Estimating Policy-Neutral Interest Rates for Canada Using a Dynamic Stochastic General Equilibrium Framework

JEAN-PAUL LAM and GREG TKACZ*

JEL-Classification: C32, E37

1. Introduction

Through the ages, some of the greatest economic thinkers (e.g., THORNTON, 1802; WICKSELL, 1898; and MEADE, 1933) have forwarded the proposition that there exists some unobservable interest rate that equilibrates supply and demand conditions in the economy. When observable market rates equal the equilibrium rate, which is roughly consistent with the marginal productivity of capital, price stability ensues. On the other hand, when observable market interest rates deviate from the unknown equilibrium rate, adjustments such as price increases or decreases occur in the economy to compensate.

In recent years, the notion of equilibrium interest rates has regained prominence, both in policy circles (e.g., JULIUS, 1998; MEYER, 2000; and NEISS and NELSON, 2003) and academia (e.g., WOODFORD, 2000; BOMFIM, 2001; and LAUBACH and WILLIAMS, 2001). In an era when the primary policy instrument is the level of the short-term interest rates, a comparison of that rate with some equilibrium, or policy-neutral, rate is a convenient method to measure the stance of monetary policy. The real interest rate gap – the difference between the real equilibrium rate and the rate set by the central bank – can thus serve as a leading indicator of future inflationary or deflationary pressures in the economy.[1]

* Department of Monetary and Financial Analysis, Bank of Canada, e-mail: jplam@bankofcanada.ca and gtkacz@bankofcanada.ca. The authors wish to thank Ed Nelson for valuable discussions on the neutral rate and to Bank of Canada seminar participants for several valuable comments and suggestions. The views in this paper are exclusively those of the authors and should not be attributed to the Bank of Canada.

1 This is especially true in models where the output gap reacts with a lag to the interest rate gap. The latter will give advance information on both the output gap and inflation.

The objective of this paper is to construct measures of equilibrium interest rates, variables that are not directly observed. To derive the equilibrium natural rates, we follow NEISS and NELSON (2003) very closely and calibrate their sticky-price dynamic stochastic general equilibrium (DSGE) model for Canada. These authors show that their derived stance measure for the UK (the difference between the actual and equilibrium rates) leads actual inflation relatively well, and thus can be useful for policy-makers.

As in WOODFORD (2000) and NEISS and NELSON (2003), we define the equilibrium rate of interest as the real rate that prevails in an economy characterized by fully flexible prices, or, equivalently, the real interest rate that returns output to its potential level in a sticky-price economy.[2] Although this definition focuses on a short horizon for price stability and thus differs from the usual medium-run definition of the equilibrium rate, the rate that corresponds to the intermediate dynamics of the economy, it is worth remembering that in the model the "averaging" of the natural rate into a medium run concept can be implemented by private agents since their pricing and spending decisions are forward looking.[3]

In our model, the equilibrium real rate (and potential output) is obtained by eliminating all nominal rigidities; that is, we assume fully flexible prices in all markets. When prices are completely flexible all markets clear and, by definition, output and the real interest rate are at their equilibrium levels. On the other hand, when markets do not clear instantaneously, output and the real rate diverge from their equilibrium values. Any attempt on the part of policy-makers to close the output and/or real interest rate gap will reduce inflationary or deflationary pressures in the economy.

Once our measure of the equilibrium rate is derived, we provide several tests to evaluate the quality of our real interest rate gap series, particularly its forecasting abilities. We find that the real interest rate gap can be a useful measure of policy stance, as fluctuations in the real interest rate gap lead real activity with the explanatory power peaking at about 4 to 5 quarters ahead. This is consistent with the view that monetary policy takes about 4 to 6 quarters to affect output. Our results also indicate that the empirical links of the real interest rate gap with real activity and inflation are strong. Our derived measure of the real

2 The equilibrium interest rate can be computed directly from any DGE structure.

3 The 'averaging' of the natural rate into a medium-run concept can also be done in the definition itself. What matters in the model is not the actual output gap but also expected output gaps.

equilibrium rate, for a given specification of the model, predicts output as accurately as the yield spread.

Moreover, our results indicate that our measure of the natural rate is not as volatile as in previous studies (Amato and Laubach, 2001, and Woodford, 2000, for example). Past studies have found the natural rate to be very volatile mostly because of the overly high calibration of the degree of intertemporal substitution of consumption. Since we choose less extreme values to calibrate the degree of intertemporal substitution in consumption, we do not obtain such a volatile response in the natural rate and thus obtain a smoother series for the equilibrium interest rate.

The remainder of the paper is organized as follows. In the next section we further motivate the notion of equilibrium rates by reviewing some of the views on this subject by current, and former, central bankers in both the United States and the United Kingdom. In Section 3 we present our DSGE model. Section 4 shows how our model is calibrated. Section 5 presents the impulse response functions of the model and we also discuss in this section how the measure of the equilibrium rate is obtained. The forecasting ability of our equilibrium series for output and inflation is discussed in Section 6. Section 7 concludes and suggests avenues for further research.

2. Equilibrium Interest Rates and Monetary Policy

In virtually all developed countries monetary policy is conducted through the control of some short-term interest rate that represents the rate at which commercial banks borrow in order to settle their overnight balances. The absolute level of this rate, however, cannot be viewed as an adequate gauge of the stance of policy, as it ignores supply and demand conditions in product and credit markets. Instead, an appropriate measure of stance would be to compare the level of the policy rate relative to some equilibrium, or policy-neutral, rate. If the policy rate is above equilibrium, then policy would be considered tight; when below equilibrium, it would be loose. The challenge, however, is to estimate such an equilibrium rate.

2.1. *United States*

Although the equilibrium rate is difficult to estimate and identify, the concept of such a rate is embraced and firmly entrenched in the thinking of some modern policy-makers. Most notably, in the United States several current and former members of the Federal Reserve Board of Governors use the equilibrium rate notion to explain and rationalize policy decisions.

JOHNSON and KELEHER (1996), for example, offer a textbook treatment on how market prices, and most notably bond prices, can be used to formulate policy. Their monograph identifies three useful indicators, and explains how these indicators can be jointly used in policy-making. In particular, they advocate prices from central auction markets as pivotal policy indicators, namely commodity prices, exchange rates and bond rates. These can be jointly used to locate the real rate of interest consistent with price stability (Wicksell's natural rate). Johnson and Kelemer (p. 6) recognize that

> (e)ach of these indicators serves as a proxy measure of the value of money from a somewhat unique perspective. Commodity prices measure the value of money in terms of traded goods. Foreign exchange measures the price of money in terms of other monies. The price of bonds represents the exchange rate between money and bonds, or to put it another way, the price of money in terms of future money.

Monetary policy can affect all these variables, but the authors warn so too can non-policy factors. Thus, "it is the careful and joint assessment of these proxies for the value of money that enables perceptive policymakers to contribute significantly to price stability" (p. 6).

BLINDER (1998), a former Fed Vice-Chairman, also advocates the use of an equilibrium rate when formulating policy. His rough estimate of the real equilibrium rate is about 2.75 per cent, representing an historical average of the real interest rate. He advocates formal estimation of such a rate by equating the steady-state IS curve to potential output each period, which would yield an interest rate for which the rate of inflation would remain constant.[4] In estimating such a rate BOMFIM (1997) finds, however, that the resulting series is highly volatile.

Most recently, Fed Board of Governor member Laurence MEYER (2000) used an equilibrium rate to rationalize the challenges currently facing policy-makers. He explains that the level of the real equilibrium rate should be tied to the level of productivity in the economy, with higher productivity resulting in higher real equilibrium rates. "The equilibrium real interest rate at full employment has to balance saving (driven by the thrift motive) and investment (responding to the

4 See also KING (2000).

productivity and, hence, profitability of capital)." (p. 8). The goal of monetary policy is to keep market real interest rates in line with the equilibrium rate in order to achieve the objective of price stability.

2.2. United Kingdom

Equilibrium interest rates form the basis of a recent *Financial Times* article by Brittan (2001), who notes that the estimate of the real equilibrium rate would "almost certainly come in the range of 3–5 per cent." Part of the motivation for the article stemed from the views of DeAnn Julius, member of the Bank of England Monetary Policy Committee (MPC). She justifies her positions on interest rate movements by drawing upon her beliefs about the level of the equilibrium rate. Consider her testimony at a recent House of Commons Select Committee on Treasury:

> *Question:* In the latest minutes that we have available, you recommended a cut in interest rates (question #302), whereas your colleagues either recommended to hold interest rates steady or recommended an increase. What do you think was the major factor determining your resolve that the interest rates should be cut (#303)?
> *Julius:* The stance of monetary policy needs to take into account of where we are in the cycle and what is to come over the next year or year and a half because... the effect is a year and a half or two years away, so it seemed to me that the stance of policy was too tight for that phase of the cycle. When I say that, I am referring particularly to the height of real short-term interest rates which... is probably somewhere around 4 per cent, and also the shape of the yield curve with short rates being higher than long rates, both of those are indicators that monetary policy is pretty tight and at this stage of the cycle it is not clear to me that is the appropriate stance for monetary policy as the economy is slowing, so I voted for a reduction in interest rates as a means of moving towards a more neutral position of monetary policy which I felt would be more appropriate for this phase of the cycle.

3. The Model

This section describes a sticky-price DGSE model with capital adjustment costs. The model includes households, firms, and a monetary authority, and is very similar to the model of Neiss and Nelson (2003). There are two types of firms: final- and intermediate-goods producers. Final-goods producers behave competitively and produce output using intermediate goods. On the other hand, intermediate-goods firms are monopolistic competitors and produce a differentiated product using capital and labour based on the existing technology. These firms set nominal prices on a staggered basis.

3.1. *Firms*

We assume that there are a large number of final-goods producers who behave competitively and who produce a homogenous good, Y_t, using intermediate goods, $Y_t(z)$. We also assume that there is a continuum of intermediate-goods producers owned by consumers, indexed by the letter z who operate in a Dixit-Stiglitz style imperfectly competitive economy.

3.2. *Final-Goods Producers*

Final-goods producers use the following production function to transform intermediate goods into final output:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1}$$

where $\theta > 1$. This production function exhibits constant returns to scale, diminishing marginal product and constant elasticity of substitution. In each period, the final-goods firm choose inputs $Y_t(z)$ for all $z \in [0,1]$, and output to maximize profits subject to equation (1). This maximization problem is given by:

$$\max P_t Y_t - \int P_t(z) Y_t(z) dz \quad \text{s.t equation (1)} \tag{2}$$

where $P_t(z)$ is the price of the intermediate good $Y_t(z)$ and P_t refers to the aggregate price level.

The solution to this problem yields the constant price elasticity demand function z that is homogenous of degree one in total final output:

$$Y_t(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\theta} Y_t \tag{3}$$

Combining this demand function with the production function (1), we obtain the following price index for intermediate goods:

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{4}$$

3.3. Intermediate-Goods Producers

Each intermediate-goods producer is indexed by $z \in [0,1]$, operate in a Dixit-Stiglitz style imperfectly competitive economy and face a downward sloping demand function for his product (equation 3). In addition, each firm produces intermediate goods subject to the following technology constraint:

$$Y_t(z) = A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \tag{5}$$

where $N_t(z)$ and $K_t(z)$ are respectively the amount of labour and capital hired by the firm to produce output and A_t is a technology shock which follows this process:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A \text{ and } \varepsilon_t^A \sim iid(0, \sigma_A) \tag{6}$$

The intermediate-goods producer is assumed to choose the optimal amount of physical capital and labour to maximize profits taking the productivity of the firm as given subject to equation (5). Thus the firm solves:

$$\min_{N_t(z), K_t(z)} \left[w_t(z) + r_t K_t(z) \right] \text{ s.t equation (5)} \tag{7}$$

where w_t is the real wage rate and r_t is the rental rate of capital.

The first-order conditions to this minimization problem are given by

$$w_t = \alpha mc_t \frac{Y_t(z)}{K_t(z)} \tag{8}$$

$$r_t = (1-\alpha) mc_t \frac{Y_t(z)}{K_t(z)} \tag{9}$$

where mc_t is the real marginal cost.

Using the above equations, one can see the link between marginal cost and the gross mark-up. Defining the mark-up as the inverse of real marginal costs and rearranging (8) and (9), one obtains a link between marginal cost and the gross mark-up.

Rearranging (8) and (9), we have,

$$\alpha \frac{Y_t(z)}{N_t(z)} = \mu_t w_t \tag{10}$$

$$(1-\alpha)\frac{Y_t(z)}{N_t(z)} = \mu_t r_t \tag{11}$$

$$\text{where } \mu_t = \frac{1}{mc_t}$$

Equations (10) and (11) imply that firms adjust their inputs to equate the marginal product of each input with the mark-up times the price of the input.

3.4. Inflexible Prices

Intermediate-goods producers set nominal prices on a staggered basis. We follow ROBERTS (1995), YUN (1996) and CLARIDA, GALI and GERTLER (1999) and use the model of CALVO (1983) and assume that firms are allowed to reset the price of their good in any given period with probability $(1-s)$.[5] The parameter s governs the degree of nominal price rigidity in the model. As s approaches zero (one), prices become more flexible (rigid).[6]

Firms that are able to adjust their price will do so optimally and will maximize expected profits, taking aggregate output (Y_t), the aggregate price level (P_t), nominal marginal cost (MC_t) and the constraint on the frequency of price adjustment as given. Firms that are not able to change their price instead adjust output to meet demand. However, all firms minimize their costs given demand.

The problem of the firm changing prices at time t consists of choosing to max the following

$$E_t \sum_{i=0}^{\infty} (s\beta)^i \left[\Lambda_{t,t+i} \left(\frac{P_t(z) - MC_{t+1}}{P_{t+i}} \right) Y_{t,t+i}(z) \right] \tag{12}$$

subject to the demand for its good $Y_{t,t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\theta} Y_{t+i}$

5 An alternative is to follow CHRISTIANO, EICHENBAUM and EVANS (2001) who modify Calvo's framework by assuming a dynamic price-updating scheme instead of a static one. One advantage of their methodology and the assumption of dynamic price-updating is that the lagged inflation term can be derived in a non- trivial way without having recourse to some adhoc assumptions.

6 If firms face a probability $(1-s)$ of changing its price, prices will remain fixed on average for $\frac{1}{1-s}$.

where $\Lambda_{t,t+1}$ is the ratio of the marginal utility of consumption at time *t+i* and *t* and $\beta\Lambda_{t,t+1}$ is the rate at which firms discount earnings at time *t+i*. Substituting the above equation into equation (12), the objective function can then be written as

$$E_t \sum_{i=0}^{\infty} (s\beta)^i \left[\Lambda_{t,t+i} \left(\left[\frac{P_t(z)}{P_{t+i}} \right]^{1-\theta} - \left[\frac{MC_{t+i}}{P_{t+i}} \right] \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\theta} \right) Y_{t+i} \right] \tag{13}$$

The first order condition to this maximization problem can be written as

$$P_t(z) = \mu E_t \sum_{i=0}^{\infty} \omega_{t,t+i} MC_{t+i} \tag{14}$$

$$\text{where } \omega_{t,t+i} = \frac{(s\beta)^i \Lambda_{t,t+i} D_{t,t+i}}{E_t \sum_{i=0}^{\infty} (s\beta)^i \Lambda_{t,t+i} D_{t,t+i}} \;,\; \mu = \frac{\theta}{\theta - 1}$$

is the steady state mark up or the inverse of the steady-state real marginal cost and

$$D_{t,t+i} = \left[\frac{P_t(z)}{P_{t+i}} \right]^{1-\theta} Y_{t+i}$$

denotes firm's revenues at *t+i* conditional on $P_t(z)$ as in Gali and Gertler (1999).

As firms know that they cannot change their prices in each period, their optimal price will be a function of past and expected future demand. As a result, the aggregate price index will be a weighted average of the optimal price and prices set in period $t-1$ to reflect the proportion of firms that are not able to change their prices. The aggregate price index can thus be expressed as

$$P_t^{1-\theta} = (1-s)\tilde{P}_t^{1-\theta} + sP_{t-1}^{1-\theta} \tag{15}$$

By log-linearizing the FOC of the firm and equation (15) above, we obtain a supply function which Roberts (1995) has labelled the "New Keynesian" Phillips curve. This supply function is given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \varphi mc_t \text{ where } \varphi_t = \left[\frac{s}{1-s} \right] \left[1 - (1-s)\beta \right] \tag{16}$$

As discussed in CLARIDA *et al.* (1999), unless a serially correlated cost-push shock is added to this equation, the "New Keynesian" Phillips curve does not generate realistic inflation dynamics. For this reason, we use a hybrid supply function as a sensitivity test in our analysis. We follow GALI and GERTLER (1999) and assume that a proportion of firms use a simple rule of thumb that is based on the recent history of aggregate price behaviour to set prices.[7] They show that when this is combined with the model of Calvo, one obtains the following hybrid supply function:

$$\pi_t = \phi \pi_t + (1-\phi) E_t(\pi_{t+1}) + \varphi mc_t \qquad (17)$$

In equations (16) and (17), the parameter governs the degree to which prices are sticky. The larger (smaller) φ is, the more flexible (rigid) prices are. The equilibrium real rate is obtained by calibrating φ to a very large number while the observed (actual) rate is obtained by assuming that prices are rigid. Once the two measures are obtained, we can construct the interest rate gap, i.e., the difference between the equilibrium rate (flexible-price value) and the observed rate (sticky price value). While the specification of the supply function and interest rate setting rule does not affect our measure of the equilibrium rate, however, the observed or actual interest rate will be dependent on how the model is specified.

3.5. Households

The economy is composed of a continuum of infinitely-lived agents where each of them consumes a final good (C_t), hold real money balances $\left(\frac{M_t}{P_t}\right)$, and supplies labour ($N_t$). They are assumed to maximize the sum of discounted expected utility by choosing the optimal quantity of goods to consume and the amount of hours to work and invest in physical capital (K_{t+1}) each period given prices (P_t), wages (W_t) and interest rates (r_t). Assuming that β is the discount rate of the consumer, the representative household chooses a sequence of consumption, nominal money balances, one period bond holdings (B_{t+i}), capital (K_{t+i}) and employment to maximize the following lifetime utility function:

7 See CHRISTIANO, EICHENBAUM and EVANS (2001) for a derivation of (17) which does not rely on the assumption os rule of thumb.

$$\text{Max } E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\xi_t\frac{\sigma}{\sigma-1}\left(\frac{C_t}{C_{t-1}^h}\right)^{\frac{\sigma-1}{\sigma}}+\frac{\gamma_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m}+a_nL_t\right]\right\} \quad (18)$$

where subject to a series of period budget constraints:

$$C_{t+i}+I_{t+i}+\frac{M_{t+i}}{P_{t+i}}+\frac{B_{t+i+1}}{P_{t+i+1}}=w_{t+i}N_{t+i}+r_{t+i}K_{t+i}+\Pi_{t+i} \quad (19)$$
$$+\frac{M_{t-1+i}}{P_{t+i}}+\frac{(1+R_{t-1+i})B_{t+i}}{P_{t+i}}-H(I_{t+i})$$

$$L_t+N_t\leq 1 \quad (20)$$

where

$$I_{t+i}=K_{t+1+i}-(1-\delta)K_{t+i}\ ,\ \delta\in[0,1]\ , \quad (21)$$

$$H(I_{t+i})=\psi I_{t+i}^{\eta} \quad (22)$$

$$\text{and } \xi_t=\rho_\xi\xi_{t-1}+\varepsilon_t^{\xi}. \quad (23)$$

Consumption in the model displays habit formation since households derive utility from current but also from past consumption. The parameter h measures the strength of habit persistence. In the limiting case where $h=0$, the function reduces to a standard time-separable utility function. We follow FUHRER (2000), NEISS and NELSON (2003) and CHRISTIANO *et al.* (2001) and set h to a fraction. In that case, households derive utility from current and also from past consumption. The parameter σ measures the curvature of the consumption function and corresponds to the intertemporal elasticity of substitution. ξ_t represents a preference shock that will be specified below.[8]

In equation (19), r_t represents the rental rate of capital, R_t is the gross nominal interest rate and Π_t denotes lump sum firm profits. I_t denotes investment and it is related to the capital stock as in equation (21) where is the depreciation rate. We assume that households cannot adjust their capital stock instantaneously (equation 22)). We follow ABEL (1983), CASARES and MCCALLUM (2000) and NEISS

8 According to MCCALLUM and NELSON (1999), this preference shock represents a demand shock in the expectational IS.

and NELSON (2003) and assume capital accumulation in the model is subject to some adjustment cost as shown. With $\eta = 2$, equation (22) amounts to quadratic costs of adjustment. However, we allow for a more general specification and let take on different values.

Letting λ_t denote the lagrangian multiplier on (18), we have the following first-order conditions for the representative household:

Consumption

$$\left[\xi_t C_{t-1}^{-h\left(\frac{\sigma-1}{\sigma}\right)} C_t^{-\frac{1}{\sigma}}\right] - \beta h E_t \left[\xi_{t+1} C_{t+1}^{\frac{\sigma-1}{\sigma}} C_t^{-h\left(\frac{\sigma-1}{\sigma}\right)-1}\right] - \lambda_t = 0 \tag{24}$$

Employment

$$-a_n + \lambda_t w_t = 0 \tag{25}$$

Money Demand

$$a_m \left(\frac{M_t}{P_t}\right)^{-\gamma_m} - \lambda_t + \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \tag{26}$$

Capital

$$\lambda_t \left(1 + H'\left(I_t\right)\right) - \beta E_t \lambda_{t+1} \left[(1-\delta)(1 + H'\left(I_{t+1}\right)) + r_{t+1}\right] \tag{27}$$

Bonds

$$-\lambda_t + \beta(1 + R_t) E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \tag{28}$$

3.6. Monetary Authority

Monetary policy in the model is characterized by a interest rate rule. Following Clarida, Gali and Gertler (1998), we estimate a forward-looking rule for Canada for the period 1990Q1 to 2000Q4. This rule is given by:

$$R_t = 0.816R_{t-1} + (1-0.816)\left[2.075E_t\pi_{t+1} + 1.08(y_t - \overline{y}_t)\right] + \varepsilon_t^R \qquad (29)$$

where ε_t^R is a policy shock and $y_t - \overline{y}_t$ is the output gap.

As a sensitivity test, we also consider an alternative rule which has been found by Côté, Kuszczak, Lam, Liu and St-Amant (2004) to be robust in some models of the Canadian economy. This rule is a Taylor-type rule but with a weight of 2 on the deviations of inflation from its target.

3.7. Market Clearing and Equilibrium

We define an equilibrium as a collection of allocation for final-goods-producers, Y_t, $Y_t^D(z)$ for all $z \in [0,1]$; allocations for intermediate-goods producers, $N_t(z)$, $K_t(z)$ for all $z \in [0,1]$; allocations for consumers, $C_t, N_t, I_t, M_t, B_{t+1}, K_{t+1}$; together with a price vector $W_t, Z_t, P_t, P_t(z), R_{t+1}^n, MC_t, H(I_t)$ for all $z \in [0,1]$ such that all agents are maximizing subject to the constraints they face, supply equals demand in each market and all resource constraints are satisfied, given the values of the predetermined and exogenous variables. To examine the dynamics of the model, we log-linearize the optimality conditions around their steady state. The log-linearization is shown in Appendix I.

4. Calibration

In this section, we describe the parameter values for our benchmark model. In calibrating the model, we follow Dib (2002) who has estimated several of these parameters for Canada. We set the discount factor β to its usual value of 0.99, δ the depreciation parameter to 0.025 and α the labour share to 0.7. The probability that a firm is able to change its price, s, is set to 0.25, implying that prices are fixed on average for a year. This value is similar to those estimated by Gagnon and Khan (2001). ϕ which measures the degree of backward-lookingness in the hybrid supply function is set at 0.5, an estimate which is similar to Fuhrer and Moore (1995). μ the steady-state mark-up is set at 1.25, ψ, η the adjustment cost parameters to 0.2 and 2 respectively (we allow for a quadratic cost adjustment process – see CASARES and MCCALLUM, 2000), *h* the habit persistence parameter to 0.6 (see FUHRER, 2000), the standard deviation of the preference and technology shock are from DIB (2002) and they are set respectively at 0.015 and 0.01. We assume that both shocks are very persistent and set the serial correlation parameter to 0.875 and 0.9 respectively.

The intertemporal elasticity of substitution, σ, is set to 0.6. This parameter is typically calibrated to a lower value in closed-economy models (DIB, 2002, for example, has an estimate of 0.45 for this parameter). However, since we want the model to capture some of the dynamics of a small-open economy model (at least on the demand side), we follow NEISS and NELSON (2003) and calibrate this parameter to a higher value.[9] In an open-economy framework, net exports are an important component of aggregate demand. The influence of net exports on aggregate demand can be partially captured in this model by increasing the value that σ takes.[10] We assume a value of 0.45 and 0.15 respectively for the interest responsiveness of consumption and net exports (hence a value of 0.6 for σ). This three-to-one ratio captures a common rule of thumb about IS coefficients (see DUGUAY, 1994, for example). In addition to our baseline model, we present three alternative models which reflect some of the sensitivity tests we have performed. These sensitivity tests are shown in Table 2.

5. Equilibrium Interest Rates

5.1. Model's Impulse Response Function

As mentioned in the introduction, the equilibrium real rate is obtained by eliminating all nominal rigidities in the model.[11] In a model where capital is exogenous and where habit formation is absent, the equilibrium real rate can be easily obtained as it is simply a function of the state vector of the model which contains only exogenous variables. On the other hand, in a model with habit formation and where the capital stock is time-varying and endogenous, calculating the

9 Although we calibrate the intertemporal elasticity of substitution to a higher value to capture some of the dynamics of a small open-economy model on the demand side, our model does not allow for such effects on the supply-side since the exchange rate is not included in the Phillips curve. However, many studies have shown that for Canada, there is no empirical justification to include the exchange rate in the Phillips curve. As a result, our baseline Calvo and hybrid supply functions are reasonable approximations of open economy Phillips curves.

10 See NEISS and NELSON (2003) for more details.

11 The equilibrium level of output is obtained by a similar methodology.

Table 1: Parameter Values for Benchmark Model

Parameters	Description	Value
β	Discount factor	0.99
δ	Depreciation rate	0.025
α	Share of labour	0.7
h	Habit persistence	0.6
μ	Steady-state mark-up	1.25
ψ	Adjustment cost parameter	0.2
η	Adjustment cost parameter	2
σ	Intertemporal elasticity of substitution	0.6
ρ_ξ	Autocorrelation of preference shock	0.875
σ_{ε^ξ}	Std deviation of preference shock	0.015
ρ^A	Autocorrelation of technology shock	0.9
σ_{ε^A}	Std deviation of technology shock	0.01

Table 2: Model Specification

Models	Parameters	Aggregate Supply	Policy rule
Model 1	Baseline	Calvo	Forward-looking
Model 2	Baseline except $\sigma = 2/3$	Hybrid	Forward-looking
Model 3	Baseline except $h = 0.8$	Hybrid	Simple rule
Model 4	Baseline except $h = 0.8$ and $\rho_\xi = 0.8$	Hybrid	Forward-looking

equilibrium rate is not as straightforward. In such a model, the state vector is no longer just a function of exogenous variables but will also contain endogenous variables: the capital stock at period t and consumption at period $t-1$. Consequently, we cannot simply express the equilibrium rate as a function of the state vector since the latter does not contain only exogenous variables.

To circumvent this problem, we employ the same methodology as in Neiss and Nelson (2003). They argue that the equilibrium rate can be expressed as a function of current and past dated exogenous variables if the two endogenous variables (capital stock and consumption) are substituted out of the state vector. Since the exogenous elements of the state vector are assumed to follow an autoregressive process, the capital stock and consumption can be also expressed as a function of the exogenous variables of the state vector. As a result, they can be substituted out of the state vector. We can then write the equilibrium rate as a linear function of current and past dated shocks of the model (a distributed lag of the innovations of the model).

To obtain the impulse response functions and empirical estimates of the equilibrium real rate, we proceed as follows: we first solve the model under flexible prices and then using stochastic simulations, generate artificial data for several variables of the model, including the innovations. Since the equilibrium interest rate and potential output are linear combination of current and past dated shocks, using the artificial data generated from each stochastic simulation, we run the following regressions for an i that is high enough for a good fit.[12]

$$r_t^* = \Phi_1 \xi_t + \Phi_2 \xi_{t-1} + ... + \Phi_i \xi_{t-i} + \Theta_1 a_t + \Theta_2 a_{t-1} + ... + \Theta_i a_{t-i} \quad (30)$$

$$y_t^* = \tilde{\Phi}_1 \xi_t + \tilde{\Phi}_2 \xi_{t-1} + ... + \tilde{\Phi}_i \xi_{t-i} + \tilde{\Theta}_1 a_t + \tilde{\Theta}_2 a_{t-1} + ... + \tilde{\Theta}_i a_{t-i} \quad (31)$$

We then calculate the average of each of the above coefficients across the simulations. The average of these coefficients will be used subsequently to generate our model-based and empirical series for the equilibrium real rate.

Second, the model is solved under the assumption of sticky prices using one of the supply functions and policy rules we have specified.[13] Third, the real interest rate and output gaps are calculated using the series derived with and without price rigidities. The impulse response functions for the output gap, real interest rate gap and the equilibrium rate are shown in Figures 1–6 for four different versions of our model.

Figures 1–3 shows the response of the economy to a preference shock. Since output increases more than potential, this shock leads to a positive output gap. The equilibrium real rate as well as the actual real rate also increases. However, since the latter increases by more than the former, a negative interest rate gap opens, signalling that policy is too loose.

A positive technology shock on the other hand, leads to a negative output gap as potential output increases by more than actual output. In this model, a positive technology shock leads to a countercyclical response of the equilibrium rate – the equilibrium interest rates fall not rise. The countercyclical response of the natural rate to a positive technology shock is explained below.

12 The fit of these equation is extremely good, implying that our regression approximates well the true measure of the equilibrium rates.

13 The choice of the policy rule and supply function does not matter when prices are fully flexible.

Figure 1: output gap – preference shock

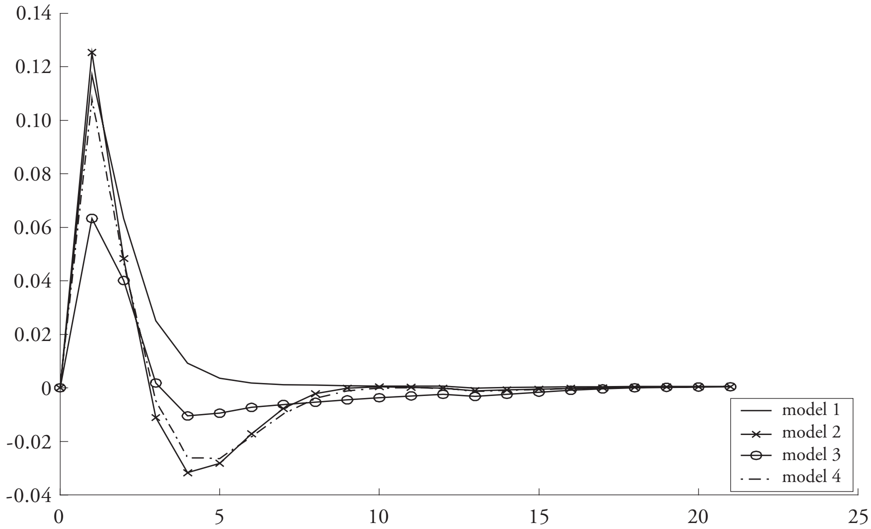


Figure 2: rgap – preference shock

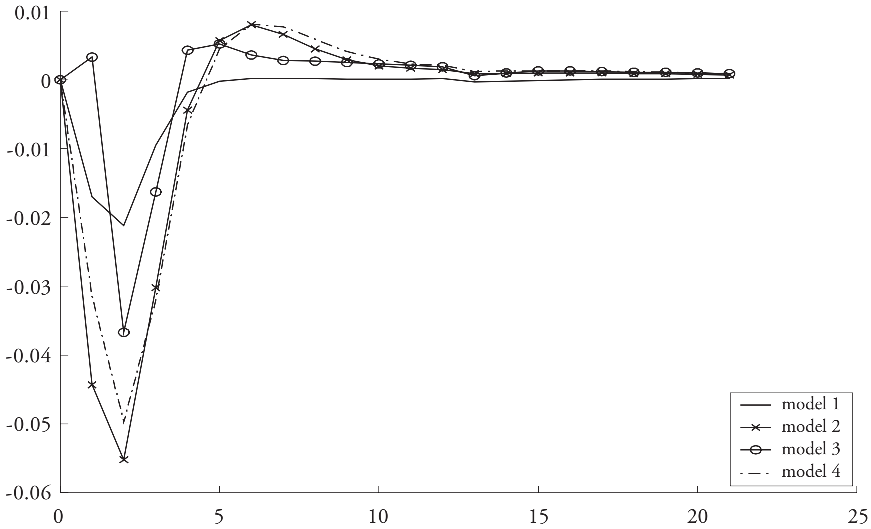


Figure 3: rstar – preference shock

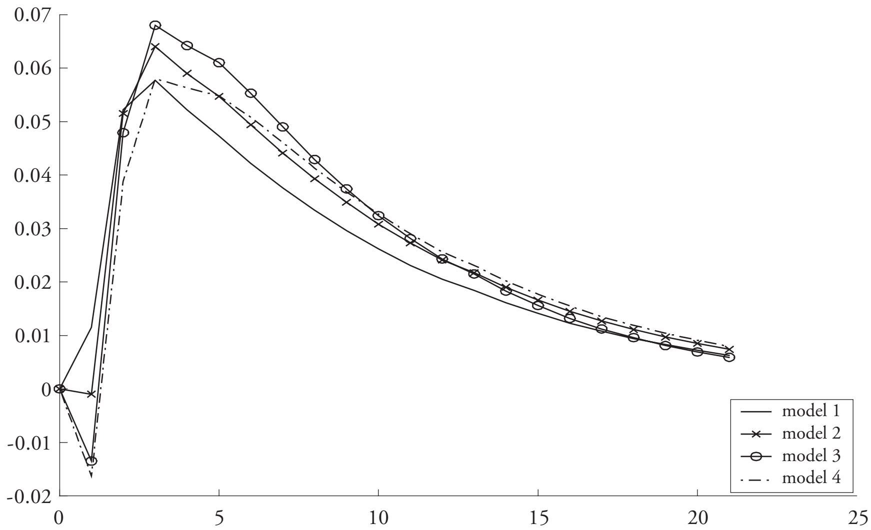


Figure 4: output gap – technology shock

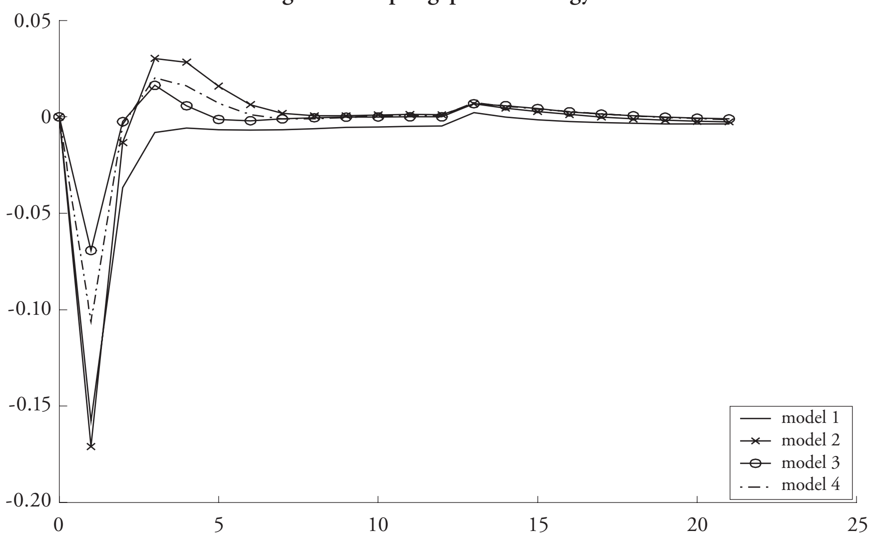


Figure 5: rgap – technology shock

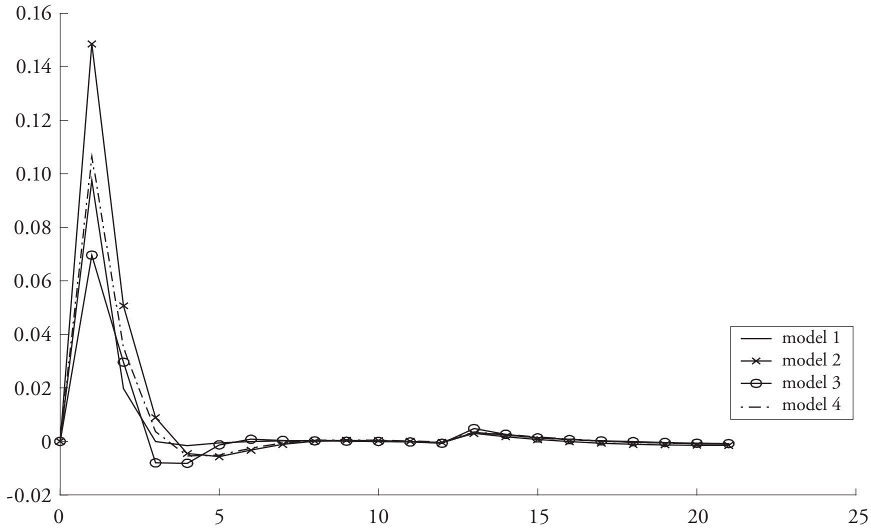
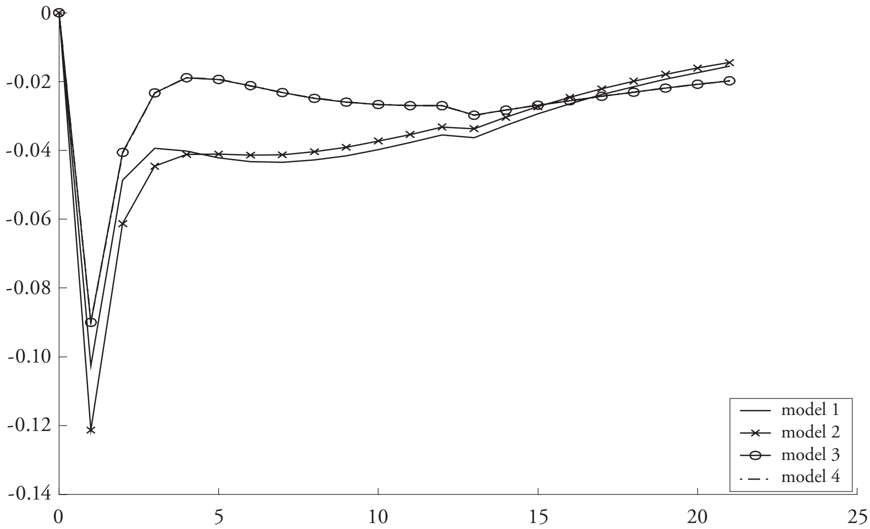


Figure 6: rstar – technology shock



A positive technology shock leads to a rise in investment and this in turn puts upward pressure on the equilibrium real interest rate. However, since it is costly to adjust capital in this model, investment does not increase by as much as it would if capital were allowed to adjust freely. As a result, the increase in the equilibrium rate is muted.

The positive technology shock also increases income and hence consumption. However, since households smooth consumption in the model, they shift resources from current to future periods and do not consume all of their higher income immediately. In other words, they increase their savings. As a result, this puts downward pressure on the equilibrium real rate. In the model, since the fall in the equilibrium rate due to higher savings more than offsets the increase due to higher investment demand, the equilibrium real rate falls following a technology shock. This is shown in Figure 6.

Before presenting our empirical estimates of the equilibrium real rate, we show in Table 3 some of the dynamic properties of the model, in particular the relationship between the real interest rate and output gap and inflation. Table 3 presents some basic correlations between the real interest rate and output gap and inflation. In all four models, it is seen that the real interest rate gap has a strong negative relationship with inflation. Moreover, it is seen also that the correlation between the observed real interest rate and the interest rate gap is very high in all four models, indicating that the variations in the interest rate gap can be mostly explained by changes in the observed real rate of interest.

5.2. Empirical Estimate of the Equilibrium Real Rate

We have already shown that the equilibrium real rate in our model can be expressed as a linear function of the technology and preference shocks. Using data on preference and technology shocks and equation (30), we construct an empirical series for the equilibrium rate. While technology shocks are typically obtained from Solow residuals, preference shocks on the other hand are much harder to measure. However, using the Euler equations of the model, we can derive an empirical series for this shock.[14] Our methodology is explained below.

In most models, technology shocks are measured by Solow residuals which are usually obtained by subtracting the log values of labour and capital inputs,

14 We are aware that identifying preference shock using first-order conditions are by no means perfect and has been criticised by many. In this paper, we make the strong assumption that the term ξ_t captures preference shock.

Table 3: Basic Correlations using Artificial Data

Variables	Model 1	Model 2	Model 3	Model 4
$Corr(\pi_t, rgap_t)$	-0.985	-0.906	-0.546	-0.906
$Corr(\pi_t, rgap_{t-1})$	-0.322	-0.724	-0.148	-0.731
$Corr(\pi_t, rgap_{t-2})$	-0.087	-0.379	-0.030	-0.393
$Corr(\pi_t, rgap_{t-3})$	-0.030	-0.119	0.014	-0.139
$Corr(\pi_t, ygap_t)$	0.989	0.580	0.471	0.560
$Corr(\pi_t, ygap_{t-1})$	0.327	0.590	0.216	0.584
$Corr(\pi_t, ygap_{t-2})$	0.090	0.361	0.049	0.369
$Corr(\pi_t, ygap_{t-3})$	0.031	0.145	0.045	0.171
$Corr(rgap_t, ygap_t)$	-0.989	-0.861	-0.894	-0.842
$Corr(r_t^*, r_t)$	0.240	0.139	0.305	0.144
$Corr(r_t^*, rgap_t)$	0.955	0.979	0.938	0.983
$Corr(y_t^*, y_t)$	0.776	0.804	0.986	0.869
$Corr(y_t^*, ygap_t)$	0.615	0.580	0.159	0.482
$Corr(r_t, y_t)$	-0.638	-0.513	-0.165	-0.429

weighted accordingly, from the log of total output. While we have estimated and derived Solow residuals using productions function, we however use the Solow residuals series from the Bank of Canada's Quaterly Projection Model (QPM).[15] In fact, we find that our results are very robust and do not depend on the methodology used to derive the Solow residuals.

To construct our preference shocks, we proceed as follows: Using the log-linearised equations for consumption (equation A1), bonds (equation A4) and equation (23), we have the following equation which relates preference shocks to consumption, prices and interest rates (more details are contained in Appendix II):

$$\xi_t = \frac{1}{v_6}\left[v_2 \Delta c_t + v_3 E_t \Delta c_{t+2} - v_4 E_t \Delta c_{t+1} - v_5 R_t + v_5 E_t \pi_{t+1}\right] \qquad (32)$$

15 As our model does not allow for sectoral growth, the series for the Solow residuals in our model corresponds to detrended total factor productivity.

In the above equation, c_t is the log of consumption excluding durables, π_t is annual core inflation, R_t is the 90-day commercial paper rate and ξ_t is a preference shock at time t. To generate expected values for consumption, we use a 3-variable VAR estimated over the period 1980q1 to 2001q4.[16] Using our calibrated values, the coefficients estimated in equation (30), our series on Solow residuals and our preference shocks series which we constructed, we can obtain an empirical series for the equilibrium real rate. Our series spans from 1984q1 to 2001q4.

Using Model 4, the equilibrium real interest rate is given by

$$\begin{aligned} r_t^* = & -0.0163\xi_t + 0.053\xi_{t-1} + 0.024\xi_{t-2} + 0.011\xi_{t-3} + 0.006\xi_{t-4} \\ & + 0.003\xi_{t-5} + 0.002\xi_{t-6} + 0.001\xi_{t-7} + 0.001\xi_{t-8} - 0.09a_t \\ & + 0.045a_{t-1} + 0.0153a_{t-2} + 0.003a_{t-3} - 0.001a_{t-4} - 0.003a_{t-5} \\ & - 0.003a_{t-6} - 0.002a_{t-7} - 0.002a_{t-8} - 0.002a_{t-9} - 0.001a_{t-10} \end{aligned} \quad (33)$$

The next section evaluates the quality of our real interest rate gap series, in particular, it evaluates the forecasting ability of this series.

6. Estimates of Equilibrium Rates

In Figure 7 we plot four different measures of policy stance, each defined as the difference between an estimate of the (time-varying) real equilibrium rate and the real 90-day Commercial Paper rate. In addition, we plot the familiar yield spread, measured as the difference between long- term (10-year and over) interest rate and the 90-day rate. As discussed, the leading indicator properties of the yield spread has been thought to originate from the fact that, through the expectations hypothesis, the long-term rate can be viewed as a proxy for the equilibrium short-term rate, and so the yield spread could be viewed as a measure of policy stance.

From the graph we notice that our four model-based measures of policy stance move in a similar fashion over time, differing only on the degree of policy stimulus in effect. With zero representing, by definition, a neutral policy stance, we note that according to our model estimates, policy has been tight for the majority of time since 1984. This is further evidenced by the fact that the average measures of policy stance (Table 4) have been negative since 1984. Estimates

16 The VAR contains ten-lags of the following variables, $\Delta c_t, \pi_t, \Delta R_t$, with a dummy at 1992q1 to capture changes in the monetary policy regime.

Figure 7: Yield Spread and Policy Stance Measures 1984q1 to 2002q1

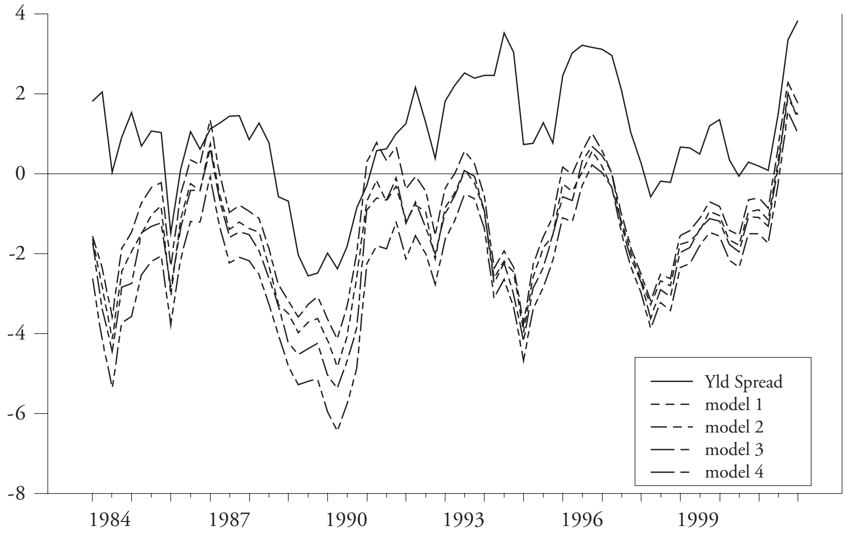


Table 4: Basic Statistics, 1984q1 to 2002q1

Variable	Mean	Std Dev	Minimum	Maximum	Latest Value (2002q1)
Real 90-day rate (R90)	4.71	2.16	0.11	10.36	0.20
Real 10-year rate (R10)	5.64	1.43	3.17	9.03	4.01
R_1^*	3.16	1.34	1.00	6.51	1.70
R_2^*	3.61	1.52	1.18	7.49	1.98
R_3^*	2.95	1.24	0.90	6.29	1.59
R_4^*	2.31	0.97	0.70	4.95	1.24
$R10-R90$	0.93	1.47	-2.55	3.82	3.81
R_1^*-R90	-1.55	1.39	-4.82	1.88	1.50
R_2^*-R90	-1.10	1.44	-4.13	2.29	1.79
R_3^*-R90	-1.76	1.57	-5.37	2.02	1.39
R_4^*-R90	-2.39	1.64	-6.44	1.56	1.05
Annual output growth ($\dot{Y}$)	2.95	2.14	-3.47	6.34	0.30
Annual inflation (π)	2.66	1.15	1.06	5.15	1.99

Note: Sample for output growth ends in 2001q4.

Table 5: In-Sample Estimates, Output Growth, 1985q1 to 2001q4

Explanatory Variables (lagged 4 quarters)	Estimated Parameters (t-stat)	R^2	Breusch- Godfrey Serial Correlation	WHITE (1980) Heterosked asticity	ANDREWS (1993) Parameter Stability	RAMSEY (1969) Non- linearity RESET (2, 3 and 4)
Constant	2.09 (4.30)	0.304	44.8*	1.06	8.27	10.5*
$R10-R90$	0.81 (3.17)				(95q1)	5.26*
						3.53*
Constant	4.13 (8.01)	0.232	47.9*	0.56	9.45	3.07
R_1^*-R90	0.79 (2.41)				(98q3)	5.75*
						3.84*
Constant	3.59 (7.76)	0.165	51.0*	0.46	9.15	4.20*
R_2^*-R90	0.65 (1.88)				(98q2)	4.74*
						3.13*
Constant	4.23 (8.38)	0.277	47.5*	0.49	9.53	5.10*
R_3^*-R90	0.76 (2.72)				(98q3)	9.21*
						6.04*
Constant	4.85 (8.59)	0.339	44.8*	0.44	8.56	3.98
R_4^*-R90	0.80 (3.27)				(98q3)	6.69*
						4.39*
Constant	2.53 (5.39)	0.392	39.8*	0.17	3.00	0.21
$R10-R90$	1.06 (5.25)				(95q4)	0.42
$(R10-R90)^2$	-0.23 (-2.64)					1.29
Constant	3.69 (6.04)	0.255	45.3	4.08*	5.61	7.23*
R_1^*-R90	-0.02 (-0.03)				(98q3)	4.44*
$(R_1^*-R90)^2$	-0.21 (-1.36)					2.92*
Constant	3.56 (7.53)	0.203	47.3*	2.50	6.06	4.89*
R_2^*-R90	-0.04 (-0.07)				(98q3)	2.42
$(R_2^*-R90)^2$	-0.25 (-1.47)					1.59
Constant	3.71 (6.31)	0.319	45.5*	6.11*	5.66	11.68*
R_3^*-R90	-0.11 (-0.21)				(88q4)	6.52*
$(R_3^*-R90)^2$	-0.193 (-1.74)					4.34*
Constant	3.94 (6.39)	0.368	43.5*	4.39*	7.26	8.70*
R_4^*-R90	-0.04 (-0.09)				(88q4)	4.95*
$(R_4^*-R90)^2$	-0.14 (-1.68)					3.75*

from our model reveal that policy rate in 2002Q4 is between 104 (Model 4) and 179 (Model 2) basis points below neutral, which is estimated to be 1.24 (Model 4) and 1.98 (Model 2) per cent, in real terms. Assuming that the equilibrium rates remain constant, our model indicates that short-term real interest rates in 2002Q4 was between 100 and 175 basis points below the estimated neutral value.

6.1. Output Indicator Models

A key indicator of success of any measure of policy stance is an ability to lead output growth and inflation. To this end, we estimate simple indicator models of the year-over-year growth rates of both real output and the core price level. In Table 5 we present in-sample estimates of output growth models, using as only regressors the yield spread and our four stance measures. The yield spread has long been shown to be a good indicator of output growth at this horizon, and so provides a strong benchmark against which other measures of policy stance are to be measured. Based purely on in-sample fit, Model 4 is the only model-based stance measure capable of improving upon the fit of the yield spread.

Examining the regression diagnostics, we find that our use of overlapping data results in serial correlation problems, and as such we adjust our estimated standard errors using the Newey-West variance-covariance matrix to account for this issue. Parameter instability, however, does not appear to be an issue with these models, as all estimated constant and slope parameters appear to be stable over the sample. Our final diagnostic, the RESET test to uncover possible nonlinearities in the residuals, reveals that there are indeed significant nonlinearities present in these simple models. To partly account for such nonlinear effects, we augment each model with the square of each stance measure. The effect of adding such a variable results in a notable jump in the fit of the yield spread model, with the adjusted-R^2 climbing from 0.30 to 0.39. The positive value of 1.06 on the spread indicates that the relationship between the lagged spread and output growth is positive, but the negative value on the parameter of the squared term indicates that the relationship is concave. Performing the RESET test on this augmented model reveals that the residuals have been cleansed of all nonlinearities, indicating that a simple quadratic specification is a simple way of explaining the relationship between the yield spread and output growth within a parametric framework. This finding of significant nonlinearities support the nonparametric neural network models estimated in Tkacz (2001), but within a more tractable parametric model.

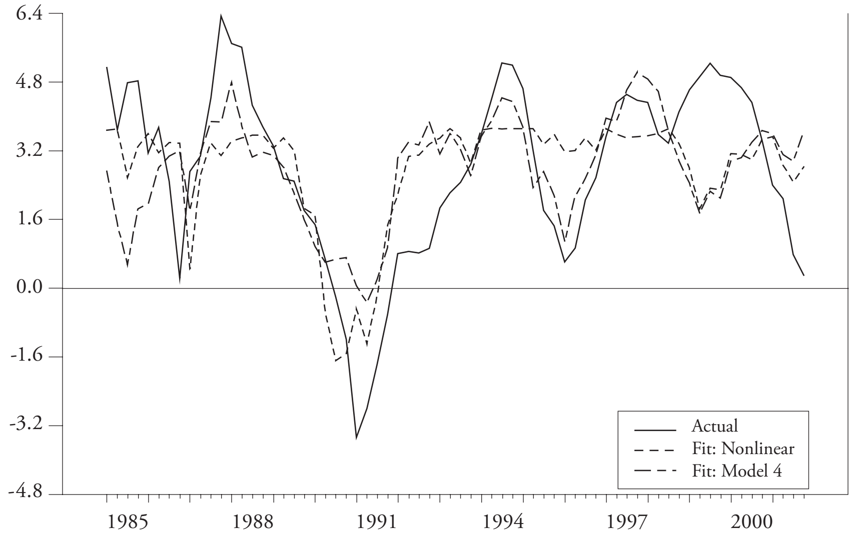
Table 6: Out-of-Sample Forecasts, Output Growth, 1993q4 to 2001q4

Explanatory Variables (lagged 4 quarters)	Root Mean Squared Error	Mean Absolute Error	Confusion Rate
Constant $R10-R90$	1.91	1.43	0.41
Constant R_1^*-R90	1.67	1.24	0.34
Constant R_2^*-R90	1.77	1.31	0.34
Constant R_3^*-R90	1.62	1.19	0.38
Constant R_4^*-R90	1.53	1.18	0.38
Constant $R10-R90$ $(R10-R90)^2$	1.74	1.44	0.69
Constant R_1^*-R90 $(R_1^*-R90)^2$	1.68	1.32	0.50
Constant R_2^*-R90 $(R_2^*-R90)^2$	1.80	1.43	0.56
Constant R_3^*-R90 $(R_3^*-R90)^2$	1.65	1.29	0.56
Constant R_4^*-R90 $(R_4^*-R90)^2$	1.53	1.19	0.44

The trick of adding squared values of the stance measures to the other models, however, does not prove to be effective in improving the modeled relationships, as the in-sample fits do not improve as much, and the models continue to fail the RESET tests, indicating that a more complex method must be used to capture the nonlinearities.

It is well-known that strong in-sample fit does not always lead to strong forecasting performance. For this reason, we also perform an out-of-sample forecasting exercise in which we estimate each model using data from 1985q1 to 1992q4, produce an output growth forecast for 1993q4, and then re-estimate each model using an additional data point and produce another forecast four quarters ahead.

Figure 8: Actual and Fitted Output, 1985q1 to 2001q4



This procedure is repeated until a forecast is obtained for 2001q4. Our forecasting sample is chosen to be sufficiently long so as to draw meaningful conclusions regarding relative forecast performance.

The results in Table 6 reveal that the model-based stance measures outperform the simple yield spread in forecasting output growth, even when the yield spread is augmented with a quadratic term. The best forecasting model is Model 4, which produces a root mean squared error of 1.53. In Figure 9 we find that both the yield spread and Model 4 forecasted output growth very well until 1998, but that the forecast performance has since deteriorated, indicating that non-policy-related factors (e.g. productivity growth) have been superior indicators of output growth.

6.2. Inflation Indicator Models

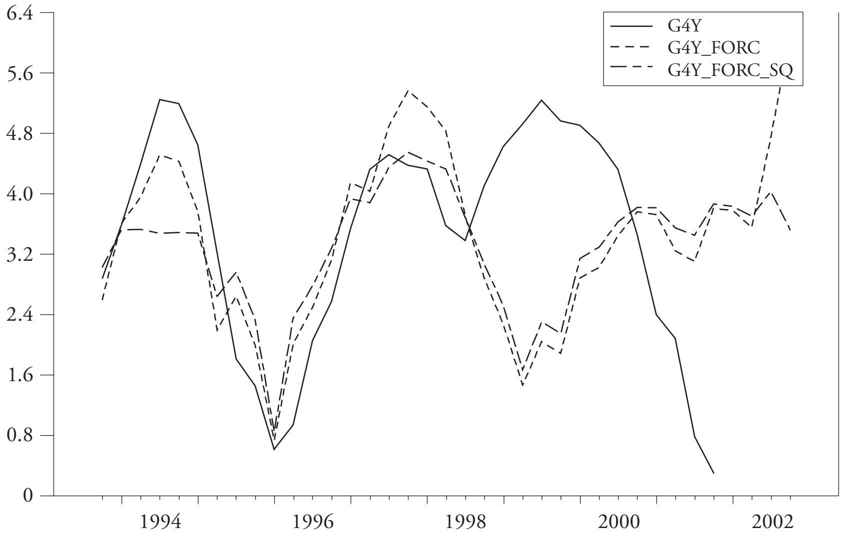
Given that the Bank of Canada has a formal inflation target range of 1 to 3 per cent, another useful feature of a measure of stance is an ability to predict the rate of inflation. In this section we compare the relative merits of the different stance measures of explaining the year-over-year growth in the core price level.

In Table 7 we present the in-sample estimates of the inflation models. The initial results indicate that the stance measures and yield spread are poor indicators

Table 7: In-Sample Estimates, Inflation, 1985q1 to 2001q4

Explanatory Variables (lagged 4 quarters)	Estimated Parameters (t-stat)	R^2	Breusch- Godfrey Serial Correlation	WHITE (1980) Heterosked asticity	ANDREWS (1993) Parameter Stability	RAMSEY (1969) Non- linearity RESET (2, 3 and 4)
Constant	2.77 (9.89)	0.053	59.0*	0.27	169.2*	0.27
$R10-R90$	-0.21 (-2.56)				(91q3)	1.09
						0.73
Constant	2.39 (6.17)	0.004	60.2*	0.06	222.1*	0.06
R_1^*-R90	-0.12 (-0.92)				(91q3)	0.05
						0.04
Constant	2.51 (7.24)	-0.008	60.4*	0.00	229.0*	0.00
R_2^*-R90	-0.07 (-0.49)				(91q3)	0.11
						0.19
Constant	2.35 (6.19)	0.013	60.2*	0.06	229.2*	0.06
R_3^*-R90	-0.13 (-1.18)				(91q3)	0.24
						0.17
Constant	2.14 (5.27)	0.043	59.7*	0.25	223.9*	0.25
R_4^*-R90	-0.18 (-1.86)				(91q3)	0.13
						0.10
Constant	3.91 (22.4)	0.839	29.4*	1.42	6.66	2.88
$R10-R90$	0.07 (1.28)				(89q3)	7.18*
Dummy	-2.22 (-10.5)					5.45*
Constant	4.16 (22.4)	0.847	28.3*	6.47*	5.50	10.9*
R_1^*-R90	0.11 (2.26)				(91q3)	7.41*
Dummy	-2.23 (-11.57)					5.37*
Constant	4.11 (24.2)	0.850	28.2*	6.85*	5.61	10.3*
R_2^*-R90	0.12 (2.47)				(91q3)	6.66*
Dummy	-2.22 (-11.8)					5.49*
Constant	4.16 (21.0)	0.846	28.3*	8.43*	6.60	14.3*
R_3^*-R90	0.09 (1.99)				(91q3)	9.85*
Dummy	-2.23 (-11.3)					7.33*
Constant	4.19 (18.5)	0.843	28.9*	9.58*	7.22	17.4*
R_4^*-R90	0.08 (1.69)				(91q3)	11.5*
Dummy	-2.24 (-11.0)					8.38*

Figure 9: Actual and Forecasted Output (Model 4), 1993q4 to 2002q4



of future inflation, as the adjusted R^2's are very low, and the slope parameters are insignificant and/or have the wrong sign.

Upon examination of the diagnostics we find, however, that a severe break occurred in 1991q3 for all models. That period is near the beginning of formal inflation targets (which were announced in February 1991) and a drop in overall inflation resulting from the 1990–91 recession. When re-estimating the models with a dummy variable to capture the break in the relationship, we find that the model fits improve dramatically, and the slope parameters are of the correct sign and in some cases significant. Furthermore, all parameters of the dummy-augmented models are found to be stable. It is also worth noting that the inflation rates that emerge in these models when policy is neutral (i.e. when the model-based stance variables equal zero, which occurs only when the policy rate equals the equilibrium rate) are consistent with the Bank of Canada's inflation target. For Models 1 through 4 the steady-state inflation rates are found to be in the range of 1.89 (Model 2) to 1.95 (Model 4) per cent.

The effects of the structural break in the inflation models are most apparent in Figures 10 and 11. The structural break is clearly discernible in Figure 10, while the requirement for a dummy variable in the forecasting model is demonstrated in Figure 11.

Figure 10: Actual and fitted Inflation, 1985q1 to 2001q4

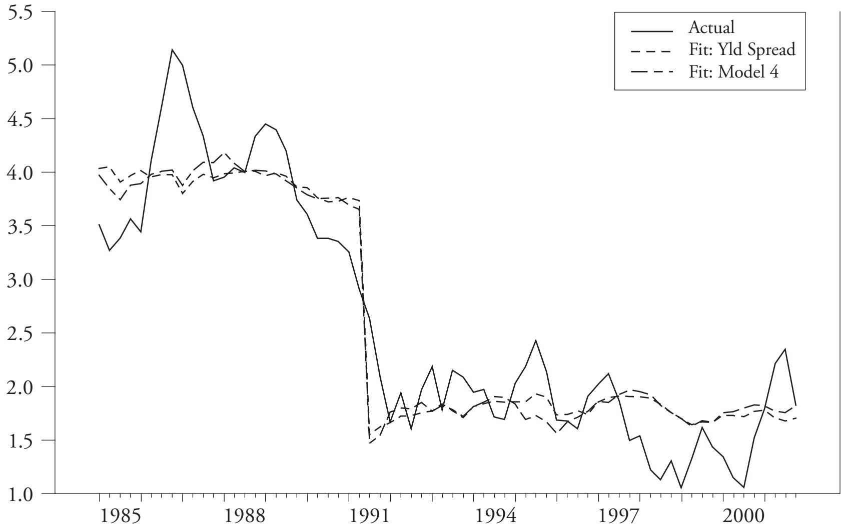


Figure 11: Actual and Forecasted Inflation (Model 4), 1993q4 to 2002q4

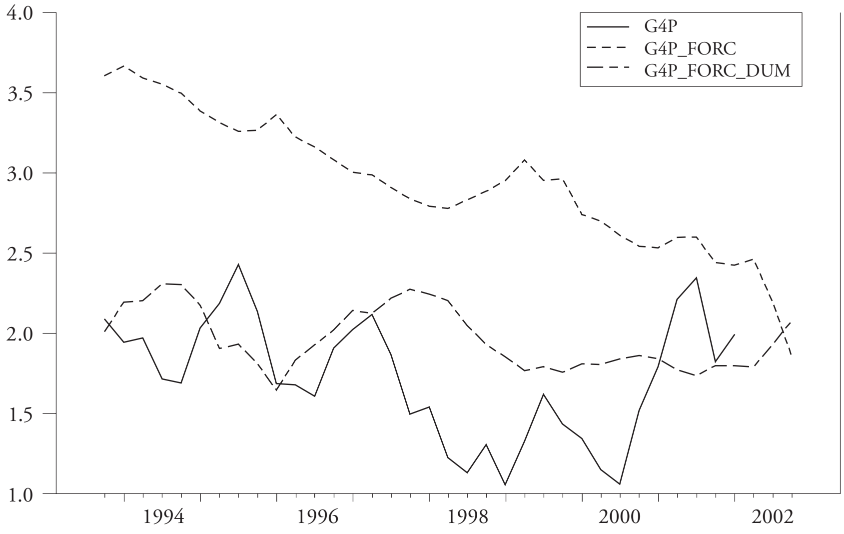


Table 8: Out-of-Sample Forecasts, Inflation, 1993q4 to 2001q4

Explanatory Variables (lagged 4 quarters)	Root Mean Squared Error	Mean Absolute Error	Confusion Rate
Constant $R10-R90$	1.36	1.28	0.47
Constant R_1^*-R90	1.38	1.32	0.53
Constant R_2^*-R90	1.40	1.34	0.53
Constant R_3^*-R90	1.38	1.31	0.50
Constant R_4^*-R90	1.36	1.29	0.53
Constant $R10-R90$ *Dummy*	0.44	0.36	0.53
Constant R_1^*-R90 *Dummy*	0.45	0.37	0.53
Constant R_2^*-R90 *Dummy*	0.44	0.36	0.56
Constant R_3^*-R90 *Dummy*	1.65	1.29	0.56
Constant R_4^*-R90 *Dummy*	1.53	1.19	0.44

7. Conclusion

The objective of this paper is to provide an estimate of the equilibrium interest rate that relies on a DSGE equilibrium model. Using a dynamic general equilibrium model with nominal rigidities, we define the equilibrium rate as the rate that would prevail each period were the rigidities removed; that is, the rate that would prevail were all markets in equilibrium. To verify the robustness of our findings, we estimate four different equilibrium rates under different calibration assumptions.

Our findings reveal that the derived stance measures are fairly robust to the calibration assumptions, in the sense that all stance measures move in similar patterns over time. Comparing the model-based stance measures to the yield spread, we detect one stance measure in particular (Model 4) that is as good as or better than the yield spread in explaining future output and inflation.

Appendix A: Log-linearised conditions

The log-linear model is given by the equations below. The lower case letters are the log deviations from their steady-state values.

Consumption

$$\left[\frac{1-\beta h\rho_{\xi}}{1-\beta h}\right]\xi_t-\left[\frac{1}{\sigma(1-\beta h)}\right]$$
$$[h(\sigma-1)c_{t-1}-\left(\beta h(\sigma-1)\right)E_t c_{t+1}+\left(\beta h^2(\sigma-1)+\beta h\sigma-1\right)c_t]-\lambda_t=0 \quad \text{(A1)}$$

Employment

$$y_t-n_t=\mu_t-\lambda_t \quad \text{(A2)}$$

Money Demand

$$-\frac{1}{\gamma_m}-\tilde{m}_t-\left[\frac{1}{\gamma_m}\frac{R_t}{R^{ss}}\right]=0 \quad \text{(A3)}$$

Euler equation

$$E_t\lambda_{t+1} - \lambda_t - r_t = 0 \tag{A4}$$

Fisher equation

$$r_t = R_t - \mathrm{E}_t\left(p_{t+1} - p_t\right) \tag{A5}$$

Production function

$$y_t = a_t + \alpha n_t + (1-\alpha)k_t \tag{A6}$$

Resource constraint

$$y_t = \frac{\bar{C}}{\bar{Y}}c_t + \left(\frac{\bar{I} + \psi\eta\bar{I}^{\eta}}{Y}\right)i_t \tag{A7}$$

Capital law of motion

$$k_{t+1} = i_t + (1-\delta)k_t \tag{A8}$$

Calvo pricing

$$\pi_t = \beta E_t(\pi_{t+1}) + \varphi mc_t \tag{A9}$$

Capital

$$\begin{aligned} i_t = {} & (1-\delta)E_t(I_{t+1}) \\ & + E_t\left[\left(\frac{1}{(\eta-1)\psi\eta\bar{I}^{\eta-1}}\right)\left(\left(\frac{(1-\alpha)\bar{Y}}{\bar{K}}\right)\left(y_{t+1} - k_{t+1} - mc_{t+1}\right) + r_t\right)\right] \end{aligned} \tag{A10}$$

Appendix B: Derivation of preference shocks

From (A1), we have

$$v_1\xi_t = v_2 c_{t-1} + v_3 E_t c_{t+1} - v_4 c_t + v_5 \lambda_t \quad \text{(B1)}$$

where

$$v_1 = \sigma(1-\beta h\rho_\xi), v_2 = h(\sigma-1), v_3 = \beta h(\sigma-1), v_4 = \beta h^2(\sigma-1) + \beta h\sigma - 1$$

and

$$v_5 = \sigma\left(1-\beta h\right)$$

Writing the above equation forward one period and taking expectations at time *t*, we have

$$v_1 E_t \xi_{t+1} = v_2 c_t + v_3 E_t c_{t+2} - v_4 E_t c_{t+1} + v_5 E_t \lambda_{t+1} \quad \text{(B2)}$$

Subtracting (B1) from (B2), and using equations (23) and (28) in the text, one obtains

$$v_1\left(\rho_\xi - 1\right)\xi_t = v_2 \Delta c_t + v_3 E_t \Delta c_{t+2} - v_4 E_t \Delta c_t + v_5\left(E_t \pi_{t+1} - R_t\right) \quad \text{(B3)}$$

which is equation (32) in the text with $v_6 = v_1\left(\rho_\xi - 1\right)$.

References

Abel, A.B. (1983), "Optimal Investment under Uncertainty." *American Economic Review*, 73, 228–233.

Andrews, D. W. K. (1993), "Tests for Parameter Instability and Structural Change with Unknown Change Point." *Econometrica*, 61, 821–856.

Blinder, A. S. (1998), *Central Banking in Theory and Practice*. Cambridge: MIT Press.

Bomfim, A. N. (1997), "The Equilibrium Fed Funds Rate and the Indicator Properties of Term-Structure Spreads." *Economic Inquiry*, 35, 830–846.

Bomfim, A. N. (2001), "Measuring Equilibrium Real Interest Rates: What Can We Learn from Yields on Indexed Bonds?" Federal Reserve Board, *Finance and Economics Discussion Series*, No 53.

BRITTAN, S. (2001), "Alternatives to inflation targets." *Financial Times*, March 1.

CALVO, G. A. (1983), "Staggered Contracts in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12, 383–398.

CASARES, M. and B. MCCALLUM (2000), "An Optimising IS-LM framework with endogenous investment." *NBER Working Paper Series*, No 7908.

CHRISTIANO, L., M. EICHENBAUM and C. EVANS (2001), "Nominal Price Rigidities and the Dynamic Effects of a Shock to Monetary Policy." Working Paper, Northwestern University.

CLARIDA, R., J. GALI and M. GERTLER (1998), "Monetary Policy Rules in Practice: Some International Evidence." *European Economic Review*, 42 (6), 1033–1067.

CLARIDA, R., J. GALI and M. GERTLER (1999), "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, 37, 1661–1707.

CÔTÉ, D., J. KUSZCZAK, J.-P. LAM, Y. LIU and P. ST-AMANT (2004), "The Performance and Robustness of Simple Monetary Policy Rules in Models of the Canadian Economy", forthcoming *Canadian Journal of Economics.*.

DIB, A. (2002), "Nominal Rigidities and Monetary Policy Rule in Canada Since 1980." mimeo, Bank of Canada.

DUGUAY, P. (1994), "Empirical Evidence on the Strength of the Monetary Transmission Mechanism in Canada: An Aggregate Approach." *Journal of Monetary Economics*, 33(1), 39–61.

FUHRER, J. C. and G. R. MOORE (1995), "Inflation Persistence." *Quarterly Journal of Economics*, 109, 127–159.

FUHRER, J. (2000), "Optimal Monetary Policy in a Model with Habit Formation." *Federal Reserve Bank of Boston Working Paper*, 00-5.

GAGNON, E. and H. KHAN (2001), "New Phillips Curve with Alternative Marginal Cost Measures for Canada, the United States, and the Euro Area." *Bank of Canada Working Paper* 2001-25.

GALI, J. and M. GERTLER (1999), "Inflation Dynamics: A structural Econometric Analysis." *Journal of Monetary Economics*, 44, 1237–70.

GERLACH, S. and F. SMETS (2002), "MCIs and Monetary Policy in Small Open Economies under Floating Exchange Rates." *European Economic Review*, Forthcoming.

IRELAND, P. N. (1997), "A Small, Structural, Quarterly Model for Monetary Policy Evaluation." *Carnegie-Rochester Conference Series on Public Policy*, 47(0), 83–108.

JOHNSON, M. H. and R. E. KELEHER (1996), *Monetary Policy, A Market Price Approach*. Westport, CT: Quorom Books.

JULIUS, D. (1998), House of Commons (U.K.) Select Committee on Treasury, Minutes of Evidence, Questions 300–319, Thursday June 18.

LAUBACH, T. and J. C. WILLIAMS (2001), "Measuring the Natural Rate of Interest," Federal Reserve Board, *Finance and Economics Discussion Series*, No 56.

MCCALLUM. B. and E. NELSON (1999), "An Optimising IS-LM Specification for Monetary Policy and Business Cycle Analysis." *Journal of Money, Credit and Banking*, 31, 296–316.

MEADE, J. (1933), *The Rate of Interest in a Progressive State.* London: Macmillan.

MEYER, L. (2000), "The Economic Outlook and the Challenges Facing Monetary Policy." Remarks before the Toronto Association for Business and Economics, April 12.

NEISS, K. S. and E. NELSON (2003), "The Real Interest Rate Gap as an Inflation Indicator." *Macroeconomic Dynamics*, 7(2), 239–262.

RAMSEY, J. B. (1969), "Tests for Specification Errors in Classical Linear Least-Squares Analysis." *Journal of the Royal Statistical Society*, 31, 350–371.

ROBERTS, J. (1995), "New Keynesian Economics and the Phillips Curve." *Journal of Money, Credit and Banking*, 27, 975–84.

ROTEMBERG, J. J. "Sticky Prices in the United States." *Journal of Political Economy*, 60, 1187–1211.

THORNTON, H. (1802), *An Enquiry Into the Nature and Effects of the Paper Credit of Great Britain*. Reprinted (1978). Fairfield, NJ: A. M. Kelley.

TKACZ, G. (2001), "Neural Network Forecasting of Canadian GDP Growth." *International Journal of Forecasting*, 17, 57–69.

WHITE, H. (1980), "A Heteroskedasticity-Consistent Covariance Matrix Estimator and Direct Test for Heteroskedasticity." *Econometrica*, 48, 817–838.

WICKSELL, K. (1898), *Interest and Prices: A Study of the Causes Regulating the Value of Money*. Translated by R. F. Kahn (1936), reprinted (1965). New York: A. M. Kelley.

WOODFORD, M. (2000), "A Neo-Wicksellian Framework for Analyzing Monetary Policy." Working Paper, Princeton University.

YUN, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics*, 37, 345–70.

SUMMARY

In an era when the primary policy instrument is the level of the short-term interest rates, a comparison of that rate with some equilibrium rate can be a useful guide for policy and a convenient method to measure the stance of monetary policy. The real interest rate gap, the difference between the real equilibrium rate and the rate set by the central bank, can thus serve as a leading indicator of future inflationary or deflationary pressures in the economy. This paper estimates equilibrium interest rates for Canada using a sticky-price dynamic stochastic general equilibrium (DSGE) model. We follow the methodology of Neiss and Nelson (2003) closely and derive measures of the interest rate gap for Canada. Our results indicate that the interest rate gap can be a useful guide for policy and is a good an indicator of future output and inflation. Moreover, we find that our measures of the interest rate gap perform as well as the yield spread, a typical measure of policy stance that is assumed to contain significant information about future economic activity.

ZUSAMMENFASSUNG

In einer Zeit wo das Hauptinstrument der Geldpolitik der kurzfristige Zinssatz ist, liefert ein Vergleich von diesem Zins mit seinem Gleichgewichtswert Informationen über die geldpolitische Haltung und künftige Inflationstendenzen. Dieser Artikel schätzt für Kanada Gleichgewichtszinsen mittels eines dynamischen stochastischen Gleichgewichtsmodelles mit Preisstarrheiten. Danach berechnen wir mit der Methode von Neiss und Nelson (2003) Masse für die kanadische Zinslücke (definiert als Gleichgewichtszins abzüglich dem tatsächlichem Zins). Unsere Resultate zeigen, dass die Zinslücke ein guter Indikator für den zukünftigen Output und die zukünftige Inflation ist und nützliche Angaben zur Politikgestaltung gibt. Ausserdem finden wir, dass unsere Masse für die Zinslücke genauso gut funktionieren wie die Fristenprämie, welche typischerweise als Indikator der geldpolitischen Haltung genommen wird.

RÉSUMÉ

Comme le niveau d'un taux d'intérêt à court terme est l'outil primaire de la politique monétaire, comparer un tel taux à son niveau d'équilibre pourrait servir de guide utile pour l'autorité monétaire ainsi que de mesure simple pour la direction

de la politique monétaire. Le différentiel entre le taux d'équilibre réel et le taux sous le contrôle de l'autorité monétaire peut donc servir d'indicateur avancé de futurs pressions inflationnistes ou déflationnistes dans l'économie. Dans la présente étude nous estimons quelques taux d'équilibres pour le Canada utilisant un modèle d'équilibre général dynamique et stochastique avec prix rigides. Nous suivons de près l'approche de NEISS et NELSON (2003) afin de dériver nos mesures du taux neutre pour le Canada. Nos résultats démontrent que l'écart entre le taux d'équilibre et le taux monétaire peut être utile pour la politique monétaire et est aussi un bon indicateur avancé de la croissance économique et de l'inflation. De plus, la performance empirique de nos mesures de la direction de la politique monétaire est comparable à celle de la pente de la courbe de rendement, une mesure couramment utilisée pour évaluer la direction de la politique monétaire.