. This hypothesis is an important building block especially for monetary models of the exchange rate. But the validity of the condition is relevant for policy makers as well. If capital moves rapidly to equalize real returns across countries, the influence of government action on real economic performance is limited.

Whether real interest parity holds or not is an empirical question. Because of the increased integration in the international goods and financial markets, one could expect that the condition is approximately in line with reality. However empirical evidence is mixed. In fact the parity is rejected in the studies of Cumby and Obstfeld (1984), Mishkin (1984), Mark (1985) and Cumby and Mishkin (1986), among others. Miller Dutton (1993) reports some evidence in favor of the condition, when inflation rates are based on prices for tradable goods. Gagnon and Unfertth (1995) compute a world interest rate in a factor analysis exercise, and the world rate is highly correlated with its national counterparts. Due to the analysis of Meese and Rogoff (1988) and Edison and Pauls (1993), real interest rate differences seem to have a unit root. In contrast, Kugler and Neusser (1993), Goodwin and Grennes (1994) and Wu and Chen (1998) report the stationarity of this variable in most cases, eventually after accounting for structural breaks (Fountas and Wu, 1999; Felmingham, Qing, and Healy, 2000). Therefore, even the long run properties of the time series are questionable.

The ambiguous results of the literature could reflect substantial deficits of the hypothesis, namely, an imperfect integration of international markets or important risk premiums. On the other hand, econometric methods may not be appropriate. For example, conventional unit root tests have low power in small or medium sized samples. However,
the power of the tests can be enlarged. In particular, in the panel unit root tests, the cross section dimension is taken into account. In this framework, the condition is tested jointly for different countries. Because of the additional long run information set, one could expect an increase in power and therefore, a more qualified judgement of the real interest parity hypothesis is obtained. Earlier papers often perform tests where the joint null hypothesis is the nonstationarity of the real interest differences for all panel members following the lines of the augmented Dickey-Fuller (ADF) tests. In addition to previous work this paper relies on tests based on KPSS principles as well (KWIATKOWSKI, PHILLIPS, SCHMIDT and SHIN, 1992). The null is the joint stationarity for all panel members. Both type of tests are applied to check for cointegration between ex post real interest rates in the group of G7 countries.

The rest of the paper is organized as follows: First, we review the conditions under which real interest parity holds. Second, panel unit root tests stemming from ADF- and KPSS-principles are discussed. They are used to examine the overall validity of the real interest parity condition. In addition standard unit root tests are performed to control for long-run heterogeneity across the panel members. The last section summarizes our results. Critical values for the panel unit root tests are obtained by simulation, and details are put together in an appendix.

2. REAL INTEREST PARITY

Real interest parity arises from perfect mobility in the international goods and financial markets. Exchange rates, prices and nominal interest rates are connected via three international parity conditions. If two of these parities are fulfilled, the third is implied. In particular, real interest parity stems from relative purchasing power parity (PPP) and uncovered interest parity (UIP). Because of relative PPP, the expected change in the exchange rate between periods $t$ and $t + j$ corresponds to the expected inflation differential

$$E_t(s_{t+j} - s_t) = E_t(\pi_{t+j} - \pi^*_{t+j})$$  \hspace{1cm} (1)

According to UIP, expected fluctuations of the exchange rate are approximately equal to the difference in nominal interest rates

$$E_t(s_{t+j} - s_t) = i_{j,t} - i^*_{j,t}$$  \hspace{1cm} (2)

where $s$ is the logarithm of the exchange rate, denoted as home currency units per foreign currency unit, $\pi$ is the inflation rate and $i$ the nominal interest rate for holding assets with a maturity of $j$ periods. Foreign variables are indexed by an asterisk. Expectations are rational and are formed on the base of the information available to the market at time $t$. The two conditions imply

$$i_{j,t} - i^*_{j,t} = E_t(\pi_{t+j} - \pi^*_{t+j})$$  \hspace{1cm} (3)
and so the difference in expected inflation, which is unobserved, will be sufficiently proxied by the observable difference of nominal interest rates. By definition, ex ante real interest rates are equal to the difference between nominal interest and expected inflation. After rearranging, real interest parity appears,

\[ i_{j,t} - E_t \pi_{t+j} = i^*_{j,t} - E_t \pi^*_t \Rightarrow r_{j,t} = r^*_{j,t} \] (4)

where \( r \) is the real interest rate for assets holding over \( j \) periods. Stated in this way, the hypothesis postulates an equalization of national real rates of return.

For the choice of the appropriate empirical strategy, some adjustments has to be considered. In the presence of transaction costs, real interest rates will differ by an amount compatible with these costs (Clinton, 1988). Real interest rates can fluctuate independently within the neutral band determined by the cost level, where arbitrage is not profitable. This issue can be addressed by the means of nonlinear models, which is behind the scope of the analysis.

Furthermore, the preconditions for real interest parity may not be fulfilled exactly. For example, due to the stickiness of prices in the goods markets, relative PPP may hold only in the long run. Moreover if international investors are not risk neutral, then a risk premium must be added in equation (2). Note that risk premiums should be partially embodied in the ex post real rates, because risk is likely correlated with inflation. However, uncovered interest parity holds only for risk adjusted interest rates. Given that the risk premium does not have a unit root, real interest parity is restored as a long run equilibrium relation. Real rates may have different levels because of the impact of risk premiums, but they should move together in the long run.

Finally, real interest parity should hold in terms of expected (ex ante) real interest rates, which are not observable. Thus in the empirical analysis, the variables have to be replaced. The proxies usually employed are the ex post real interest rates, calculated on the base of realized inflation. The proxies are incomplete measures for the variables of interest, since ex post inflation includes both the expected inflation and the forecasting error, where the latter is assumed to be stationary. Apart from this problem, the forecast error of inflation will follow a moving average process of order \( j-1 \), when forecasts are done over a horizon of \( j \) periods. As a consequence, real interest parity can only be tested as a long run condition.

3. PANEL UNIT ROOT TESTS

Real Interest Parity is justified as a long run condition, when ex post rates are pairwise cointegrated. But here, the cointegrating vector is known and so, a more efficient test is available. According to equation (4), the difference between two interest rates must be stationary in each case. Hence one can evaluate the hypothesis by the means of unit root tests.
The standard unit root tests are expected to have low power, in particular against the alternative of a stationary process with an autoregressive coefficient near unity (CAMPbell, PERRON, 1991). One way to increase power is to expand the sample period. But due to the presence of structural breaks, the benefits of the strategy are limited. Also a higher frequency of the data does not resolve the problem, because no long-run information is added to the data set (SHILLER, PERRON, 1985; PIERSE, SNELL, 1995). A more promising way to restore power is to pool across countries, and this is done in panel unit root tests (LEVIN and LIN, 1993).

There are several panel unit root tests available in the literature (BANERJEE, 1999). However in the present study the Im, Pesaran, Shin (IPS) and Maddala, Wu (MW) test are preferred (IM, PESARAN and SHIN, 1997; MADDALA and Wu, 1999). Both strategies combine the evidence of standard unit root tests which are performed independently on the cross section level. Therefore they allow for heterogeneity across the panel members. As a principle the IPS-statistic is based on t-values, while the MW-statistic is set up on the grounds of the appropriate p-values. As an example, consider the conventional augmented Dickey Fuller (ADF)-test regression for the i-th panel member

$$\Delta y_{it} = \alpha_i + \delta_i y_{i,t-1} + \sum_{j=1}^{k} \beta_{ij} \Delta y_{i,t-j} + e_{it}$$

(5)

where the fixed effect $\alpha_i$ the short run dynamics and the autoregressive coefficient $\delta_i$ may differ across the $N$ panel members, $i = 1, \ldots, N; t = 1, \ldots, T$. The ADF-test evaluates the significance of the t-value of $\delta_i$. Then the IPS-test

$$IPS(ADF) = \frac{1}{N} \sum_{i=1}^{N} t_i(\delta_i)$$

(6)

is build upon the average of the individual ADF-tests, while the MW-test

$$MW(ADF) = -2 \sum_{i=1}^{N} \log p_i(\delta_i)$$

(7)

relies on the corresponding p-values. According to the conventional ADF-test, the null hypothesis in (6) and (7) is the nonstationarity of the series for all individuals, while under the alternative, it is stationary at least for one panel member. Provided that the cross sections are independent, the statistics (6) and (7) are distributed as standard normal and chi-squared with $2N$ degrees of freedom, respectively. For the IPS-test a normalization is needed in order to meet the required asymptotic distribution (IM, PESARAN and SHIN, 1997).

In principle these procedures can be carried out with any type of unit root test on the individual level (BANERJEE, 1999). Therefore in the analysis both tests are computed on
the grounds of individual ADF- and KPSS-settings. In the latter case, a panel unit root test for the null of stationarity for all cross sections is easily obtained (Choi, 2000).

The gain in efficiency arising from the cross section dimension will be not a free lunch. In particular contemporaneous correlation may appear due to common shocks which affect jointly all the individuals. Note that a correlation can be expected here even if shocks are absent. This stems from the fact that real interest differences are computed relative to the same foreign country. Due to the contemporaneous correlation, independent long run information is lost. Even more important, the test statistics will suffer from substantial size distortions (O’Connell, 1998). In fact, the true size of the unit root tests can be far above the nominal level even if the correlation is moderate. Therefore the results are highly questionable when cross section dependencies are not modelled appropriately. Hence the asymptotic distributions are suspended and critical values have to be generated by simulation.

In addition panel unit root tests do not reveal any individual specific information in respect to the unit root feature. Consider for example the procedures build on ADF principles. If the null is rejected, the series may be stationary for all cross sections or only for a subgroup. The null might be rejected even if there is only one stationary individual in an otherwise unit root environment, see the Monte Carlo evidence presented by Taylor and Sarno (1998). Thus the results of the panel unit root tests should be checked by the means of conventional unit root tests in order to get robust results.

4. EMPIRICAL ANALYSIS

Real Interest Parity is examined as a long run equilibrium condition for the countries in the G7 using monthly data. The sample period is from 1980.01 to 1998.12, so for each cross section unit, 228 observations are available. The parity is tested by the means of ex post rates. They are defined as differences between nominal interest rates and realized inflation, which replaces expected inflation. Nominal interest rates are 3 month Euro-currency rates (United States, Germany, United Kingdom, Japan). For France, Italy and Canada the Euro-currency rates are not available over the whole sample period, and 3 month Treasury Bill rates are employed instead. Annualized inflation is calculated on the base of consumer price indices. Prices are seasonally adjusted by the means of the Census X11-method. Nominal interest data are taken from the DRI-WEFA database (DRIINTL), and prices are taken from the OECD Main Economic Indicators. Differences of ex post real rates are plotted in figure 1. In order to get robust results, they are calculated alternatively with the United States and Germany as the foreign

1. Euro money market rates are quoted for currencies on deposits outside their country of origin. They are free market rates not affected by capital controls, reserve requirements etc.
2. Due to the 3 month real interest rates and the monthly frequency of the data, an MA(2) component is introduced in the forecast error of inflation. Since this process is stationary, it will not affect the long run properties of real interest rate differences. The analysis was also re-done with non-overlapping data on the quarterly frequency, leaving the results unchanged.
countries. For example, it may be argued that the economic performance might be somewhat tighter in the Eurozone and the former European Monetary System. Therefore the test outcomes could depend on the choice of the foreign country.

**Figure 1: Differences of ex post real interest rates in the G7 countries, 1980–1998**

**A. United States as the foreign country**

- Germany
- France
- Italy
- United Kingdom
- Canada
- Japan
B. Germany as the foreign country

United States

France

Italy

United Kingdom

Canada

Japan
If real interest parity holds as a long run equilibrium condition, ex post real rates are pairwise cointegrated. Because the cointegration vector is known, this request is equivalent with the stationarity of ex post real interest differences. The stationarity of the series is examined by the means of panel and conventional unit root tests. The contemporaneous correlation between the residuals of separate ADF regressions is 0.4 on the average and so, critical values for the panel tests have to be derived by simulation. Details about this procedure are given in the appendix. Moreover conventional unit root testing is needed to uncover long run heterogeneity across the individual real interest rate differences.

The lag length \(k\) in the ADF-regressions is determined by the procedure suggested by Campbell and Perron (1991). Specifically an upper bound of \(k = 18\) is set for all cross sections. Then \(k\) is reduced sequentially by 1 until the last lag becomes significant. To access the significance of the lags, the 10% value of the normal distribution is used. Moreover all insignificant lags are removed in order to avoid a dependency of the results from nuisance regressors. The KPSS-test requires a consistent estimator of the long run variance of the residuals, which is obtained using the method suggested by Newey and West (1987). For the computation of the long run variance, covariances up to 18 periods are considered. In addition all models include a constant, but no trend. When real interest parity is supported by the data, pairwise differences must be stationary just at the level, but not stationary around some deterministic time trend. The panel unit root tests are based exactly on the above settings for the individuals. The outcome of the tests are reported in Table 1.

The results of the panel based tests are mixed. Generally, real interest parity is supported by the panel tests relying on the ADF principle, but rejected by the KPSS settings. Therefore both joint null hypothesis – namely the nonstationarity and the stationarity for all panel members – are rejected by the data. Note that this result is not a contradiction. Not all series in the panel are nonstationary or stationary, that is, the panel unit root tests might indicate a mixture of stationary and unit root processes.

However a panel analysis does not uncover any individual specific information with respect to the unit root feature and therefore, standard unit root tests are also employed. With the US as the foreign country, all differences of real interest rates are nonstationary, according to the ADF-test. In contrast, real interest parity is mostly confirmed, when the differences are computed between European economies. Hence the results might reflect a tighter economic performance in Europe.

3. As an exception from this rule the null of nonstationarity is rejected by the MW(ADF) only at the 7 percent level of significance, when the US serves as the base country.

4. In order to check the robustness of the results, the univariate tests were adopted to allow for a structural break in the data, according to Perron (1997). The evidence is broadly unchanged. In particular, a unit root in most real interest rate differences in the US sample is detected even after controlling for a break.
Table 1: Panel and univariate unit root tests

Panel unit root tests

<table>
<thead>
<tr>
<th>Foreign Country</th>
<th>IPS (ADF)</th>
<th>IPS (KPSS)</th>
<th>MW (ADF)</th>
<th>MW (KPSS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>-2.28*</td>
<td>0.43*</td>
<td>20.42</td>
<td>26.53*</td>
</tr>
<tr>
<td>GER</td>
<td>-3.54*</td>
<td>0.39*</td>
<td>59.92*</td>
<td>42.12*</td>
</tr>
</tbody>
</table>

Univariate unit root tests

<table>
<thead>
<tr>
<th>ADF</th>
<th>Foreign Country</th>
<th>KPSS</th>
<th>Foreign Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>GER</td>
<td>US</td>
<td>GER</td>
</tr>
<tr>
<td>GER</td>
<td>-2.54</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>FRA</td>
<td>-2.17</td>
<td>0.51*</td>
<td></td>
</tr>
<tr>
<td>ITA</td>
<td>-2.01</td>
<td>0.60*</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>-2.50</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>-2.54</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>-2.56</td>
<td>0.48*</td>
<td></td>
</tr>
<tr>
<td>JAP</td>
<td>-1.87</td>
<td>0.22</td>
<td></td>
</tr>
</tbody>
</table>

Notes: GER = Germany, FR = France, ITA = Italy, UK = United Kingdom, US = United States, CAN = Canada, JAP = Japan. The IPS- and MW-panel tests are based either on ADF or KPSS principles. P-values for the KPSS-test values are calculated by a low order polynomial due to the approximation in Hansen (1992), Table 3. A * indicates that the null hypothesis is rejected at least at the the 5 percent level.

5. CONCLUSIONS

In this paper we have examined the empirical performance of real interest parity as a long run equilibrium condition. According to this hypothesis, ex post real interest rates must be pairwise cointegrated, and so, the differentials of real interest rates must be stationary. We have tested this condition on the basis of standard and panel unit root tests relying on ADF and KPSS principles. The panel tests suggest a mixture of stationary and nonstationary series. In order to identify individual specific features, standard unit root tests were performed. They indicate that the condition is broadly supported within the European countries. Therefore the results may be traced to a tighter economic performance in Europe.
APPENDIX

Due to the dependence between the cross section units, the finite sample distribution of the panel unit root statistics under the null have to be derived by simulation. This is done for the US and Germany as the base country. In general the critical values depend on the cross section and time series dimension of the panel and on the contemporaneous correlation structure as well. Testing relies on individual autoregressive models describing the fluctuations of the real interest differences. After performing OLS on the cross sections separately, the contemporaneous covariance matrix of the residuals is computed. The matrix is employed in the Monte Carlo experiments.

Specifically 6 error series with an equal length of 328 are drawn from the multivariate gaussian distribution with predefined covariance matrix. In the ADF settings pseudo observations are generated. For the tests in the KPSS style, the error series are employed to estimate the long run variances. In order to reduce a possible sensitivity of the results on initial conditions the first 100 observations were discarded, leaving 228 observations for estimation (1980:01–1998:12). In order to find the empirical distribution relevant in this application, the process is repeated 5000 times. Simulation results are presented in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Critical values for panel unit root tests</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. United States as the foreign country</strong></td>
</tr>
<tr>
<td>IPS (ADF)</td>
</tr>
<tr>
<td>10%</td>
</tr>
<tr>
<td>5%</td>
</tr>
<tr>
<td>1%</td>
</tr>
<tr>
<td><strong>B. Germany as the foreign country</strong></td>
</tr>
<tr>
<td>IPS (ADF)</td>
</tr>
<tr>
<td>10%</td>
</tr>
<tr>
<td>5%</td>
</tr>
<tr>
<td>1%</td>
</tr>
</tbody>
</table>

Note that the critical values are slightly different in the two samples due to a different correlation structure. For example at the 5% level of significance and 228 observations for each of the 6 series in the panel the critical value of the MW(ADF) test is 21.73, when the US serves as the foreign country. The appropriate value with Germany as the base economy is 21.90. If the test exceeds this level, the joint null of nonstationarity for all panel members is rejected. The TSP386 program which generates the critical values is available from the authors upon request.
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**SUMMARY**

We examine the validity of real interest parity as a long run condition for the G7 countries. If real interest parity holds, differences of real interest rates are stationary. This is investigated by the means of conventional and panel unit root tests, where heterogeneity and contemporaneous correlation across the panel members is taken into account. Performing ADF- and KPSS-style panel tests on ex post rates, the evidence suggests a mixture of stationary and nonstationary series. However strong linkages between individual real interest rates can be found in the European economies.

**ZUSAMMENFASSUNG**


**RÉSUMÉ**

Cet article examine pour les pays du G7 l'hypothèse de parité des taux d'intérêt réels en tant que condition d'équilibre à long terme. Les différences de taux d'intérêt réels sont stationnaires dès que la parité est atteinte. Cette conséquence est soumise à l'épreuve de tests conventionnels et de racine unitaire de panel, dans lesquels sont pris en compte l'hétérogénéité et une corrélation contemporaine entre les composantes du panel. Fondés sur les principes ADF et KPSS et appliqués aux taux ex-post, les tests dénotent le caractère à la fois stationnaire et non stationnaire des séries. Des liens robustes entre les taux d'intérêt individuels apparaissent toutefois, surtout pour les économies européennes.